Machine Learning Essentials SS25 - Exercise Sheet 7

Instructions

- `TODO` 's indicate where you need to complete the implementations.
- You may use external resources, but **write your own solutions**.
- Provide concise, but comprehensible comments to explain what your code does.
- Code that's unnecessarily extensive and/or not well commented will not be scored.

In [7]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.sparse import dok_matrix, coo_matrix
from scipy.sparse.linalg import lsqr

DATA_DIR = "hs_tomography"  # Change this to your data directory
```

Task 1

In [8]:
```
size = 77
sensor_size = 109


alphas = np.load(f'./{DATA_DIR}/alphas_{size}.npy')
X_example = np.load(f'./{DATA_DIR}/X_example.npy')


alphas_195 = np.load(f'./{DATA_DIR}/alphas_{195}.npy')
```

In [9]:
```
def construct_X(M, alphas, Np=None):
    # TODO: implement vectorised sparse projection matrix X, according to
    if Np==None:
        Np = int(np.ceil(np.sqrt(2)*M))

    D = M*M


    # C matrix with pixel centers of shape (2, D)

    a, b = np.mgrid[-(M-1)/2:(M-1)/2+1, (M-1)/2:-(M-1)/2-1:-1]
    C = np.vstack([a.ravel(order='F'), b.ravel(order='F')])

    # unit vectors
    alphas = np.deg2rad(alphas)
    unit_vectors = np.array([np.cos(alphas), np.sin(alphas)])
    s0 = (Np-1)/2

    row_indices = []
    col_indices = []
    weights = []
```

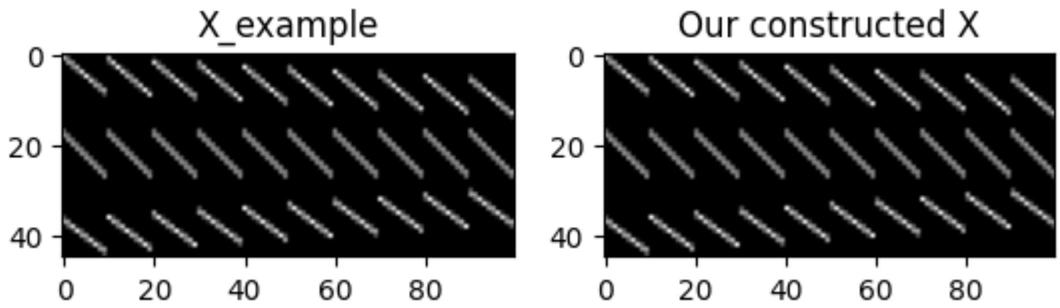
```
    for i, alpha in enumerate(alphas):

        n = unit_vectors[:, i].reshape(2, 1)

        P = (n.T @ C).flatten() + s0

        idx_left = np.floor(P).astype(int)
        idx_right = idx_left + 1

        frac = P - idx_left

        w0 = (1 - frac)
        w1 = frac

        mask0 = (idx_left >= 0) & (idx_left < Np)
        mask1 = (idx_right >= 0) & (idx_right < Np)

        row0 = idx_left[mask0] + i * Np
        row1 = idx_right[mask1] + i * Np

        col0 = np.where(mask0)[0]
        col1 = np.where(mask1)[0]

        row_indices.append(row0)
        row_indices.append(row1)
        col_indices.append(col0)
        col_indices.append(col1)
        weights.append(w0[mask0])
        weights.append(w1[mask1])

    if weights:
        row_indices = np.concatenate(row_indices)
        col_indices = np.concatenate(col_indices)
        weights = np.concatenate(weights)

    else:
        row_indices = np.array([], dtype=int)
        col_indices = np.array([], dtype=int)
        weights = np.array([], dtype=np.float32)

    return coo_matrix((weights, (row_indices, col_indices)), shape=(len(a
```

Quick sanity check

In [28]:
```
# TODO: Check if your image matches `X_example.npy` (Figure 2) up to mirr
X = construct_X(10, np.array([-33, 1, 42])).toarray()
print(f'The images match numerically: {np.allclose(X, X_example, atol=1e-
```

```
The images match numerically: True
```

In [38]:
```
fig, axs = plt.subplots(1,2)
axs[0].imshow(X_example, cmap = 'gray')
axs[0].set_title('X_example')
axs[1].imshow(X, cmap = 'gray')
axs[1].set_title('Our constructed X')
plt.show()
print('The Xs match visually')
```



The Xs match visually

In [11]:
```
X_77 = construct_X(size, alphas, sensor_size)
X_195 = construct_X(195, alphas_195, 275)
```

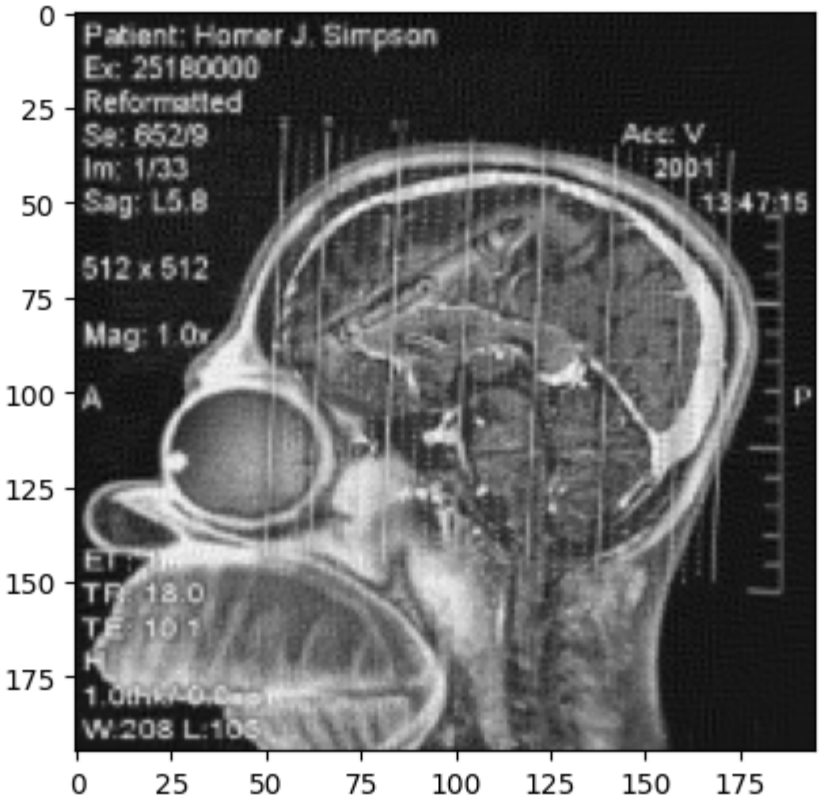
Task 2 – Reconstruct the tomogram

In [ ]:
```
# TODO: Reconstruct the tomogram and plot it as a 2D image. Use scipy.spa
M = 195 # Choose between 77 & 195

X = X_195
No=179
Np=275
print(f'Number of nonzero elements in sparse matrix X for M = {M} is: {X.

# Load response vector
y = np.load(f'./{DATA_DIR}/y_{M}.npy')
```

```
Number of nonzero elements in sparse matrix X for M = 195 is:13535933
Number of cells is 1871780625: orders of magnitude larger
```

In [ ]:
```
# Solve using efficient least squares with sparse matrix
beta = lsqr(X, y)[0]
```

In [53]:
```
# Unflatten to 2d image and display
mu = beta.reshape(M,M)

plt.imshow(mu, cmap = 'gray')
plt.show()
```



Diagnosis

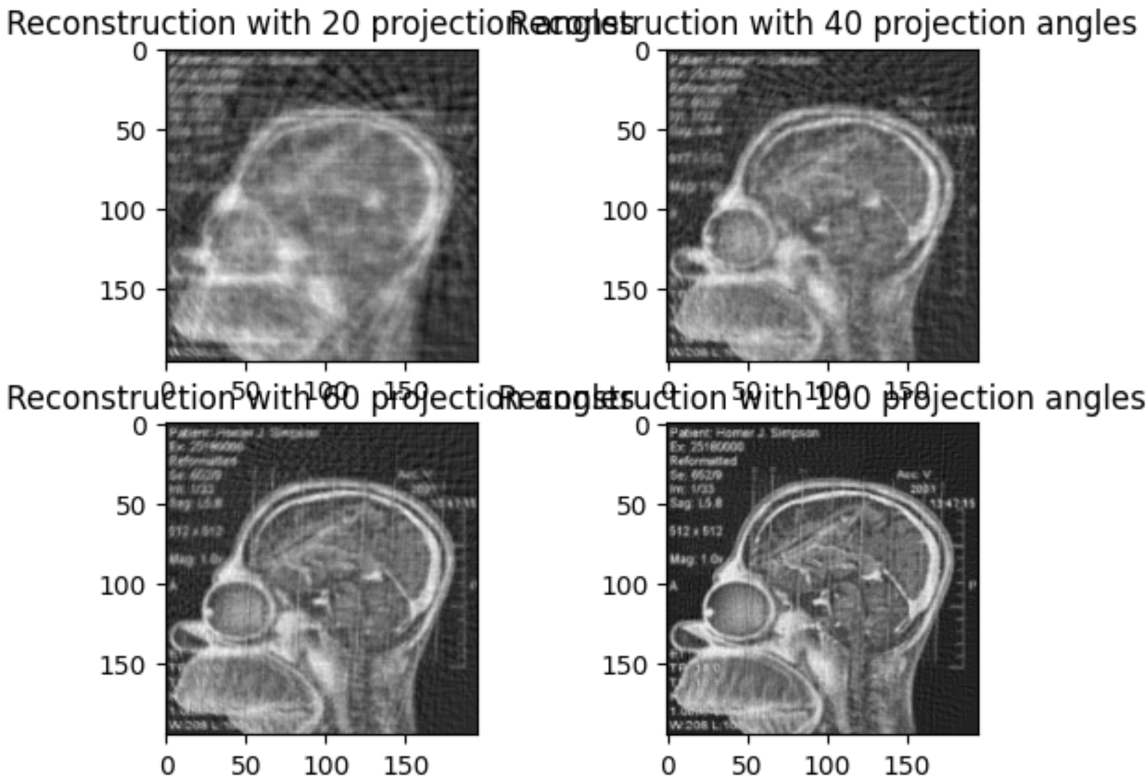
TODO: The issue likely stems from the crayon stuck in Mr Simpson's brain. We would recommend removing it.

Task 3

In [110…

```
def reconstruct_with_subset():
    # TODO: Reconstruct X using only a subset of projection angles.
    M = 195 # Image resolution along one axis

    # Set reduced numbers of angles and set up plotting
    N_subs = [20,40,60,100]
    fig, axs = plt.subplots(int(np.ceil(len(N_subs)/2)),2)
    axs = axs.flatten()

    # Other relevant arrays and values
    alphas = np.load(f'./{DATA_DIR}/alphas_{M}.npy')
    y = np.load(f'./{DATA_DIR}/y_{M}.npy')

    No = len(alphas) # = 179
    Np = 275

    y_sino = y.reshape(No, Np)

    for i, N_sub in enumerate(N_subs):
        # Obtain mask extracting N_sub regularly spaced indices to extrac
        mask = np.linspace(0, No-1, N_sub, dtype = int)

        # Extract subset of projection angles, and reduce sinogram accord
        alphas_sub = alphas[mask]
```

```
        y_sub = y_sino[mask].flatten()

        # Create reconstructed X and solve problem.
        X_sub = construct_X(M, alphas_sub, Np)
        beta = lsqr(X_sub, y_sub)[0]
        mu = beta.reshape(M,M)

        # Plot
        axs[i].imshow(mu, cmap = 'gray')
        axs[i].set_title(f'Reconstruction with {N_sub} projection angles'

    plt.show()


# TODO: Reduce the number of projection angles in a sensible way and visu
reconstruct_with_subset()
```



TODO: state the smallest number of projections that still resolves the pathology clearly enough to give a diagnosis and propose a treatment. 100 projections is certainly enough, at 60 one can just make out the crayon, while it's not clear at 40. The minimum lies between 40 and 60, and depends on the radiologist viewing the image. We would give the number 60.