# Statistics

## Univariate Distributions

### Normal distribution

Suppose $X \sim N(\mu, \sigma^2)$.

- $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$
- $E(X) = \mu$ and $var(X) = \sigma^2$

### $t$ distribution

### $\chi^2$ distribution

Suppose $X \sim \chi^2_k$.

- Support: $x \in (0, \infty)$ if $k = 1$ otherwise $x \in [0, \infty)$
- $E(X) = k$ and $var(X) = 2k$
- $\sum_{i=1}^k Z_i^2 \sim \chi^2_k$ where $Z_i \sim N(0,1)$
- $\sum_{i=1}^k X_i^2 \sim \sigma^2\chi^2_k$ where $X_i \sim N(0, \sigma^2)$
- $\sum_{i=1}^k (X_i - \hat{\mu})^2 \sim \sigma^2\chi^2_{k-1}$ where $X_i \sim N(\mu, \sigma^2)$
- $\frac{1}{m}\sum_{i=1}^k (X_i - \hat{\mu})^2 \sim \frac{\sigma^2}{m}\chi^2_{k-1}$ where $X_i \sim N(\mu, \sigma^2)$

### $F$ distribution

- $\frac{X_1/d_1}{X_2/d_2} \sim F_{d_1,d_2}$ where $X_1 \sim \chi^2_{d_1}$ and $X_2 \sim \chi^2_{d_2}$
- $\frac{S_1^2/d_1}{\sigma_1^2} / \frac{S_2^2/d_2}{\sigma_2^2} \sim F_{d_1,d_2}$
- If $X \sim t_n$ then $X^2 \sim F_{1,n}$ and $X^{-2} \sim F_{n,1}$

## Multivariate Distributions

### Normal Distribution

Suppose $\boldsymbol{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $|\boldsymbol{X}| = k$.

- $f(\boldsymbol{x}) = \frac{1}{\sqrt{(2\pi)^k|\boldsymbol{\Sigma}|}}\exp\left(-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^\top\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right)$

#### Special Cases

$$\begin{bmatrix}\boldsymbol{X}_1\\ \boldsymbol{X}_2\end{bmatrix} \sim N\left(\begin{bmatrix}\boldsymbol{\mu}_1\\ \boldsymbol{\mu}_2\end{bmatrix}, \begin{bmatrix}\boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12}\\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22}\end{bmatrix}\right)$$

- $E(\boldsymbol{X}_2|\boldsymbol{X}_1 = \boldsymbol{x_1}) = \boldsymbol{\mu}_2 + \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}(\boldsymbol{x}_1 - \boldsymbol{\mu}_1)$
- $var(\boldsymbol{X}_2|\boldsymbol{X}_1 = \boldsymbol{x_1}) = \boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}$

## Univariate Tests

## References