1. Give the multiplication tables for S_3 and Δ_3.

We know that the Symmtric Group of Degree 3 has 3! = 6 elements.

Then let us define S_3 on the set Z_3 as follows.

primary_table: { '1,2,3': 'a', '1,3,2': 'b', '2,3,1': 'c', '2,1,3': 'd', '3,1,2': 'e', '3,2,1': 'f' },

a	b	c	d	e	f
b	a	d	c	f	e
c	f	e	b	a	d
d	e	f	a	b	c
e	d	a	f	c	b
f	c	b	e	d	a

Likewise, Δ_3 is the finite set {1, R, R^2,D_1,D_2,D_3}

1 = {1,2,3}

R = {3,1,2}

R^2= {2,3,1}

D_1= {1,3,2}

D_2= {3,2,1}

D_3 = {2,1,3}

1	R	R^2	D_1	D_2	D_3
R	R^2	1	D_3	D_1	D_2
R^2	1	R	D_2	D_3	D_1
D_1	D_2	D_3	1	R	R^2
D_2	D_3	D_1	R^2	1	R
D_3	D_1	D_2	R	R^2	1

2. Exhibit an isomorphism of the additive groups $Z_6$and Z_2xZ_3.

In other words, show that there is a function f:$\{Z_6,+,1\}$-> $\{Z_2xZ_3,+',1\}$ such that f(x+y) = f(x)+'f(y) and f is bijective.

We can construct a simple function table as follows

Z_6	Z_2xZ_3
0	0x0
1	0x1
2	0x2
3	1x0
4	1x1
5	1x2

So f(n) = q x r for n = q(3)+r

Clearly, this function is bijective.

Now we have to show that it is a morphism. We can do this informally.

First, observe that the ρ function defined in Chapter 1,Theorem 16 is a morphism.

But, all f is is a function that carries Z_6into it's remainder and quotient mod 3.

So, f is a morphism.

Thus, f is bijective, as required.

4. Show that the group $Z_2 x Z_2$is not isomorphic to Z_4but is isomorphic to the group of all symmetries of a rectangle.

First, we must show that there is no function f:$\{Z_2 x Z_2, +, 1\} -> \{Z_4, +', 1\}$ such that f is bijective and f(x+y) = f(x)+'f(y).

First, we can observe that +' is not bijective– (a+a) = (b+b) = (c+c) = 0

Then, suppose we have an f that is bijective and morphism.

Suppose in our biject that f(x) = 0, f(y) = a, f(z) = b, f(i) = c, for some x,y,z,i in Z_4

Then f(y+y) = f(z+z) = f(i+i) = f(x) = 0

Then (y+y) = (z+z) = (i+i) = x, since f is bijective

But for + in Z_4only two pairs of numbers add to the same sum, 2 and 4

Then y = z or z = i or y=i. either choice is absurd.

Therefore f cannot be both bijective and a morphism, as required.

For the second part–I don't see how there can be an isomorphism between the two groups, since the domain $Z_2 x Z_2$hsa four elements, while the codomain of the all symmetries of a rectangle has eight elements.