Assignment 3

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February 3, 2016

time.lower	time.upper	Fhat	SE_Fhat	Lower	Upper
0	33	0.00	0.00	0.00	0.00
33	46	0.08	0.08	0.02	0.34
46	50	0.17	0.11	0.05	0.42
50	59	0.25	0.12	0.10	0.50
59	62	0.33	0.14	0.15	0.58
62	71	0.42	0.14	0.21	0.65
71	74	0.50	0.14	0.28	0.72
74	75	0.58	0.14	0.35	0.79
75	78	0.67	0.14	0.42	0.85
78	80	0.83	0.11	0.58	0.95

Table 1: Summary of link failure times (in 1000s of cycles) including estimated probabilities with corresponding standard errors and 90% confidence intervals.

3.4 (a) See Table 1.

(b) See Table 1. Each interval is constructed by a method that covers the truth at least 95% of the time over many realizations of data generated by the assumed model.

(c) They are the same since there are no censored observations before the 3rd failure.

(d) The observer may not have been present at all times, so there could have been interval censoring. Another possibility is that the data were simply rounded. This is not likely to have a large impact on the analysis since most failures occured at different times and there are a relatively large number of possible failure times before right censoring occurred.

(e) We might expect less variability in the failure times if only one or two heats were used.

(f) It seems that the state of the test stand may change through the testing process in a way so as to increase or decrease the failure times. If we notice more failures at the beginning or end of the process, this possibility ought to be considered.

C3.24 (a)

$$L(\pi) \propto \pi_1^4 \pi_2^5 \pi_3^2 \pi_4^{95} (\pi_2 + \pi_3 + \pi_4)^{99} (\pi_3 + \pi_4)^{95}$$

(b)

$$L(p) \propto p_1^4(1-p_1)^{296} p_2^5(1-p_2)^{192} p_3^2(1-p_3)^{95}$$

4.3

$$\Phi_{sev}(z) = p \iff 1 - \exp[-\exp(z)] = p \iff z = \log[-\log(1-p)]$$
$$\implies \Phi_{sev}^{-1}(p) = \log[-\log(1-p)]$$

4.7 The first and second derivatives of $h(t;\beta,\eta)$ are:

$$h'(t) = \frac{\beta(\beta-1)}{\eta^\beta} t^{\beta-2}, \quad h''(t) = \frac{\beta(\beta-1)(\beta-2)}{\eta^\beta} t^{\beta-3}$$

(a) If $0 < \beta < 1$, then $1-\beta < 0$, so $h'(t) < 0$ for $t > 0$. So $h(t)$ is decreasing.

(b) If $1 < \beta < 2$, then $\beta - 1 > 0$ but $\beta - 2 < 0$, so $h'(t) > 0$ and $h''(t) < 0$ for all $t > 0$, so $h(t)$ is concave increasing.

(c) If $\beta > 2$, then $h'(t) > 0$ and $h''(t) > 0$, so $h(t)$ is convex increasing.

3.12 (a) We can make inference about the distribution of failure times for these detectors under conditions that are more severe than normal operating conditions. In particular, this data will be somewhat informative about the left tail of the distribution and less informative about the rest. That is because 3/4 of the failure times are right censored. On the other hand, the intervals for the interval censored observations are fairly small, so the seven failures are observed with reasonable precision.

(b) See Figure 1.

(c) See Table 2.

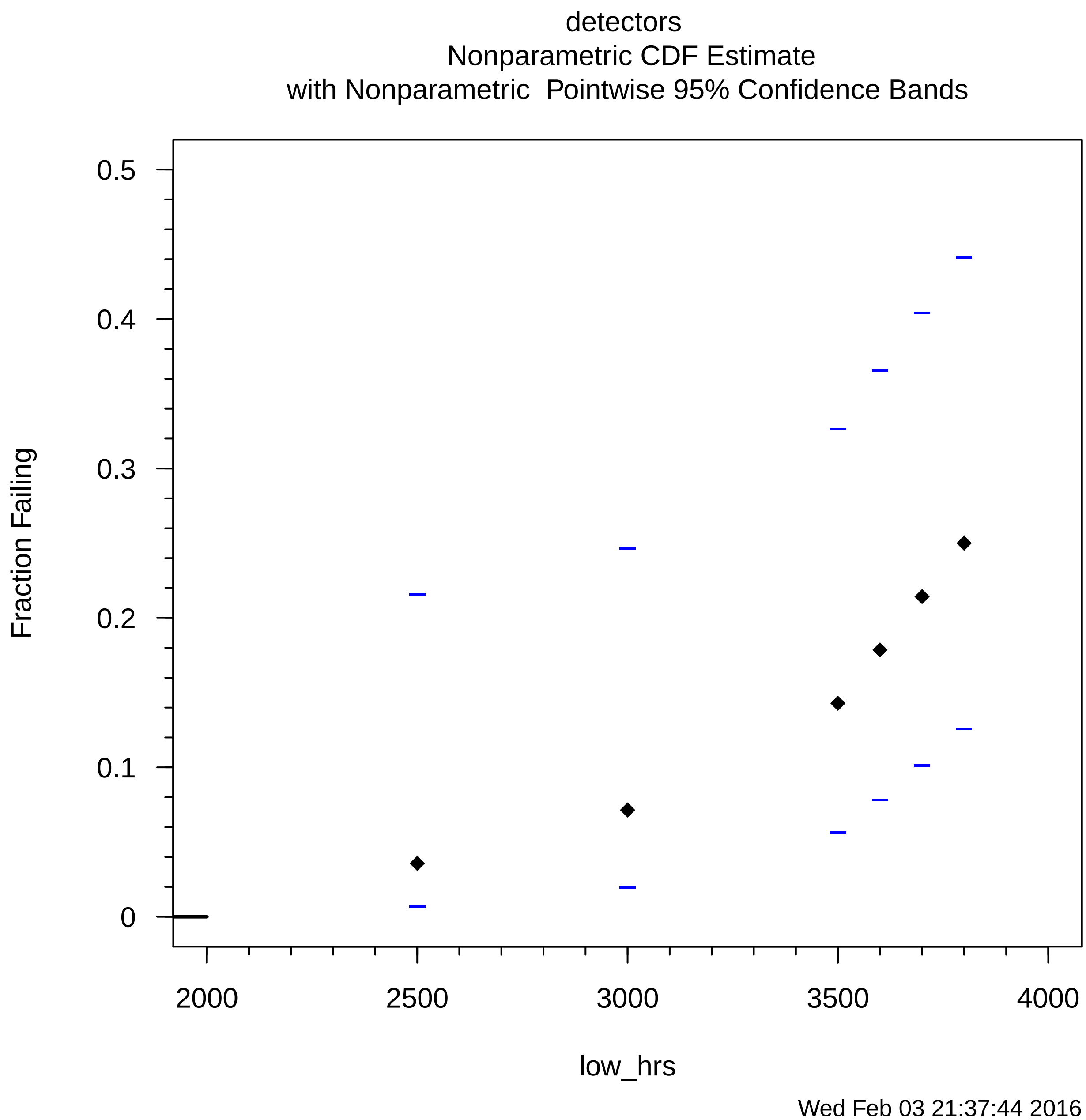
time	Fhat	SE_Fhat	L_pw	U_pw	L_simul	U_simul
2500	0.04	0.04	0.01	0.21	0.00	0.49
3000	0.07	0.05	0.02	0.24	0.01	0.49
3500	0.14	0.07	0.05	0.32	0.03	0.52
3600	0.18	0.07	0.08	0.36	0.04	0.54
3700	0.21	0.08	0.10	0.40	0.05	0.57
3800	0.25	0.08	0.12	0.44	0.07	0.60

Table 2: Summary of detector failure times (in hours) including estimated probabilities, standard errors, pointwise and simultaneous 95% confidence intervals.

(d) See Table 2 and Fig 1.

(e) See Table 2.

(f) The simultaneous intervals are wider to acheive 95% confidence in covering all relevant quantities simultaneously. This could be useful in identifying departures from our model assumptions. If we fit a parametric model to these data, we would reasonably expect that the curve (conditional expectation) should stay within the simultaneous confidence band.


detectors
Nonparametric CDF Estimate
with Nonparametric Pointwise 95% Confidence Bands
Fraction Failing
0.5
0.4
0.3
0.2
0.1
0
2000
2500
3000
3500
4000
low_hrs
Wed Feb 03 21:37:44 2016

4.8 Let $Y = \log(T), T \sim WEIB(\eta, \beta)$.

$$P(Y < y) = P(T < \exp(y))$$

$$= 1 - \exp\left\{-\frac{\exp(\beta y)}{\eta^{\beta}}\right\}$$

$$= 1 - \exp\left\{-\frac{\exp(\beta y)}{\exp(\beta \log \eta)}\right\}$$

$$= 1 - \exp\{-\exp(\beta(y - \log \eta)\}$$

$$= \Phi_{SEV}(\log(\eta), 1/\beta)]$$

C4.22 $V = -\log(1 - U) \sim EXP(1)$. To see this, note that

$$P(V < v) = P(-\log(1 - U) < v)$$

$$= P(U < 1 - \exp(-v))$$

$$= 1 - \exp(-v),$$

which is the distribution function of an exponential(1) random variable.