We used a Bayesian modeling approach to estimate the Weibull parameters for each model. Rather than modeling each model independently, we modeled the Weibull parameters hierarchically, to pool information and to provide better inferences, particularly for the brands that produced few failures. There were 5 models among the 21 with fewer than 10 failures (models 10, 12, 15, 16, 18). The model was fit using the rstan package in R [1].

We parameterized the Weibull in terms of a lower log quantile, t_p (where $p = 0.01$), and scale, σ. These were modeled by a Student's t with 5 degrees of freedom and a lognormal, respectively.

$$Y_{mi} \overset{ind.}{\sim} \text{Weibull}(\eta_m, \beta_m)$$

$$\sigma_m = \frac{1}{\beta_m}, \quad t_{p,m} = \exp\{\log(\eta_m) + \sigma_m \Phi_{sev}(p)\}$$

$$\log(t_{p,m}) \overset{i.i.d}{\sim} \text{t}(\nu = 5, \mu_1, \tau_1)$$

$$\sigma_m \overset{i.i.d}{\sim} \text{log-normal}(\mu_2, \tau_2^2)$$

The following uninformative and vague priors were used for the hyperparameters.

$$p(\mu_1, \mu_2) \propto 1$$

$$\tau_1, \tau_2 \overset{ind.}{\sim} \text{half-Cauchy}(0, 10)$$

The model for Y_{mi} must be modified to accomodate the censoring and truncation patterns in the data. In the case of a failure that was observed and left-censored at t_L,

$$Y_{mi} \sim \text{TWeibull}(\eta_m, \beta_m, t_L, \infty),$$

where $\text{TWeibull}(\eta, \beta, a, b)$ is a truncated Weibull distribution with left and right truncation points a and b, respectively. For untruncated, right censored observations with censoring point t_c,

$$Y_{mi} \sim \text{Bernoulli}(1 - \Phi(t_c)),$$

Where Φ is the cdf for a $\text{Weibull}(\eta_m, \beta_m)$ distribution. For observations that are both right censored and truncated,

$$Y_{mi} \sim \text{Bernoulii}(1 - \Phi_{t_L}(t_c)),$$

where Φ_{t_L} is the cdf for a left truncated Weibull distribution.

References

[1] Stan Development Team. Stan: A c++ library for probability and sampling, version 2.8.0, 2015.