A Short Lecture on Basic Finance

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Risk and Return: Foundation

Central Question

- How do investors price risky assets?

Key Insight

- Expected return must compensate for risk
- Trade-off: Higher risk $\Rightarrow$ Higher required return

Mathematical Foundation

$$E(R_i) = R_f + \text{Risk Premium}_i$$
$$\text{Risk Premium}_i = f(\text{Risk measures})$$

Implications

- Risk premium depends on systematic risk, not total risk
- Diversifiable risk earns no premium in equilibrium

Types of Risk

Systematic Risk (Non-diversifiable)

- Market-wide factors
- Economic cycles
- Interest rate changes
- Inflation
- Political events

Unsystematic Risk (Diversifiable)

- Company-specific events
- Management decisions
- Product failures
- Legal issues
- Industry-specific shocks

Key Insight: Only systematic risk matters for pricing!

$$\sigma^2_{total} = \sigma^2_{systematic} + \sigma^2_{unsystematic}$$

Equity Risk Premium

Definition

- Expected excess return of stocks over risk-free bonds

$$\text{Equity Risk Premium} = E(R_{market}) - R_f$$

Historical Evidence (US, 1926-2020)

- Geometric mean: $\approx 10\% - 3\% = 7\%$ annually

The Equity Premium Puzzle

- Premuium: ‘too high’ given standard risk aversion models
- Requires unrealistically high risk aversion coefficients
- Alternative explanations: behavioral biases, rare disasters, liquidity

Implication for Trading: Stocks have historically outperformed bonds, but with significant volatility

Market Equilibrium

Starting Point for CAPM: The following conditions lead to the Capital Asset Pricing Model

Equilibrium Condition

- Supply = Demand for each asset

Key Assumptions

- Investors are rational and risk-averse
- Perfect information
- Homogeneous expectations
- No transaction costs or taxes

Result

- Market portfolio is mean-variance efficient
- All investors hold combinations of risk-free asset & market portfolio

$$\text{Market Portfolio} = \sum_{i=1}^{N} w_i \cdot \text{Asset}_i, \quad w_i = \frac{\text{Market Value}_i}{\text{Total Market Value}}$$

Beta: Measuring Systematic Risk

Definition

- Beta measures an asset's sensitivity to market movements

$$\beta_i = \frac{\mathsf{Cov}(R_i, R_m)}{\mathsf{Var}(R_m)} = \frac{\sigma_{i,m}}{\sigma_m^2}$$

Interpretation

- $\beta = 1$: Asset moves with market
- $\beta > 1$: Asset more volatile than market (amplifies market moves)
- $\beta < 1$: Asset less volatile than market (dampens market moves)
- $\beta = 0$: Asset uncorrelated with market

Estimation via Regression

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i(R_{m,t} - R_{f,t}) + \varepsilon_{i,t}$$

Capital Asset Pricing Model (CAPM)

Big Idea

- Expected return depends only on systematic risk (beta)

$$E(R_i) - R_f = \beta_i(E(R_m) - R_f)$$

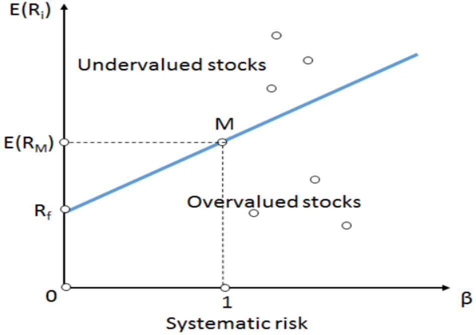
Components

- R_f: Risk-free rate (e.g., US Treasury bills)
- β_i: Systematic risk measure
- $E(R_m) - R_f$: Market risk premium

Key Predictions

- Linear relationship between beta and expected return
- Alpha (α) should be zero for all assets in equilibrium
- Only systematic risk is priced—diversifiable risk earns no premium

CAPM: Security Market Line (SML)



Key Point: All assets should lie on the SML in equilibrium

What the SML shows:

- x-axis: Beta (systematic risk); y-axis: Expected return
- Slope: Market risk premium; Intercept: Risk-free rate
- Assets above SML are undervalued; below are overvalued

CAPM: Strengths and Limitations

Strengths

- Simple, intuitive framework
- Single factor (market), easy to measure
- Widely used in practice for cost of capital calculations
- Provides clear testable predictions

Empirical Problems

- Weak relationship between beta and returns in actual data
- Size effect: Small stocks outperform prediction
- Value effect: High book-to-market stocks outperform
- Momentum: Past winners continue winning (short-term)

Theoretical Issues

- Restrictive assumptions (no taxes, perfect information, etc.)
- Market portfolio not observable (Roll's Critique)

Beyond CAPM: Multi-Factor Models

Problem

- CAPM doesn't fully explain cross-sectional variation in returns
- Other factors beyond market risk matter
- Need richer framework to capture multiple sources of systematic risk

Solution

- Extend to multiple risk factors

General Form:

$$E(R_i) - R_f = \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + \cdots + \beta_{iK}\lambda_K$$

where:

- β_{ik}: Asset i's loading (exposure) on factor k
- λ_k: Risk premium for factor k
- This form is justified by Arbitrage Pricing Theory (to be discussed)

Fama-French Three-Factor Model

Model

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{im}(R_{m,t} - R_{f,t}) + \beta_{iSMB}SMB_t + \beta_{iHML}HML_t + \varepsilon_{i,t}$$

Three Factors

- Market Factor $(R_{m,t} - R_{f,t})$: Excess return on market
- SMB (Small Minus Big): Size factor
  - Return difference between small & large cap stocks – captures size premium
- HML (High Minus Low): Value factor
  - Return difference between high & low book-to-market stocks – captures value premium

Key Insight

- Three factors explain about 95% of diversified portfolio returns
- Much better than CAPM alone (which explains about 70%)

Further Extensions

Fama-French Five-Factor Model (2015)

- Market, Size, Value (from three-factor)
- $+$ Profitability factor (RMW: Robust/high Minus Weak/low)
  - Robust (high) profitability firms earn higher returns
- $+$ Investment factor (CMA: Conservative Minus Aggressive)
  - Conservative (low) investment firms earn higher returns

Carhart Four-Factor Model (1997)

- Fama-French three factors
- $+$ Momentum factor (WML: Winners Minus Losers)
  - Past winners outperform in short/medium term

Other Factor Models

- Macro-economic factors (GDP growth, inflation, term spread, default spread)
- Statistical factors (PCA-based, extracted from returns)

Factor Models: Practical Applications

Risk Management

- Decompose portfolio risk by factor exposures
- Monitor and control factor tilts
- Understand sources of portfolio volatility

$$\sigma_p^2 = \sum_{k=1}^{K} \beta_{pk}^2 \sigma_k^2 + \sigma_{\varepsilon,p}^2$$

Performance Attribution

- Decompose returns into factor exposures vs idiosyncratic
- Identify sources of outperformance (skill vs factor exposure)
- Calculate risk-adjusted performance (alpha)

Portfolio Construction

- Factor tilting strategies (overweight certain factors)
- Risk budgeting by factors
- Factor timing (varying exposures over time)

Big Picture

Central Question in Asset Pricing

- Why do some assets earn higher returns than others?

Historical Evolution

- Equilibrium models attribute return differentials to risk (Sharpe 1964; Ross 1976)
- Early efforts focused on plausible risk sources:
  - Market fluctuations (Sharpe 1964)
  - Consumption risk (Breeden 1979)
  - Macroeconomic factors (Chen, Roll, and Ross 1986)
  - Statistical risk components (Connor and Korajczyk 1988)

Modern Approach

- More prolific: Identify empirical return predictors first
- Then infer their risk implications
- Firm characteristics robustly explain returns

Characteristics vs Factors

Empirical Success of Characteristics

- Firm characteristics explain cross-sectional return variation
  - Size, book-to-market, profitability, investment, momentum
  - Fama & French 1992, 2015; Harvey, Liu, & Zhu 2016; Chen & Zimmermann 2022

Challenges

- How to translate characteristics into risk-based models?
- Factor construction is critical
- Need theoretical foundation for why these factors should be priced

Two Giants Provided the Foundation:

- Stephen Ross: Arbitrage Pricing Theory
- Harry Markowitz: Mean-Variance Analysis

Arbitrage Pricing Theory: Stephen Ross

Stephen Ross (1944-2017)

Pioneered APT in 1976, revolutionizing asset pricing theory

Essence of APT

Assumption: Asset returns follow a factor structure

$$r_i = \mu_i + \beta_{i1} f_1 + \cdots + \beta_{iK} f_K + e_i$$

where

- r_i: excess return from asset i with $\mathbb{E}(r_i) = \mu_i$
- f_k: systematic factor k with $\mathbb{E}(f_k) = 0$ (w.l.o.g.)
- e_i: idiosyncratic error with $\mathbb{E}(e_i) = 0$
- β_{ik}: loading of asset i on factor k

Conclusion: Under no-arbitrage, expected returns satisfy

$$\mu_i = \beta_{i1}\lambda_1 + \cdots + \beta_{iK}\lambda_K$$

for some constants $\lambda_1, \ldots, \lambda_K$ (risk premia for each factor)

Key Insight: Expected returns determined entirely by factor loadings

APT: Risk Premia Interpretation

Rewriting the Factor Model

- We can express returns as:

$$r_i = \beta_{i1}\big(f_1 + \lambda_1\big) + \cdots + \beta_{iK}\big(f_K + \lambda_K\big) + e_i$$

where λ_k is the risk premium for factor f_k

Vector Notation

- Let $\boldsymbol{\mu} = N$-vector of expected returns $(\mu_1, \ldots, \mu_N)'$
- Let $\boldsymbol{\beta}_k = N$-vector of loadings $(\beta_{k1}, \ldots, \beta_{kN})'$
- Then APT claims:

$$\boldsymbol{\mu} = \boldsymbol{\beta}_1 \lambda_1 + \cdots + \boldsymbol{\beta}_K \lambda_K$$

i.e., $\boldsymbol{\mu}$ lies in the span of $\{\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_K\}$

APT: No-Arbitrage Argument

Proof by Contradiction

- Suppose $\boldsymbol{\mu}$ is not in the span of $\{\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_K\}$
- Then we can write:

$$\boldsymbol{\mu} = \boldsymbol{\alpha} + \boldsymbol{\beta}_1 \lambda_1 + \cdots + \boldsymbol{\beta}_K \lambda_K$$

  where $\boldsymbol{\alpha}$ is orthogonal to all $\boldsymbol{\beta}_k$

- Construct portfolio: $\mathbf{w} = (1/N)\boldsymbol{\alpha}$
- This portfolio has:
  - Expected return: $\mathbf{w}'\boldsymbol{\mu} = (1/N)\boldsymbol{\alpha}'\boldsymbol{\alpha} > 0$
  - Zero factor exposure: $\mathbf{w}'\boldsymbol{\beta}_k = 0$ for all k
  - Vanishing idiosyncratic risk as $N \to \infty$
- This is an arbitrage: positive return with no risk!
- Contradiction $\Rightarrow$ $\boldsymbol{\mu}$ must be in span of factors

Practical Implication of APT

Core Message

- Find systematic factors that explain expected returns $\boldsymbol{\mu}$
- Any return variation not explained by factors represents arbitrage

This Guides

- Factor Selection: Which factors should we include?
- Model Testing: Does our model eliminate arbitrage opportunities?
- Strategy Development: How to exploit deviations from APT?

Connection to Multi-Factor Models

- Fama-French and other models are implementations of APT
- They propose specific factors (SMB, HML, etc.)
- Test whether these factors satisfy APT's predictions

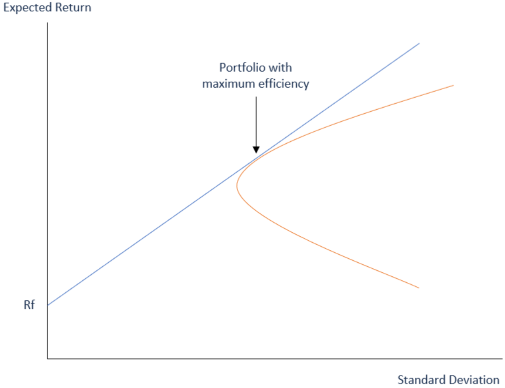
Mean-Variance Analysis: Harry Markowitz

Harry Markowitz (1927-2023)

Nobel Prize 1990 for Modern Portfolio Theory

Essence of Mean-Variance Analysis

The Efficient Frontier



Key Result

- Optimal portfolios lie on the efficient frontier
- Tangency portfolio maximizes Sharpe ratio: $\frac{E[r_p]-r_f}{\sigma_p}$

MVA: Pricing Implications

Maximum Sharpe Ratio Portfolio

- In equilibrium, this portfolio prices all assets
- Expected return of any asset i is proportional to its covariance with this portfolio:

$$\mathbb{E}(r_i) \propto \text{Cov}(r_i, r_{SR})$$

where r_{SR} is the return on the max Sharpe ratio portfolio

Connection to CAPM

- When the market portfolio is mean-variance efficient, it is the max SR portfolio
- This gives us CAPM: $\mathbb{E}(r_i) = \beta_i \lambda_m$ where $\beta_i = \frac{\text{Cov}(r_i, r_m)}{\text{Var}(r_m)}$

Key Insight: Assets are priced by their contribution to portfolio risk, not their individual risk

Implications of APT + MVA

The Combined Framework

- APT: Expected returns lie in span of factor loadings
- MVA: Assets priced by covariance with max SR portfolio
- Together: Factor portfolios should maximize Sharpe ratios

This Guides Empirical Asset Pricing

(1) Identify systematic factors related to average returns
(2) Construct portfolios that maximize Sharpe ratio of those factors
(3) These factor portfolios should explain the entire market

Practical Strategy

- Find characteristics that predict returns (size, value, etc.)
- Build long-short portfolios based on these characteristics
- These become your factors (SMB, HML, etc.)
- Test if they satisfy APT (no arbitrage) and MVA (max SR)

How Do We Evaluate Factor Models?

Key Question

- Which factor model best explains asset returns?

Evaluation Criteria

- Pricing Errors (Alphas): Should be close to zero
- Cross-Sectional R^2: How well factors explain average returns
- Statistical Tests: GRS test, t-statistics on alphas
- Economic Interpretation: Do factors make economic sense?

Implementation:

- Time-series regressions → estimate betas (factor loadings)
- Cross-sectional regressions → estimate risk premia λ
- Compare models on pricing errors and explanatory power

Evaluating Models: Time-Series Regressions

Step 1: Estimate Factor Loadings

For each asset i, run the regression:

$$r_{i,t} = \alpha_i + \boldsymbol{\beta}_i' \mathbf{f}_t + \varepsilon_{i,t}$$

- $r_{i,t}$: excess return on asset i at time t
- $\mathbf{f}_t = (f_{1t}, \ldots, f_{Kt})'$: vector of factor returns
- $\boldsymbol{\beta}_i = (\beta_{i1}, \ldots, \beta_{iK})'$: factor loadings
- α_i: pricing error (should be zero if model is correct)

What We Learn

- Each asset's exposure to systematic factors
- Whether assets have significant alphas (mispricing)
- Time-series fit (R^2): How much variance explained by factors?

Evaluating Models: Pricing Errors

Average Pricing Errors Across Assets

$$\text{Mean Absolute Alpha} = \frac{1}{N}\sum_{i=1}^{N} |\hat{\alpha}_i|$$

or

$$\text{Root Mean Squared Alpha} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} \hat{\alpha}_i^2}$$

GRS Test (Gibbons, Ross, Shanken 1989)

- Tests joint hypothesis: $H_0 : \alpha_1 = \cdots = \alpha_N = 0$
- Statistic follows F-distribution under the null
- Reject $\rightarrow$ model has systematic pricing errors

Interpretation: Lower pricing errors $\Rightarrow$ Better model

Evaluating Models: Cross-Sectional Fit

Fama-MacBeth Two-Step Procedure

Step 1: Time-Series Regressions (as before)

- For each asset: $r_{i,t} = \alpha_i + \boldsymbol{\beta}_i' \mathbf{f}_t + \varepsilon_{i,t}$
- Obtain $\hat{\boldsymbol{\beta}}_i$ for each asset $i = 1, \ldots, N$

Step 2: Cross-Sectional Regression

$$\bar{r}_i = \lambda_0 + \boldsymbol{\lambda}' \hat{\boldsymbol{\beta}}_i + \zeta_i$$

- $\bar{r}_i = \frac{1}{T} \sum_{t=1}^{T} r_{i,t}$: time-series average return of asset i
- $\hat{\boldsymbol{\lambda}} = (\hat{\lambda}_1, \ldots, \hat{\lambda}_K)'$: estimated risk premia
- This tells us: Do assets with higher factor loadings earn higher returns?

Cross-Sectional R^2

Measuring Explanatory Power

$$R^2_{cross} = 1 - \frac{\sum_{i=1}^{N} (\bar{r}_i - \hat{r}_i)^2}{\sum_{i=1}^{N} (\bar{r}_i - \bar{r})^2}$$

where $\hat{r}_i = \hat{\lambda}_0 + \hat{\boldsymbol{\lambda}}' \hat{\boldsymbol{\beta}}_i$ is the model's prediction

Interpretation

- Measures how well factor loadings explain cross-sectional return variation
- Higher $R^2 \Rightarrow$ factors better explain why different assets earn different returns
- Typical values: CAPM $\sim 70\%$, Fama-French 3-factor $\sim 90\%$

Example: If $R^2 = 0.90$, then 90% of the variation in average returns across assets is explained by their factor loadings

Model Comparison Example

Hypothetical Results

Model	Mean $|\alpha|$	R^2_{cross}	GRS p-value
CAPM	0.40%	0.70	0.01
Fama-French 3-factor	0.20%	0.95	0.18
Fama-French 5-factor	0.15%	0.96	0.25

Interpretation

- CAPM rejected (low p-value), significant pricing errors
- FF3 much better: lower alphas, higher R^2, not rejected
- FF5 marginal improvement over FF3

Trade-off: More factors $\rightarrow$ better fit, but increased complexity and overfitting risk

Stochastic Discount Factor (SDF)

Fundamental Pricing Equation

$$1 = \mathbb{E}(m_{t+1} R_{i,t+1})$$

- m_{t+1}: stochastic discount factor (SDF), aka pricing kernel
- $R_{i,t+1}$: gross return on asset i from t to $t+1$
- This says: \$1 today = expected discounted payoff tomorrow

For Excess Returns

$$\mathbb{E}(m_{t+1} r_{i,t+1}) = 0 \quad \text{for all assets } i$$

where $r_{i,t+1} = R_{i,t+1} - R_{f,t+1}$ is the excess return

Key Insight:

- The SDF is the "weight" that prices all assets correctly
- Under no arbitrage, there exists a positive SDF
- Different asset pricing models specify different forms for m

Linear SDF and Factor Models

Linear SDF Representation

$$m_{t+1} = 1 - \boldsymbol{\lambda}'\mathbf{f}_{t+1}$$

- $\mathbf{f}_{t+1}$: vector of factor realizations
- $\boldsymbol{\lambda}$: vector of factor prices (risk premia)

Connection to Factor Models

- Start with pricing condition: $\mathbb{E}(m_{t+1} r_{i,t+1}) = 0$
- Assume factor structure: $r_{i,t+1} = \boldsymbol{\beta}_i'\mathbf{f}_{t+1} + \varepsilon_{i,t+1}$
- Plug in linear SDF: $\mathbb{E}[(1 - \boldsymbol{\lambda}'\mathbf{f}_{t+1})(\boldsymbol{\beta}_i'\mathbf{f}_{t+1} + \varepsilon_{i,t+1})] = 0$

Result:

$$\mathbb{E}(r_{i,t+1}) = \boldsymbol{\beta}_i'\boldsymbol{\lambda}$$

This is exactly the multi-factor pricing equation!

Understanding the Result

Why Does $\mathbb{E}(r_{i,t+1}) = \boldsymbol{\beta}_i'\boldsymbol{\lambda}$?

Starting from: $\mathbb{E}[(1 - \boldsymbol{\lambda}'\mathbf{f}_{t+1})(\boldsymbol{\beta}_i'\mathbf{f}_{t+1} + \varepsilon_{i,t+1})] = 0$

Expand:

$$\mathbb{E}(\boldsymbol{\beta}_i'\mathbf{f}_{t+1}) - \mathbb{E}(\boldsymbol{\lambda}'\mathbf{f}_{t+1} \cdot \boldsymbol{\beta}_i'\mathbf{f}_{t+1}) + \mathbb{E}(\varepsilon_{i,t+1}) - \mathbb{E}(\boldsymbol{\lambda}'\mathbf{f}_{t+1}\varepsilon_{i,t+1}) = 0$$

Under the assumptions

- $\mathbb{E}(\varepsilon_{i,t+1}) = 0$ (idiosyncratic error has zero mean)
- $\mathbb{E}(\mathbf{f}_{t+1}\varepsilon_{i,t+1}) = 0$ (factors uncorrelated with idiosyncratic risk)
- $\mathbb{E}(\mathbf{f}_{t+1}\mathbf{f}_{t+1}') = \boldsymbol{\Sigma}_f$ (factor covariance matrix)

it follows

$$\mathbb{E}(r_{i,t+1}) = \boldsymbol{\beta}_i'\mathbb{E}(\mathbf{f}_{t+1}\mathbf{f}_{t+1}')\boldsymbol{\lambda} = \boldsymbol{\beta}_i'\boldsymbol{\Sigma}_f\boldsymbol{\lambda} \equiv \boldsymbol{\beta}_i'\boldsymbol{\lambda}^*$$

where $\boldsymbol{\lambda}^* = \boldsymbol{\Sigma}_f\boldsymbol{\lambda}$ are the factor risk premia

Why the SDF Framework Matters

Theoretical Advantages

- Unifying framework: All asset pricing models are special cases
- Weaker assumptions: Based on no-arbitrage, not utility maximization
- Rigorous foundation: Provides formal justification for factor models
- Hansen-Jagannathan bounds: Limits on SDF volatility

Practical Applications

- Guides factor selection: Which factors should enter the SDF?
- Model testing: Does proposed SDF price all assets?
- Strategy evaluation: Does strategy exploit SDF mispricing?

Connection to Maximum Sharpe Ratio

- The SDF is proportional to the return on max SR portfolio
- This links back to Markowitz mean-variance analysis

Fundamental vs Technical Analysis

Fundamental Analysis (Factor-Based)

- Grounded in economic theory and asset pricing models
- Uses firm characteristics: value, size, profitability, investment
- Based on factors that explain cross-sectional returns
- Focus: Why assets should earn different returns
- E.g.: Long-short portfolios, factor tilting, smart beta

Technical Analysis

- Based on price patterns, trends, and market microstructure
- Uses chart patterns, moving averages, volume, momentum indicators
- Does not rely on asset pricing theory
- Focus: When to enter/exit positions
- Trend following, mean reversion, breakout strategies

Key Difference: Fundamental asks "Why?"; Technical asks "When?"

Bridge to This Afternoon

This afternoon, Joon will demonstrate EML Lab's trading strategies.

What You'll See:

- Primarily technical analysis strategies
  - Price patterns and trend identification
  - Market microstructure signals
  - Machine learning applied to technical indicators
- One alpha strategy using factor models
  - Based on the theoretical framework we discussed
  - Combines fundamental factors with customized ML

The EML Lab Advantage:

- Customized ML tools designed for financial data to deal with weak signals and high noise effectively
- Apply sophisticated econometric methods to both approaches

Summary

What We Covered

- Risk & Return: Only systematic risk is priced in equilibrium
- CAPM: Single-factor model using market beta
- Multi-Factor Models: Fama-French and extensions
- APT & MVA: Theoretical foundations (Ross & Markowitz)
- Model Evaluation: How to test and compare factor models
- SDF Framework: Unified approach to asset pricing
- Two Approaches: Fundamental vs technical trading

Key Takeaways

- Factor models explain returns through systematic risk exposures
- APT and MVA provide rigorous theoretical foundation
- Model valuation requires both statistical tests and economic interpretation
- Different approaches use these foundations in different ways

Next: Joon demonstrates how EML Lab implements these ideas!