## **Contents**

- 1 Ablation study of IPNN training with and without maximizing the robust margin  $\gamma$  1
- 2 Illustrative Examples where H-Proj fails while B-Proj works 1
- 3 Illustrative Example of Benefits of "Central" Interior Points

## 1. Ablation study of IPNN training with and without maximizing the robust margin $\gamma$

To demonstrate the effectiveness of the proposed IPNN training with eccentricity minimization (approximated by maximizing  $\gamma$ ), we conduct the following ablation study.

We add random noise to the NN prediction to create more infeasible test cases (10,000 points). Given different initial infeasible NN predictors (with varying constant violations), we apply the IPNN trained with and without maximizing  $\gamma$  to conduct the bisection projection.

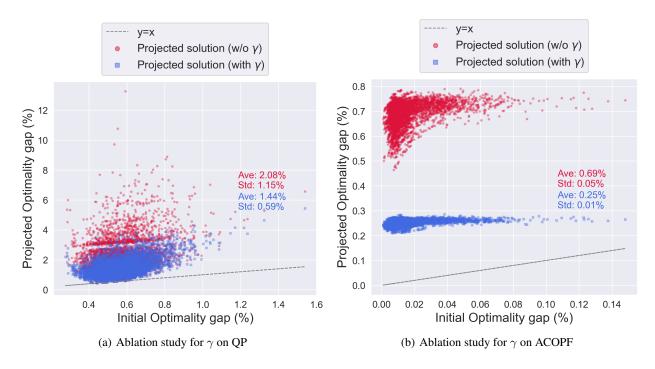
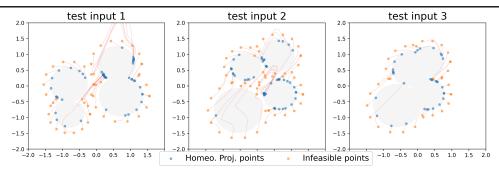


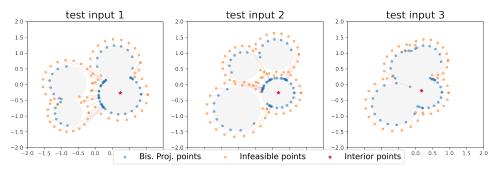
Figure 1: We apply IPNN with or without  $\gamma$  for different initial NN predictions. Results show: (i) the average performance and variance for projection to IPNN (with  $\gamma$ ) is lower than projection to IPNN (w/o  $\gamma$ ), and (ii) the worst-case of projection to IPNN (w/o  $\gamma$ ) significantly deviates from the initial prediction (reaching a **worst-case gap of 12**% for QP), while projection with IPNN (with  $\gamma$ ) has a **worst-case gap of only about 5**% for QP.

## 2. Illustrative Examples where H-Proj fails while B-Proj works

To visualize the failure case of H-Proj over non-ball homeomorphic sets (e.g., disconnected set), we construct a simple constraint set as:  $C_{\theta} = \bigcup_{i=1}^{4} \mathcal{B}(x_i, r_i)$  with input parameter as  $\theta = \{x_i, r_i\}_{i=1}^{4}$ . It is the union of 4 balls that can be disconnected. After training the invertible neural network in H-Proj to approximate the constraint set, we apply the H-Proj to project infeasible solutions. Meanwhile, we also train an IPNN to learn interior points and apply B-Proj for comparison.



(a) H-Proj: the red curve indicates the mapped projection trajectory from bisection over transformed space.



(b) B-Proj: straightforward bisection between the predicted interior point and infeasible solutions.

Figure 2: **Comparison between H-Proj and B-Proj**: The visualization demonstrates that H-Proj struggles with disconnected constraint sets since invertible neural networks cannot properly model non-homeomorphic topologies. In contrast, our IPNN with bisection projection successfully handles these disconnected regions by learning appropriate interior points and projecting them to the feasible boundary.

## 3. Illustrative Example of Benefits of "Central" Interior Points

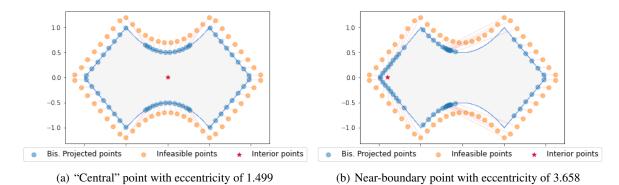


Figure 3: **Bisection projection with different interior points**: The "central" interior point with low eccentricity (a) helps to reduce the worst-case projection distance and preserves a margin to the boundary. In contrast, the near-boundary interior point (b) induces significantly larger worst-case projection distances. This illustration demonstrates why our approach of training IPNN to predict central points with low eccentricity provides more robust performance across diverse constraint scenarios.