Getting Proper Standard Errors From R Stan:

The Hessian Stan's optimization program returns, results in a covariance matrix in terms of:

$$[\log(t_{p_1}), \log(t_{p_2}), \log(\sigma_1), \log(\sigma_2), p]$$

Let's use the Δ method to get a covariance matrix for: $[\mu_1, \mu_2, \sigma_1, \sigma_2, p]$. Not the following two relationships between μ and the quantiles.

$$\Phi^{-1}_{SEV}(p) = \log(-\log(1-p))$$
$$\log(t_p) = \mu + \log(\sigma)\Phi^{-1}_{SEV}(p)$$

Solving for μ we get:

$$\mu = \log(t_p) - \exp(\log(\sigma))\Phi^{-1}_{SEV}(p)$$

Taking partials with respect to $\frac{d\mu}{d\log(t_p)}$ and $\frac{d\mu}{d\log(\sigma)}$ we get:

$$\frac{d\mu}{d\log(t_p)} = 1$$
$$\frac{d\mu}{d\log(\sigma)} = -\exp(\log(\sigma))\Phi^{-1}_{SEV}(p)$$

It is easy to transform $\log(\sigma)$ and as far as I can tell, p doesn't need to be transformed, but we might want to check if the standard errors should be changed. Putting these pieces together we get a Δ matrix:

$$\Delta = \begin{bmatrix} 1 & 0 & -\exp(\log(\sigma_1)\Phi^{-1}_{SEV}(p_1) & 0 & 0 \\ 0 & 1 & 0 & -\exp(\log(\sigma_2)\Phi^{-1}_{SEV}(p_2) & 0 \\ 0 & 0 & \exp(\log(\sigma_1)) & 0 & 0 \\ 0 & 0 & 0 & \exp(\log(\sigma_2)) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, following Meeker and Escobar (Section 8.4.3), we need to get the standard errors for the GFLP CDF to make Wald bands. Recall, the CDF: $g(\mu_1, \mu_2, \sigma_1, \sigma_2, p) = 1 - (1 - pF_1)(1 - F_2)$, which we can re-write as:

$$\begin{aligned} \Pr(t < T) &= 1 - (1 - p\Phi_{SEV}(\mu_1, \sigma_1)(z_1))(1 - \Phi_{SEV}(\mu_2, \sigma_2)(z_2)) \\ &= \Phi_{SEV}(\mu_2, \sigma_2)(z_2) + p\Phi_{SEV}(\mu_1, \sigma_1)(z_1) - p\Phi_{SEV}(\mu_1, \sigma_1)(z_1)\Phi_{SEV}(\mu_2, \sigma_2)(z_2) \end{aligned}$$

Note that the SEV distribution is standardized in terms of $z = (\log(t) - \mu)/\sigma$. Let's again take partials of the GFLP CDF with respect to the 5 parameters. Note: z_1 or z_2 should be plugged into the proper SEV function for each term.

$$\frac{dg}{d\mu_1} = -p\phi_{SEV}(\mu_1, \sigma_1)\frac{1}{\sigma_1} + p\phi_{SEV}(\mu_1, \sigma_1)\Phi_{SEV}(\mu_2, \sigma_2)\frac{1}{\sigma_1}$$
$$\frac{dg}{d\mu_2} = -\phi_{SEV}(\mu_2, \sigma_2)\frac{1}{\sigma_2} + p\Phi_{SEV}(\mu_1, \sigma_1)\phi_{SEV}(\mu_2, \sigma_2)\frac{1}{\sigma_2}$$
$$\frac{dg}{d\sigma_1} = p\phi_{SEV}(\mu_1, \sigma_1)\left(\frac{-log(t) + \mu_1}{\sigma_1^2}\right) - p\Phi_{SEV}(\mu_2, \sigma_2)\phi_{SEV}(\mu_1, \sigma_1)\left(\frac{-log(t) + \mu_1}{\sigma_1^2}\right)$$
$$\frac{dg}{d\sigma_1} = \phi_{SEV}(\mu_2, \sigma_2)\left(\frac{-log(t) + \mu_2}{\sigma_2^2}\right) - p\Phi_{SEV}(\mu_1, \sigma_1)\phi_{SEV}(\mu_2, \sigma_2)\left(\frac{-log(t) + \mu_2}{\sigma_2^2}\right)$$
$$\frac{dg}{dp} = \Phi_{SEV}(\mu_1, \sigma_1) - \Phi_{SEV}(\mu_1, \sigma_1)\Phi_{SEV}(\mu_2, \sigma_2)$$

Putting these into a 5 x 5 matrix we can once again use the Δ method for the standard error of g. Then with this standard error we can use a normal approximation and get $\hat{g} \pm z_{1-\alpha/2}\hat{g_{se}}$

Taking this a step further (Meeker and Hong, p. 169) we can use a tangent method to get a CI that respects the bounds of a CDF.

Let $y_e = log(t)$. The lower and upper bounds are calculated as:

$$y_l = y_e - \frac{z_{1-\alpha/2}\hat{g_{se}}}{\frac{dF(t)}{log(t)}}$$

$$y_u = y_e + \frac{z_{1-\alpha/2}\hat{g_{se}}}{\frac{dF(t)}{log(t)}}$$

Taking the derivative of the GFLP in terms of $log(t)$, which it is already parameterized in terms of is:

$$\frac{dg(t)}{dlog(t)} = p\phi_{SEV}(\mu_1, \sigma_1)(z_1)(1 - \Phi_{SEV}(\mu_2, \sigma_2)(z_2))(1/\sigma_1) + \phi_{SEV}(\mu_2, \sigma_2)(z_2)(1 - p\Phi_{SEV}(\mu_1, \sigma_1)(z_1))(1/\sigma_2)$$

Finally, we plug these end points back into the GFLP CDF to get our final confidence intervals.

$$[\hat{F}(\exp(y_l)), \hat{F}(\exp(y_u))]$$

Phew. I have checked this a few times, but it's kind of tedious so there very well be some errors and or typos.