

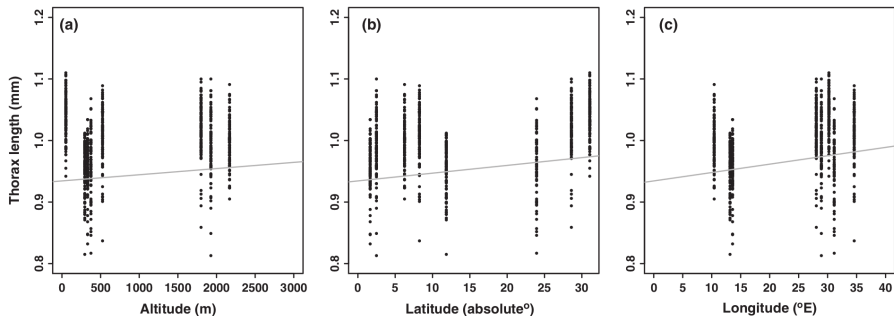
# Neutral quantitative genetic variation in structured populations

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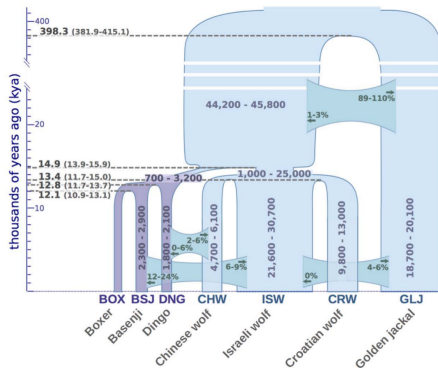
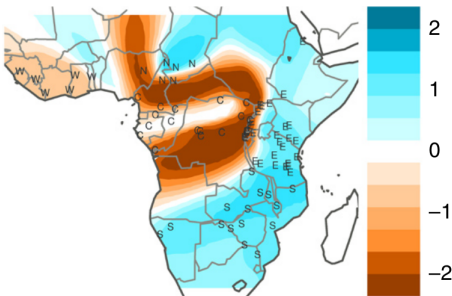
# Is spatial variation in quantitative traits adaptive?

Fabian et al. 2015

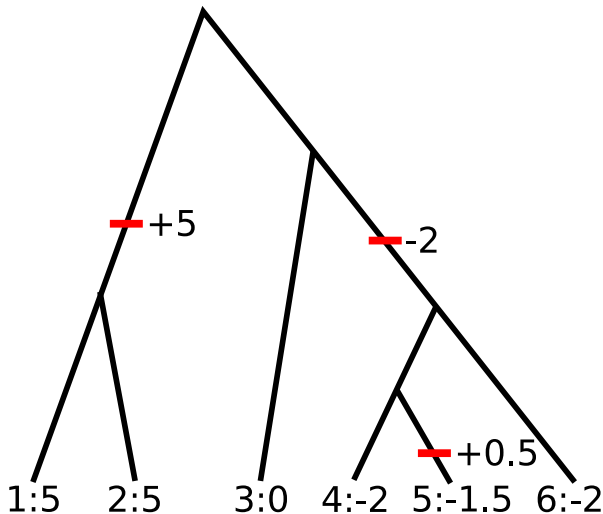


# Population structure can take different forms

Petkova et al. 2015, Freedman et al. 2014



# How can we describe phenotypic variation in a sample?



# Probability distribution $\Leftrightarrow$ moment generating function (MGF)

$$\mathbf{Y} = \{Y_a, Y_b, Y_c\} \quad (1)$$

$$\varphi_{\mathbf{Y}}(\mathbf{k}) = E \left[ e^{\mathbf{k} \cdot \mathbf{Y}} \right] = \int e^{\mathbf{k} \cdot \mathbf{Y}} P(\mathbf{Y} = \mathbf{y}) d\mathbf{y}$$

# Phenotypic MGF can be obtained from genealogy MGF

Compound poisson process on a tree, Schraiber and Landis 2015

$$\mathbf{T} = \{T_a, T_b, T_c, T_{a,b}, T_{a,c}, T_{b,c}\} \quad (2)$$

$$\varphi_{\mathbf{T}}(\mathbf{s}) \Big|_{s_{\omega} = \frac{\theta}{2}(\psi(\sum_{a \in \omega} k_a) - 1)}$$

$\psi$  is the generating function of the mutational distribution.

# This becomes complicated very quickly

Lohse et al. 2011

$$\varphi_{\mathbf{Y}}^{\Omega}(\mathbf{k}) = \left( \sum_{i=1}^M \binom{|\Omega_i|}{2} \eta_i + \sum_{(i,j): i \neq j} m_{i,j} |\Omega_i| - \sum_{i=1}^M \sum_{\omega \in \Omega_i} \frac{\theta}{2} \left( \psi \left( \sum_{a \in \omega} k_a \right) - 1 \right) \right)^{-1} \\ \times \left( \sum_{i=1}^M \eta_i \sum_{(a,b) \in \Omega_i: a \neq b} \varphi_{\mathbf{Y}}^{\Omega(i:a \cup b)}(\mathbf{k}) + \sum_{(i,j): i \neq j} m_{i,j} \sum_{\omega \in \Omega_i} \varphi_{\mathbf{Y}}^{\Omega(i:-\omega, j: +\omega)}(\mathbf{k}) \right)$$

# The infinitesimal limit

- Number of loci gets big
- Effect of each locus gets small

Let the variance of mutational effects be:  $\text{Var}[U] := \tau^2$

$$\lim_{L \rightarrow \infty} \tau^2 L \rightarrow \sigma^2$$

$$\lim_{L \rightarrow \infty} E[U^k] L \rightarrow 0$$

for  $k > 2$ .



# Differences become multivariate normal

All that matters are pairwise coalescent times.

$$E[Y_1 - Y_2] = 0 \quad (3)$$

$$\text{Cov}[Y_1 - Y_2, Y_3 - Y_4] \propto E[t_{1,4}] + E[t_{2,3}] - E[t_{1,3}] - E[t_{2,4}]$$

# Phenotypic divergence can be summarized using $Q_{ST}$

$$Q_{ST} = \frac{V_{between}}{V_{within} + V_{between}}$$

Can derive the sampling distribution using the previous theory.

# Example $Q_{ST}$ sampling distributions

