Ocean Halos Model Proof

Model description

We use a two-patch discrete-time meta population bioeconomic model to illustrate the interactions between enforcement, illegal fishing, and revenue generated from MPA spillover in a reserve system. Our system is divided into two patches, where patch F is the general fishing area and patch R is the reserve. Vessels do not need to pay in order to fish in F, and the fishing effort in this patch is assumed to be in open-access. Some fraction, L, of the patch R can be designated as a lease area, where vessels must pay an access fee in order to fish. The remaining fraction, N, is designated as a no-take area ($N = 1 - L$). Fishing effort in the lease area is driven by biomass available for harvest and the per-unit-effort access fee for fishing. We also assume that illegal fishing occurs in the reserve, which is driven by biomass in the reserve and the per-unit-effort expected fine associated with being caught fishing illegally. The stock of fish in each area is stationary within a single year, but dispersal between the patches occurs at the beginning of each time step.

Analytical proof

Fishery Dynamics

Revenue for vessels in patch F is given by pqE_FX_F where p the price of fish, q is catchability, E_F is fishing effort (days) in patch F, and X_F is stock size. The cost of fishing for vessels in patch F is given by cE_F, where c is the variable cost of fishing, and $\beta = 1.3$ matches commonly used cost-coefficients (values of $\beta > 1$ imply that additional units of effort are increasingly costly to apply). Profits from fishing in patch F are given by:

$$\Pi_F(E_F, X_F) = pqE_FX_F - cE_F^{\beta}$$

In patch R, in the absence of a lease area, the only revenue for vessels comes from illegal fishing. This revenue is given by pqE_IX_R, where E_I is illegal fishing effort (days) and X_R is stock size. The cost of illegally fishing in patch R is given by $cE_I^{\beta} + \theta E_I \psi$, where θ is the probability of detecting each unit of illegal fishing effort and ψ is the per-unit-effort fine. Together $\theta\psi$ is the per-unit-effort expected fine for fishing illegally.

If some fraction of the reserve, L, is designated as a lease area, revenue for vessels fishing in the lease area is given by pqE_FX_RL. The cost of fishing in the lease area is given by $cE_L + \chi E_L$, where χ is the per-unit-effort access fee. The parameterization of lease area size implies that profit from fishing in the lease area is given by:

$$\Pi_L(E_L, X_R, L) = pqE_LX_RL - cE_L^{\beta} - \chi E_L$$

Upon implementation of a lease area, we assume that vessels fishing in the lease area will self enforce, and illegal fishing therefore only persists in the no-take area of the reserve. The profit from fishing illegally in the reserve can then be generalized to:

$$\Pi_I(E_I, X_R, L) = pqE_IX_R(1 - L) - cE_I^{\beta} - \theta E_I \psi$$

We can derive the marginal profits for fishing in each area by taking the derivatives of the equations above:

$$\pi_F(E_F) = \frac{\partial \Pi_F}{\partial E_F} = pqX_F - \beta cE_F^{\beta-1}$$

$$\pi_L(E_L) = \frac{\partial \Pi_L}{\partial E_L} = pqX_R L - \beta c E_L^{\beta-1} - \chi$$
$$\pi_I(E_I) = \frac{\partial \Pi_I}{\partial E_I} = pqX_R(1-L) - \beta c E_I^{\beta-1} - \theta\psi$$

Assuming that effort will continue to enter each area until the profit from the last unit of effort is exactly zero, we can solve for equilibrium effort in each a rea:

$$E_F^* = \left(\frac{pqX_F}{\beta c}\right)^{\frac{1}{\beta-1}}$$
$$E_L^* = \left(\frac{pqX_R L - \chi}{\beta c}\right)^{\frac{1}{\beta-1}}$$
$$E_I^* = \left(\frac{pqX_R(1-L) - \theta\psi}{\beta c}\right)^{\frac{1}{\beta-1}}$$

Stock Dynamics

Harvest in each area is determined by fishing effort and stock size. Harvest from fishing in patch F (H_F) is given by $qE_F X_F$, harvest from fishing in the lease area (H_L) is given by $qE_L X_R L$, and harvest from illegal fishing in the reserve (H_I) is given by $qE_I X_R(1-L)$. Therefore, total harvest is given by:

$$\overline{H}(E_F, E_L, E_I, X_F, X_R, L) = H_F + H_L + H_I = qE_F X_F + qE_L X_R L + qE_I X_R(1-L)$$

The stock grows logistically according to the Gordon-Schaefer model such that growth depends on the total initial biomass in time t ($\overline{X}_t$), the intrinsic growth rate of the stock (r), and the environmental saturation level or carrying capacity (K), expressed as maximum equilibrium biomass the environment can hold:

The total initial biomass in time t in our model is given by $X_{t,F} + X_{t,R}$. Therefore total biomass in time $t+1$ is:

$$\overline{X}_{t+1} = \overline{X}_t + r\overline{X}_t\left(1 - \frac{\overline{X}_t}{K}\right) - \overline{H}_t$$

After the stock grows, a constant fraction, f, of the total stock redistributes to the reserve, and the remainder $(1-f)$ redistributes to the general fishing area. Thus biomass in time period $t+1$ in the reserve and in the fishing area respectively are given by:

$$X_{t+1,R} = f\overline{X}_{t+1}$$

$$X_{t+1,F} = (1-f)\overline{X}_{t+1}$$

Equilibrium biomass as a function of lease-area size

In equilibrium, we assume that total biomass must be unchanging. Thus, $\overline{X}_{t+1} = \overline{X}_t = \overline{X}^*$. Solving for the steady state harvest then yields:

$$\overline{H^*} = r\overline{X^*}\left(1 - \frac{\overline{X^*}}{K}\right)$$

Since total harvest is equal to the sum of the harvests in the three areas, we can rewrite this as

$$qE_F^*X_F^* + qE_L^*X_R^*L + qE_I^*X_R^*(1-L) = r\overline{X^*}\left(1 - \frac{\overline{X^*}}{K}\right)$$

Substituting for biomass in each area in terms of total biomass, we then get

$$qE_F^*\overline{X^*}(f-1) + qE_L^*\overline{X^*}fL + qE_I^*\overline{X^*}f(1-L) = r\overline{X^*}\left(1 - \frac{\overline{X^*}}{K}\right)$$

Simplifying and substituting in each of our equilibrium efforts written in terms of total biomass then gives

$$(1-f)\left(\frac{pq\overline{X^*}(1-f)}{\beta c}\right)^{\frac{1}{\beta-1}} + fL\left(\frac{pq\overline{X^*}fL-\chi}{\beta c}\right)^{\frac{1}{\beta-1}} + f(1-L)\left(\frac{pq\overline{X^*}f(1-L)-\theta\psi}{\beta c}\right)^{\frac{1}{\beta-1}} = \frac{r}{q}\left(1 - \frac{\overline{X^*}}{K}\right)$$

We can set this equal to 0:

$$(1-f)\left(\frac{pq\overline{X^*}(1-f)}{\beta c}\right)^{\frac{1}{\beta-1}} + fL\left(\frac{pq\overline{X^*}fL-\chi}{\beta c}\right)^{\frac{1}{\beta-1}} + f(1-L)\left(\frac{pq\overline{X^*}f(1-L)-\theta\psi}{\beta c}\right)^{\frac{1}{\beta-1}} - \frac{r}{q}\left(1 - \frac{\overline{X^*}}{K}\right) = 0$$