Cyclic Groups

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1. The generators of a cyclic group Z_n are 1 and the elements coprime to n. The number of generators then is Euler's totient $\phi(n)$. For Z_6, the generators are 1 and 5, and $\phi(6) = 2$.

2. For each i in Z_n, $if(x + y) = ifx + ify \pmod{n}$ gives an endomorphism $x \mapsto ix$. For any m not in Z_n, $x \mapsto mx$ is the same morphism $x \mapsto jx$ where $j \equiv m \bmod n$.

3. Since 5 is prime, 1, 2, 3, and 4 are coprime. For any prime n, Z_n has $\phi(n) = n - 1$ generators.

4. 1, 3, 5, 9, 11, and 13 are coprime to 14.

5. 1 and -1 generate Z. Remember that a g is a generator for a group G if for all $x \in G$, $g^y = x$ for some $y \in Z$.