

Assignment 6

Eric Mittman

February 23, 2016

8.12 (a) See figure 1.

(b) See figure 2.

8.13

$$L(\eta, \beta) = \prod_{i=1}^r \beta \phi_{sev}(\beta \cdot \log(t_i/\eta)) [1 - \Phi_{sev}(\beta \cdot \log(t_c/\eta))]^{n-r}$$

8.21 (a) Different values of the shape parameter can lead to very different results, particularly when extrapolation is required; misspecification can lead to incorrect results. Also, specification of the shape parameter will lead to more precise estimates, but there is the danger that eliminating uncertainty in this parameter will lead to overconfidence in estimates derived from the fitted model.

(b) A hypothesis test could be used ensure that the data are consistent with the engineer's claim. If no problems are found, then analysis can proceed under that assumption. Another strategy would be to do a Bayesian analysis with an informative prior constructed with guidance from the engineer (I would be more comfortable with this choice.)

8.23

$$\begin{aligned}\hat{\eta}_{lower} &= \left(\frac{2 \sum_{i=1}^n t_i^\beta}{\chi_{1-\alpha;2}^2} \right)^{1/\beta} \\ &= \left(\frac{2 \cdot 10 \cdot 500^{2.3}}{5.991} \right)^{1/2.3} \\ &= 844.4852\end{aligned}$$

This gives us a straightforward calculation:

$$\begin{aligned}\hat{t}_{0.01,lower} &= (-\log(1 - 0.01))^{1/2.3} \hat{\eta}_{lower} \\ &\approx 114\end{aligned}$$

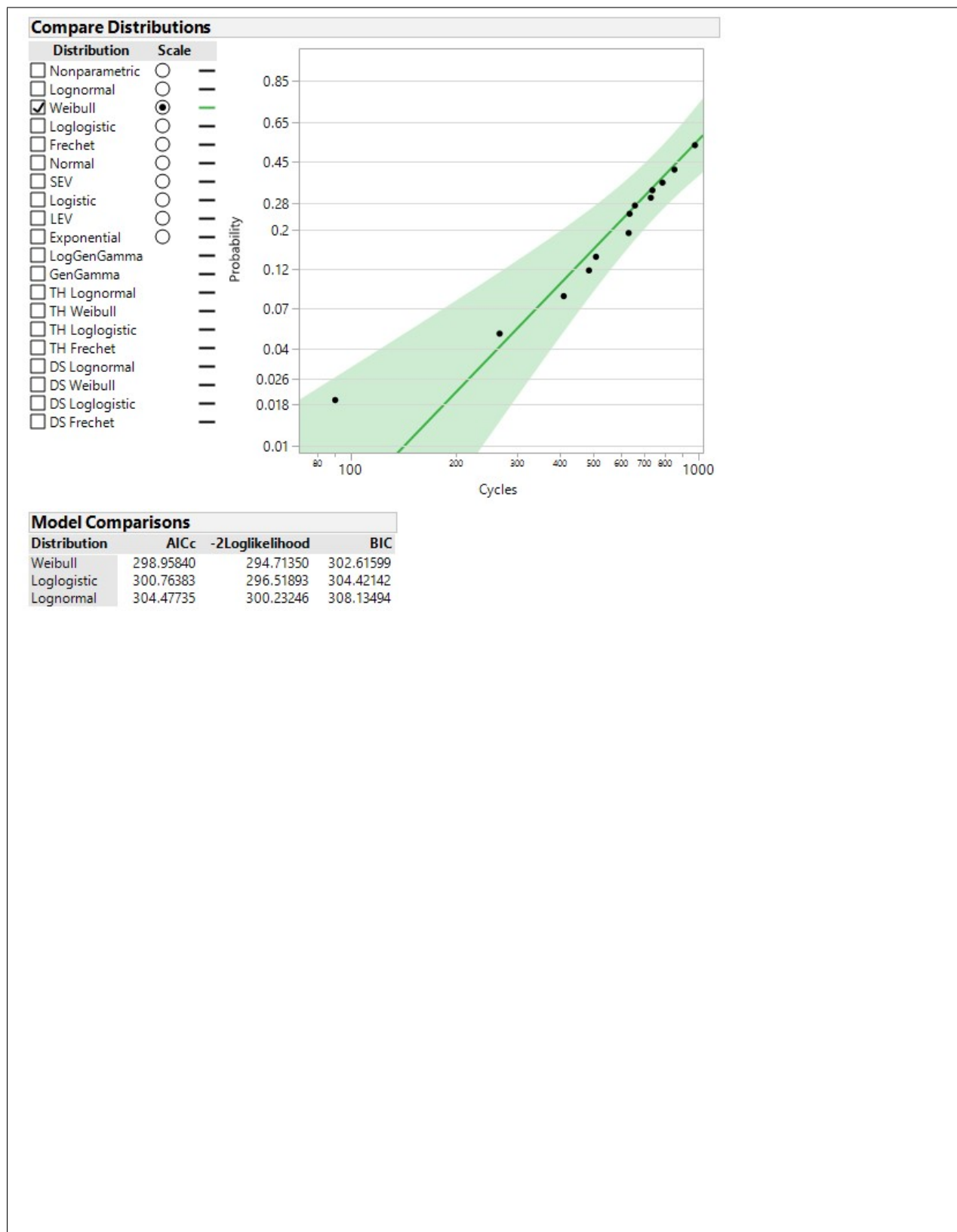


Figure 1: By AIC (or BIC) the best distributional form for these data is Weibull.

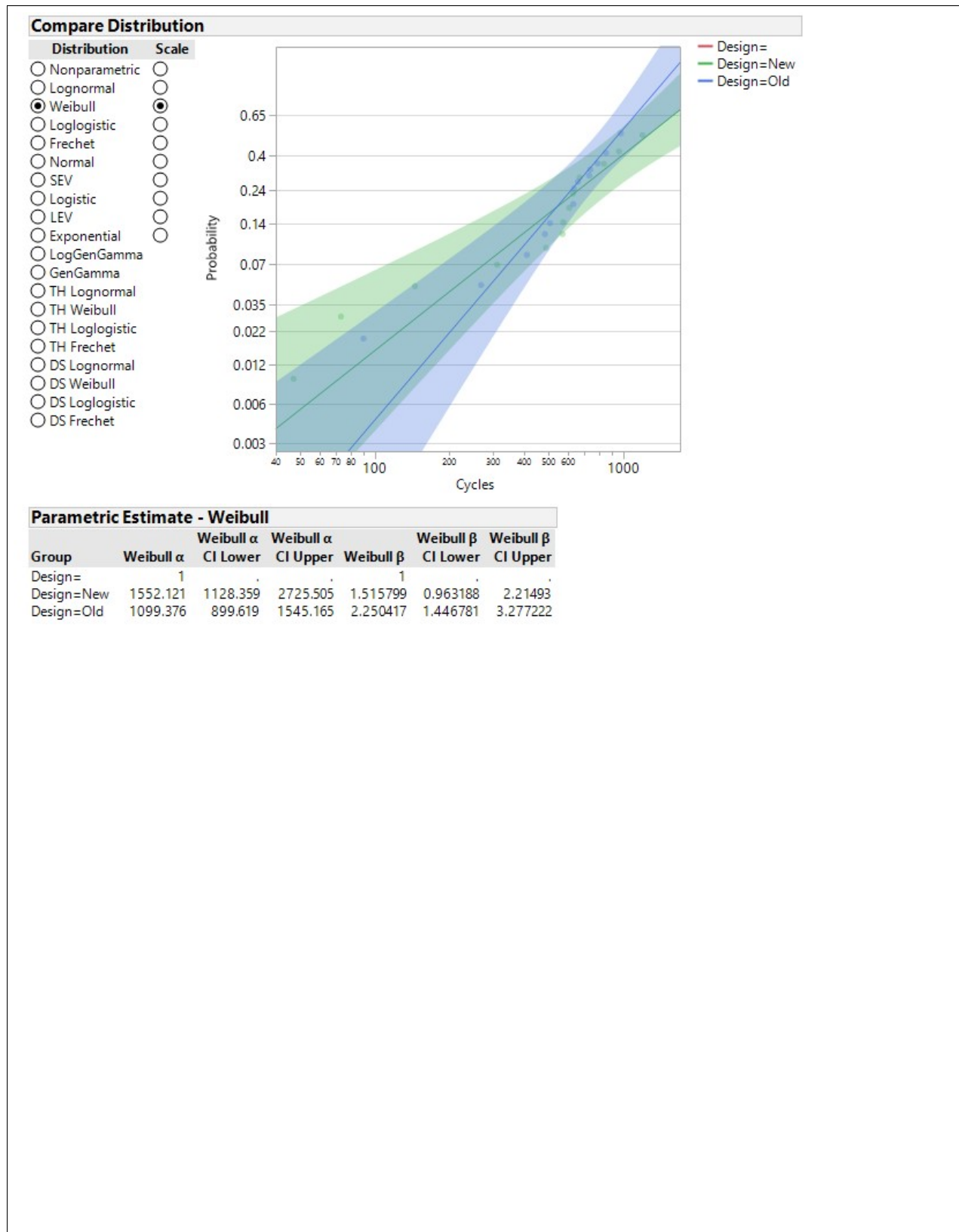


Figure 2: The estimated β for the newer design is smaller, suggesting better reliability, although the CIs for the two designs overlap, so this finding is not conclusive.

C8.25 A normal interval is constrained to be symmetric about an estimate, whereas a likelihood ratio based interval respects the parameter space. If we think of likelihood as “the probability of the data,” then the likelihood ratio interval will not exclude adjacent sections of the parameter space that which make the data more probable than some value included in the interval. This is not true for the normal interval; due to the symmetry requirement, there may well be values included in the interval where the probability of the data is much lower than adjacent values on the other side of the interval.

8.10 (a) (b) (c) See figure 3.

(d) Setting β too low will lead to overestimating proportion surviving especially when extrapolating. Setting β too high may lead to underestimating proportion surviving.

8.14 (a)

$$\begin{aligned}
L(\eta) &= \prod_{i=1}^r \frac{\beta}{\eta^\beta} t_i^{\beta-1} \exp\left(\left[\frac{-t_i}{\eta}\right]^{beta}\right) \cdot \left[\exp\left(\frac{-t_c}{\eta}\right)\right]^{n-r} \\
\Rightarrow l(\eta) &= r \log \beta - \beta r \log \eta + (\beta - 1) \sum_{i=1}^r \log t_i = \sum_{i=1}^r -\left(\frac{t_i}{\eta}\right)^\beta + (n-r) \left(\frac{t_c}{\eta}\right)^\beta \\
\Rightarrow \frac{d}{d\hat{\eta}} l(\hat{\eta}) &= \frac{-\beta r}{\hat{\eta}} + \sum_{i=1}^r \left(\frac{t_i}{\hat{\eta}}\right)^{\beta-1} \frac{t_i}{\hat{\eta}^2} + \beta(n-r) \left(\frac{t_c}{\hat{\eta}}\right)^{\beta-1} \frac{t_c}{\hat{\eta}^2} \\
&= \frac{\beta r}{\hat{\eta}} + \sum_{i=1}^r \beta t_i^\beta \frac{1}{\hat{\eta}^{\beta+1}} + \beta(n-r) t_c^\beta \frac{1}{\hat{\eta}^{\beta+1}} := 0 \\
\Rightarrow \hat{\eta}^\beta r &= \sum_{i=1}^r t_i^\beta + (n-r) t_c^\beta \\
\Rightarrow \hat{\eta} &= \left(\frac{\sum_{i=1}^r t_i^\beta + (n-r) t_c^\beta}{r}\right)^{1/\beta}
\end{aligned}$$

(b) It is analogous except instead of TTT, the L_β norm is used instead of the L_1 norm.

8.15 (a)

$$\frac{\partial}{\partial \beta} l(\eta, \beta) = \frac{r}{\beta} - r \log \eta + \sum_{i=1}^r \log t_i - \sum_{i=1}^r \log \frac{t_i}{\eta} \left(\frac{t_i}{\eta}\right)^\beta - (n-r) \log \frac{t_c}{\eta} \left(\frac{t_c}{\eta}\right)^\beta$$

The partial derivative for η is given in 8.14(a)

(b) Setting $\frac{\partial}{\partial \beta} l(\eta, \beta) = 0$ and factoring gives

$$0 = \frac{1}{\eta^\beta} \left[\left(\sum_{i=1}^r t_i^\beta \log t_i + (n-r) t_c^\beta \log t_c \right) - \log \eta \left(\sum_{i=1}^r t_i^\beta + (n-r) t_c^\beta \right) \right] + r \log \eta - \frac{r}{\beta} - \sum_{i=1}^r \log t_i$$

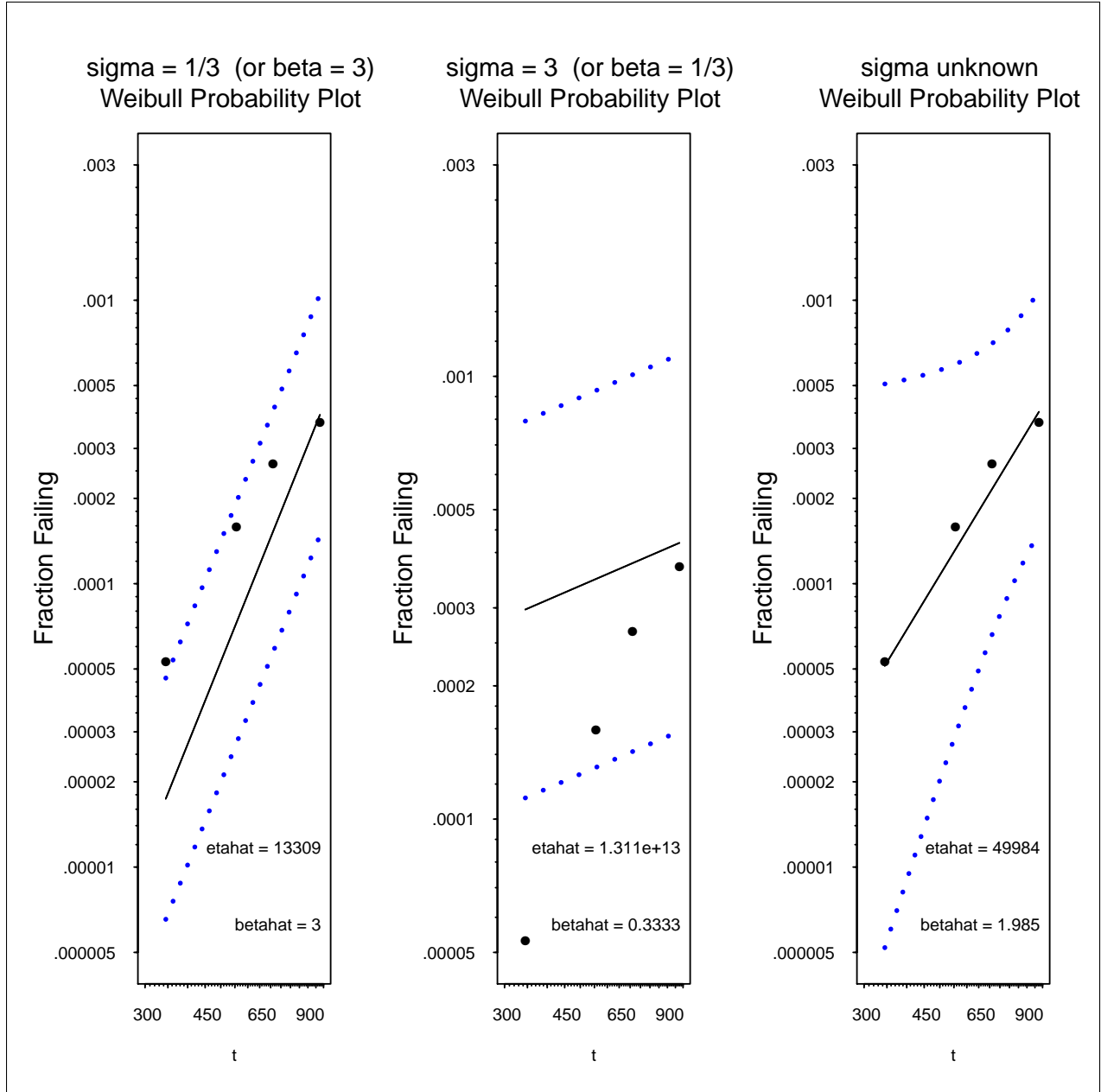


Figure 3: Different MLE estimates varying β

Substituting the solution for the η equation for η (second to last line 8.14(a)):

$$\begin{aligned}
0 &= \frac{r}{\sum_{i=1}^r t_i^\beta + (n-r)t_c^\beta} \left[\left(\sum_{i=1}^r t_i^\beta \log t_i + (n-r)t_c^\beta \log t_c \right) - \log \eta \left(\sum_{i=1}^r t_i^\beta + (n-r)t_c^\beta \right) \right] \\
&\quad + r \log \eta - \frac{r}{\beta} - \sum_{i=1}^r \log t_i \\
\Rightarrow 0 &= \frac{r \left(\sum_{i=1}^r t_i^\beta \log t_i + (n-r)t_c^\beta \log t_c \right)}{\sum_{i=1}^r t_i^\beta + (n-r)t_c^\beta} - r \log \eta + r \log \eta - \frac{r}{\beta} - \sum_{i=1}^r \log t_i \\
\Rightarrow 0 &= \frac{\left(\sum_{i=1}^r t_i^\beta \log t_i + (n-r)t_c^\beta \log t_c \right)}{\sum_{i=1}^r t_i^\beta + (n-r)t_c^\beta} - \frac{1}{\beta} - \frac{1}{r} \sum_{i=1}^r \log t_i
\end{aligned}$$