

3.1 List all the possible generators of the cyclic group Z_6 .

1($1^0 = 1, 1^1 = 1, 1^2 = 2, 1^3 = 3, 1^4 = 4, 1^5 = 5, 1^6 = 0$)

5($5^0 = 1, 5^1 = 5, 5^2 = 4, 5^3 = 3, 5^4 = 2, 5^5 = 1, 5^6 = 0$)

3.2 Show that group Z_n has exactly n endomorphisms.

To put this more clearly, we must show that Z_n has exactly n functions $f: Z_n \rightarrow Z_n$ such that $f(x+y) = f(x)+f(y)$, assuming Z_n is an additive group.

How many functions are distributive over addition in Z_n ? Just multiplication, which is a binary operator.

So the number of functions $f(n)$ which multiply a number n by a fixed amount constitute the number of endomorphisms in Z_n . But there can only be n such functions.

3.3 Show that Z_5 is generated by any element not the identity.

In general Z_n can be generated by any element $x \in Z_n$ such that it's not the case that $x \equiv n(mod n)$.

Thus, 2, 3, 4 can all be used to generate Z_5 .

3.4 Show that the cyclic group Z_{14} can be generated by any one of six generators.

1, 3, 4, 5, 6, 8—why won't 9, 10, 11, 12, 13 also generate it?

13(0, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1)