A Hierarchical Bayesian Approach for Modeling Infant-Mortality and Wearout Failure Modes

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Abstract

The text of your abstract. 100 or fewer words.

Keywords: 3 to 6 keywords, (don't reuse words appearing in title)

1 Introduction

Failure distributions of high-reliability products, can be difficult to assess. Accelerated testing tries to address this problem, but requires strong assumptions to extrapolate to use conditions in the field (citation). When field data are available, information may be very limited. In this paper, we propose the use of Bayesian hierarchical modeling to borrow information to improve inferences on lifetime distributions where information is limited by right-censoring and left-truncation.

Failure data for series systems may be collected at the system level by end-users interested only in the lifetime of the system. Chan and Meeker [1999] proposed the generalized limited failure population model (GLFP), which provides a generative model for failure times in a population where some units are susceptible to infant mortality. This model covers a subset of the class of distributions known as bathtub distributions Rajarshi and Rajarshi [1988]. Bathtub distributions are characterized by a U-shaped hazard function, where early failures are due to infant mortality and late failure are due to wearout. While complex systems actually exhibit more than two modes of failure, suitable parametric models which can suitably approximate the overall lifetime distribution are useful.

In the case of complex, high-reliability systems where cause of failure is not available, a model which

a. has parameters that can be estimated from the observed data

b. is flexible enough to fit the observed data

c. in interpretable with respect to available theory of the underlying process

is desired. We present a Bayesian, hierarchical modeling approach for grouped failure data. As a benefit of working with Monte Carlo samples, we can easily make inference on a wide range of quantities of interest while accounting for uncertainty. We demonstrate this approach on real data hard drive failure data from a cloud-based storage company.

1.1 Background

In engineering applications, a product can often fail from one out of a set of possible malfunctioning components. For example, a computer system can fail if the mother board, disc drive or power supply stop working. Circuit boards (CB) can fail due to a manufacturing defect or later as a result of wearout. The general name for such products is a series system where the lifetime of the product is the minimum failure time across s different components or risks Nelson [2005]. A common assumption in series systems is the time to failure for each risk, s, is statistically independent. Thus, the overall reliability of a unit is modeled using the product rule across all s risks. Parameter estimation is straightforward if the cause of failure is known for each observation. With engineering systems data, however, the exact cause of failure is frequently unknown or masked from the researcher.

Previous papers have employed various data assumptions and methodologies to model masked lifetime failure data. When modeling computer system failures Reiser et al. assumed each observed failure came from a known subset of failure modes, and estimation was performed using a Bayesian approach Reiser et al. [1995]. Chan and Meeker labeled the cause of circuit board failures as infant mortality, unknown, or wearout based on the time of observed failures. This helped identify parameters when using maximum likelihood (ML) estimation. Extending Chan and Meeker's analysis, Basu et. al performed a Bayesian analysis with informative priors to better identify early versus late failure modes without making any data assumptions Basu et al. [2003]. Berger and Sun introduced the Poly-Weibull distribution where cause of failure is the minimum of a several Weibull distributions Berger and Sun [1993]. More recently, Ranjan et al. considered a competing risk model for infant mortality and wearout as a mixture of Weibull and exponential failure distributions Ranjan et al. [2015]. Treating the unknown failure modes as incomplete data, an expectation maximization algorithm with ML was used, in addition to Bayesian estimation.

1.2 Motivation

The goal of this paper is to model, and compare, the lifetime distribution of different hard drive brands using the GLFP lifetime model. Product populations often contain a mixture of defective and non defective units. The hazard function for this type of population is often described as a bathtub curve: the beginning of the curve corresponds to defective units failing early, followed by a constant hazard, and then and upswing as units fail from wearout. Ignoring this hazard structure, which exists for series systems, could lead to spurious inference when comparing the reliability of hard drive brand models Rajarshi and Rajarshi [1988]. The GLFP combines different failure time distributions to account for bending hazard functions; for example, one parametric model for defective units (infant mortality) and a second model for wearout Chan and Meeker [1999].

While the GLFP model allows for multiple hazard functions, it does not account for the group structure of the hard drive data. A naive approach would combine all drive models and fit an overall GLFP model. This model can be easily estimated, but a common GLFP model for all drive brand groups fits poorly: the model captures drive models with lots of observed failures, while the predicted time to failure for drive brand models with less information is over or under estimated. At the other extreme, fitting individual models to each hard drive brand excludes many drive brand models with fewer than 5 failures. The GLFP model applied to these data produces very imprecise parameter estimates or can only be fit if highly informative priors are imposed.

We therefore propose to fit the GLFP model using a hierarchical Bayesian approach. With over 60 brand drive models in testing a hierarchical model is advantageous as it pools information from across brands to get more precise estimates of the GLFP parameters for each drive model group. Moreover, the hierarchical model is a nice compromise between the aggregate and individual modeling approaches: drive models with lots of failures are estimated precisely; and in cases where little data is available the hierarchical model borrows information from other drive brand models to obtain stable estimates. This allows us to include all of the hard drive data and more importantly, make valid inferences for each

group of hard drive brands.

1.3 Overview

The structure of the paper is as follows. Section 1 summarizes the Backblaze data and the unique modeling challenges it presents. Section 2 introduces the GLFP model as a mixture of two Weibull distributions. In Section 3 we fit multiple specifications of the GLFP model to best describe the Backblaze data. We also discuss computational and modeling issues that suggest a final model with a common Weibull distribution for defective units and individual Weibull distributions for the wearout failure mode. In Section 4 we compare hard drive brand models based on our final GLFP model. Finally, in Section 5, we review the GLFP model and propose future extensions.

2 Data

Backblaze is a company that offers backup storage to protect against data loss. While selling storage space is Backblaze's main business, since 2013 it has been collecting data on hard drives operating at its facility. The purpose is to provide consumers and businesses with reliability information on different hard drive brands and models. The hard drives continuously spin in controlled storage pods where they run until failure. When a hard drive fails it is permanently removed, and new hard drives are regularly added to the storage pods. In addition, the number of storage pods is increasing as Backblaze adds new hard drive brands to the sample. Every quarter Backblaze makes its data publicly available through their website bac. In addition, Backblaze publishes summary statistics of the different hard drive models currently operating. No other analysis or modeling of the failure data is provided; however, Backblaze does encourage the public to further analyze its data, which for this paper goes through the first quarter of 2016.

As of the first quarter of 2016, Backblaze was collecting data on 63 different hard drive models. Some drive models have been running since 2013, while others have been added at a later date. There are a total of 75,297 hard drives running. The distribution of drives by model varies: some models have only have a service record for a single drive; the maxi-

mum number of service records for a single drive model is 35,860. We can also look at the distribution of total number of failures and total time on test across all drive models. As seen below, some drive models contain a lot of information, while others have few failures or a little time in operation. [INSERT FIGURE 1].

Probability plotting is an effective method to check the adequacy of various parametric models to the data. Identifying whether failure data follows a specific distribution is difficult to do by eye: especially when looking at pdfs or other non-linear plots. However, if we can linearize the cumulative distribution function it is easier to visually assess a distributional goodness of fit; if a model is appropriate, a non parametric estimate of $\hat{F(t)}$ graphed on linarized probability scales should approximately follow a straight line.

For the hard drive data we first estimate the empirical cdf for each model using the Kaplan-Meier estimator E. L. Kaplan [1958]. With left truncation, however, the Kaplan-Meier estimate, $\widehat{F(t)}_{KM}$, is conditional based on hard drive i surviving up to time τ_i^L where L is the amount of time the hard drive was running before Backblaze starting monitoring the drive. To get the unconditional distribution we correct the non-parametric estimates using a parametric adjustment outlined by Turnbull, and given in more detail by Meeker and Escobar Turnbull [1976], Meeker and Escobar [1998]. For each hard drive model we select $\tau_{\min}^L$, the smallest left truncated time in the sample. Then, using the parametric estimates of the parameters, we calculate $Pr(T > \tau_{\min}^L)$ the probability a hard drive has survived up to $\tau_{\min}^L$. We then apply this correction to the non parametric estimates, which results in the unconditional distribution of time to failure.

In Figure 1 we plot the Kaplan-Meier adjusted cdf for hard drive model X on Weibull paper. Each point on the plot corresponds to a hard drive failure. Censored drives are not plotted. As mentioned in the introduction, the population of hard drives exhibits two primary failure modes. One mode is a result of manufacturing defects, which cause early failures, known as infant mortality. The second mode is non-defective hard drives that eventually fail due to wearout. Evidence of at least two failure modes is seen in the Kaplan
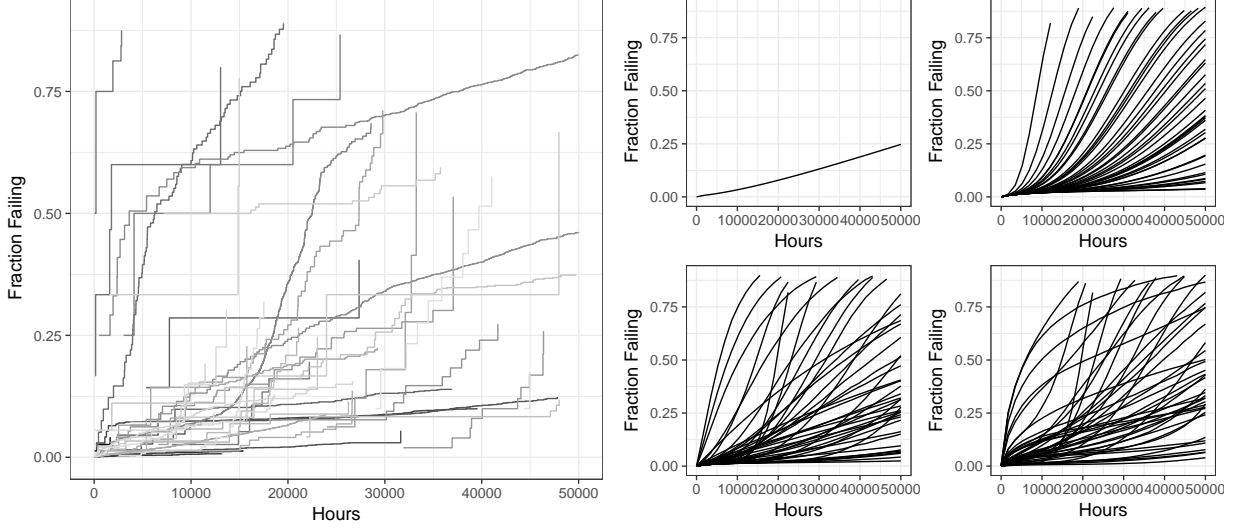
6

Figure 1: Left: Kaplan-Meier estimates of the time to failure for each of the drive-models in the Backblaze data. Right: Pointwise posterior median time to failure curves for Models 1-4, orderered left to right, top to bottom.

Meier plot with a kink occurring around hour Z. Therefore, fitting a single Weibull model would not be flexible enough to model the failure distribution.

3   Hierarchical GLFP model

For modeling the lifetime of hard-drives, we select the Generalized Limited Failure Population model of Chan and Meeker [1999]. Let $T_{d,i}$ be the time of failure for the ith drive of drive-model d. We assume that $T_{d,1}, \ldots T_{d,n_d}$ are independent and have a probability distribution with cdf given by

$$P(T_{d,i} \leq t) = 1 - (1 - \pi_d F_{d1}(t))(1 - F_{d2}(t)), \text{ for } t > 0, \text{ and where } \pi_d \text{ is in } (0,1).$$

As in Chan and Meeker [1999], we assume F_{dj} is a member of the Weibull family of cdfs and parameterize in terms of a log-location parameter μ_{dj} and log-scale parameter σ_{dj} so that

$$F_{dj}(t) = 1 - \exp\left\{-\exp\left\{\frac{\log(t) - \mu_{dj}}{\sigma_{dj}}\right\}\right\}, \ t > 0, j = 1, 2$$

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For the purpose of exposition, we will refer to the probability distributions F_{d1} and F_{d2} as "failure modes." Specifically, we will refer to F_{d1} as the "early failure mode", and F_{d2} as the "main failure mode." The interpretation of the parameter π_d is to represent the proportion of units susceptible to early failure, hence susceptible to both failure modes. Here the cause of failure is not assumed to be known, thus units of the same drive-model are exchangable.

To borrow strength across models, we can either share parameters across drive-models, or model the drive-model specific parameters hierarchically, allowing the data to inform the hyperparameters. For the second option, we model the scales, σ_j, quantiles, $t_{p_j,d,j} = \Phi^{-1}(\mu_j)$, and proportions, π_d, of the component distributions as follows:

$$\sigma_{d,1} \overset{ind.}{\sim} \text{Lognormal}\left(\eta_{\sigma,1}, \tau_{\sigma,1}^2\right) \text{ for } d = 1, \ldots, D$$

$$\sigma_{d,2} \overset{ind.}{\sim} \text{Truncated-Lognormal}\left(\eta_{\sigma,2}, \tau_{\sigma,2}^2, 0, 1,\right) \text{ for } d = 1, \ldots, D$$

$$t_{p_j,d,j} \equiv \mu_{d,j} + \sigma_{d,j}\, \Phi^{-1}(p_j) \overset{ind.}{\sim} \text{Normal}\left(\eta_{t_{p_j},j}, \tau_{t_{p_j},j}^2\right) \text{ for } j = 1, 2\ d = 1, \ldots, D$$

$$\text{logit } \pi_d \overset{ind.}{\sim} \text{N}(\eta_p i, \tau_p i) \text{ for } d = 1, \ldots, D.$$

Here, Φ^{-1} is the quantile function of the standard log-Weibull distribution. We truncate $\sigma_{d,2}$ at 1, appealing to the logic that wearout implies an increasing hazard function. The decision to parameterize in terms of a quantile other than the log-location parameter, $\mu = \Phi^{-1}(.5)$, is that lifetime data often features heavy right-censoring where inferences about the location parameter are extrapolations beyond the range of the data. For this data we selected $p_1 = 0.5$, (the median), and $p_2 = 0.2$.

We consider the models with the following set of restrictions (from most to least restrictive):

1. $\pi_d = \pi, \quad \mu_{d1} = \mu_1, \quad \sigma_{d1} = \sigma_1, \quad \mu_{d2} = \mu_2, \quad \sigma_{d2} = \sigma_2$

2. $\pi_d = \pi, \quad \mu_{d1} = \mu_1, \quad \sigma_{d1} = \sigma_1, \quad \sigma_{d2} = \sigma_2$

3. $\pi_d = \pi, \quad \mu_{d1} = \mu_1, \quad \sigma_{d1} = \sigma_1$

4. $\mu_{d1} = \mu_1, \quad \sigma_{d1} = \sigma_1$

The set of model specifications were chosen based on the data, interpretation of the model, as well as estimation considerations. Drive brand specific parameters for the the wearout failure mode (μ_{d2}, σ_{d2}), and the proportion defective (π_d), are considered as a means to account for heterogeneity across brands in the right tails of the failure distribution. Going from a common model for all hard drive brands and gradually increasing the complexity of the model, we end up at Model 4. Model 4 allows for the probability of infant mortality as well as the shape and scale parameters for the 2nd failure mode to vary by brand drive model.

For all of the models we consider, the parameters for the infant mortality failure mode are held in common across drive brand models. We found that there was insufficient information in the data to model these parameters hierarchically. Moreover, assuming a common distribution for infant mortality allows for straightforward interpretation of the quantities like $\frac{\pi_d}{\pi_{d'}}$.

3.1 Priors

To complete the full probability model, we need to select prior distributions for the parameters governing the hierarchical model. We select proper priors to ensure a proper posterior. For the scale hyper-parameters we follow the recommendations of Gelman et al. [2014] and use half-Cauchy priors. As for the location hyper-parameters, we select weakly informative priors consistent with our prior information on hard-drives. The prior for η_π puts 95% of prior mass on the interval $(0.006, 0.27)$ for the median proportion defective (What justification? Should maybe be lower.) For $\eta_{\sigma,2}$, 95% of prior mass is on values greater than 0.037. Since σ_d is the reciprocal of the Weibull shape parameter, this correponds roughly to an assumption that the median Weibull shape parameter is less than $1/0.037 = 27$. The prior for $\eta_{t_{p_2},2}$ implies that the median 20th percentile for non-defective units is less than greater than 3 days and less than 24 years.

$$\eta_\pi \sim \text{Normal}(-3, 1)$$

$$\tau_\pi \sim \text{Cauchy}^+(0, 1)$$

$$\eta_{\sigma,2} \sim \text{Normal}(0, 2)$$

$$\tau_{\sigma,2} \sim \text{Cauchy}^+(0, 1)$$

$$\eta_{t_{.2_2},2} \sim \text{Normal}(9, 2)$$

$$\tau_{t_{.2_2},2} \sim \text{Cauchy}^+(0, 1)$$

3.2 Computation

Each model was fit in $\hat{\text{R}}$ using Hamiltonian Monte Carlo Betancourt and Girolami [2015] as implemented in `rstan`Guo et al. [2014]. Multiple chain were run with warmup iterations. Convergence was assessed by examining posterior plots and checking that Gelman-Rubin diagnostics were close to 1.

4 Data analysis

4.1 Results

The right side of Figure 1 shows that the set of estimated failure curves for all four models that we fit. The upper left panel shows the estimated failure curve fit to all data ignoring drive model as a factor (Model 1). The upper left plot, which represents Model 2, demonstrates that there is appreciable differentiation between drive models; note that, since the scale parameter, $\tau_{t_{p2}}$ is learned hierarchically from the data, we can ascertain that there is evidence in the data that this variability is real.

The lower left panel, representing Model 3, suggests differing shape parameters in the wearout failure mode which mean that the ranking of models with respect to fraction failing differs as a function of time.

Finally, the lower right panel, corresponding to Model 4, shows that, by allowing π_d to vary by drive model, there is greater variability in the early rate of failure and, for some drive models, the forecasted fraction failing is adjusted downward.

4.2 Model Comparisons

First look at the overlaid posterior credible bands.

In addition to graphically examining model fits via posterior credible bands, we compared the predictive accuracy of the 4 different models using an approximate leave-one-out cross validation (LOO) method. As outlined in Vehtari et. al, the predictive accuracy from a fitted Bayesian model can be estimated simply with posterior draws of the model parameters Vehtari et al. [2016b]. For each observation, i, the log point-wise predictive density is calculated over the full set of posterior samples with the ith data point removed. The final expected point-wise predictive density is the sum over all observations (elpd $= \sum \log p(y_i|y_{-i})$). We computed the elpd for all 4 models using the R package loo Vehtari et al. [2016a]. When n is large, the distribution of the elpd is approximately normal and models can be statistically compared. We calculated the difference in expected predictive accuracy for Model 4 vs 3, Model 3 vs 2, and Model 2 vs 1, as well as the standard error of the difference (Table 1). As the model complexity increased, the expected predictive accuracy improved. Of all the models, Model 4 had the best predictive accuracy and was significantly better than the other 3 models.

	elpd	Diff(elpd)	SE(Diff)
Model 4	-13309.5	40.7	11.3
Model 3	-13350.2	458.8	31.0
Model 2	-13809.0	3674.6	96.2
Model 1	-17483.6		

Table 1: Model Comparison Based on LOO

4.3 Brand Comparisons

Ideas for plots: A graph comparing the pi's for all drive models.

I think a graph looking at the B10 using the full model could be good, too. Something like the graph below. Perhaps we could sort drive model based on sample size?

This should probably be truncated at something like eight years, since the right tails

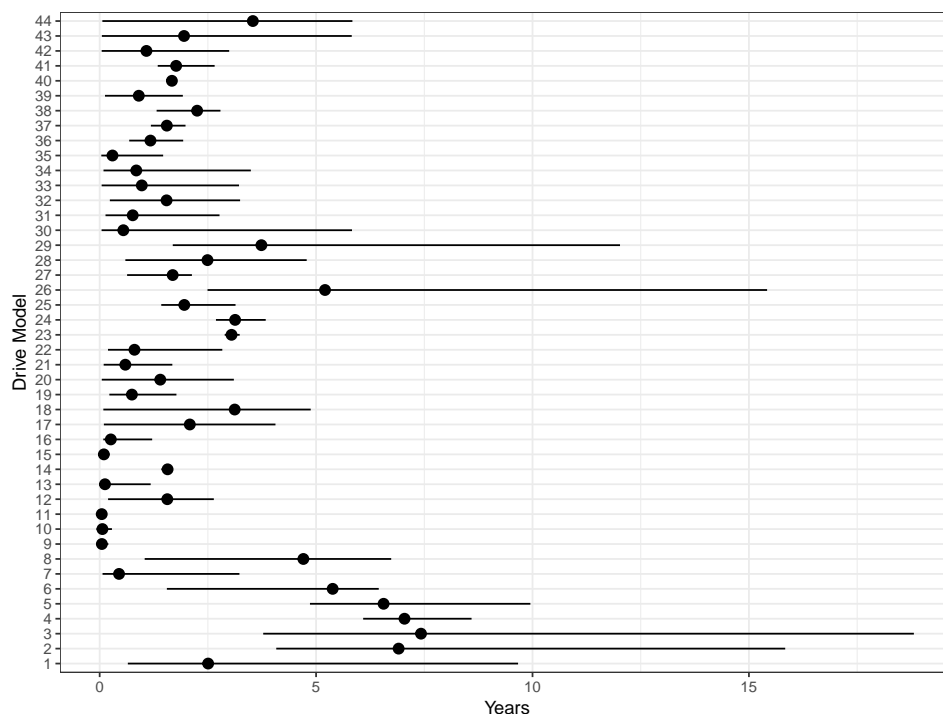seem unlikely. Sort by left endpoint?

Figure 2: B10 in Years for All Drive Models

5 Concluding Remarks and Extensions

Review the advantages of fitting the GFLP model and offer future ideas.

SUPPLEMENTARY MATERIAL

Put R Stan code here

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