Subgroups

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1. Recall the four group $Z_2 \times Z_2$:

+	0,0	0,1	1,0	1,1
0,0	0,0	0,1	1,0	1,1
0,1	0,1	0,0	1,1	1,0
1,0	1,0	1,1	0,0	0,1
1,1	1,1	1,0	0,1	0,0

   Now denote the elements of Δ_6 by its rotational symmetries: $\{1, R, R^2, R^3, R^4, R^5\}$ and its flip symmetries: $\{D_1, ..., D_6\}$. If the vertices are labeled from 1 to 6 clockwise, consider the diagonal from 2 to 5. The symmetries that preserve the diagonal are $1, R^3, D_2, D_5$. The forms a subgroup isomorphic to $Z_2 \times Z_2$ with $1 \leftrightarrow (0,0)$, $R^3 \leftrightarrow (0,1)$, $D_2 \leftrightarrow (1,0)$ and $D_5 \leftrightarrow (1,1)$.

2.

   (a) The elements of Δ_3 are $\{1, R, R^2, D_1, D_2, D_3\}$. Its non-trivial subgroups are $\{1, R, R^2\}$, $\{1, D_1\}$, $\{1, D_2\}$ and $\{1, D_3\}$, which are mutually exclusive. $S_3 \cong \Delta_3$.

   (b) $2Z_{18} \subset Z_{18}$, $3Z_{18} \subset Z_{18}$,
   $6Z_{18} \subset 2Z_{18}$, $6Z_{18} \subset 3Z_{18}$,
   $9Z_{18} \subset 3Z_{18}$,
   $18Z_{18} \subset 6Z_{18}$, $18Z_{18} \subset 9Z_{18}$