# Solving Chance-Constrained AC-OPF Problem by Neural Network with Bisection-based Projection


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## ABSTRACT

Power grid security faces multiple threats, including cyber attacks, renewable generation uncertainty, and load variations. These uncertainties challenge grid operators to maintain both security and efficiency. The chance-constrained AC optimal power flow (CC-ACOPF) problem offers a valuable approach for maintaining grid reliability while optimizing operational costs under complex uncertainties. Despite its importance for reliable grid operation, existing solution methods either compromise model accuracy or face computational barriers that limit their practical implementation in security-critical environments where rapid response is essential. We present a neural network (NN) based approach to efficiently solve CC-ACOPF through two key phases: (i) an approximation phase leveraging NN techniques to obtain initial deterministic AC-OPF solutions; and (ii) a projection phase employs our recently developed bisection-based algorithm to recover solutions satisfying chance constraints with a pre-specified confidence level, leveraging random sampling to evaluate solution feasibility. We establish solution feasibility guarantees and optimality bounds of our approach. Extensive simulations on IEEE 30-/118-/200-bus test systems demonstrate that our method generates feasible CC-ACOPF solutions under diverse settings with minor optimality loss, and achieves a two-order-of-magnitude speedup compared to state-of-the-art alternatives.



## CCS CONCEPTS

• **Hardware → Power networks**.

## KEYWORDS

ACOPF, Chance Constraint, Neural Network, Projection

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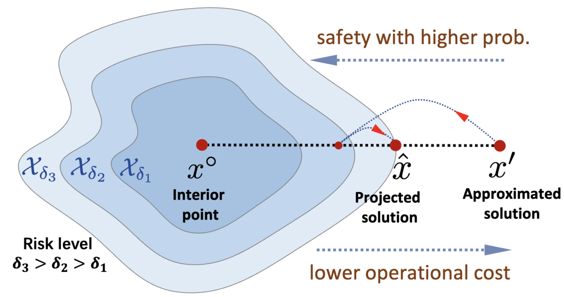


**Figure 1: The approximate-then-project framework.**

## 1 INTRODUCTION

Power grid security is increasingly threatened by different sources of uncertainty, including cyber attacks, renewable generation variation, and load fluctuations. These uncertainties challenge the fundamental task of power system operation - AC optimal power flow (AC-OPF), which determines optimal power generation and voltage settings to maintain both security and economic efficiency.

Approaches for solving AC-OPF under uncertainties fall into several categories: (i) robust formulation, which minimizes worst-case/expected costs while guaranteeing robust constraint satisfaction within defined uncertainty sets [29]; (ii) chance-constrained formulation, which minimizes expected costs while maintaining operational constraints at a specified probability level [39]. Chance-constrained AC-OPF (CC-ACOPF) provides a particularly valuable framework for power systems operation under stochastic uncertainty, e.g., the renewable uncertainty and load variation. It enables operators to manage the risk of constraint violations and maintain sufficient generation reserves to compensate for potential disruptions while avoiding overly conservative operations.

However, non-convex probabilistic constraints make CC-ACOPF computationally challenging [16]. Existing approaches, including *deterministic approximation* (converting probabilistic constraints into a tractable closed-form formulation) [39], and *data-driven methods* (iteratively refining the problem formulation with data) [31], struggle to balance solution robustness with computational efficiency–a critical barrier for rapid response in security-critical environments, see Sec. 2 for discussions.

In this paper, we present an **approximate-then-project** framework to solve CC-ACOPF efficiently. As shown in Fig. 1, our approach consists of two key steps: (i) **Approximation** - obtaining initial approximated solutions by solving a deterministic AC-OPF problem using existing neural-network (NN)-based techniques, e.g., DeepOPF [36, 37, 49], and (ii) **Projection** - applying our bisection-based projection method to adjust these solutions to satisfy chance constraints [22], with a pre-specified confidence level, leveraging random sampling to evaluate solution feasibility. Simulations on IEEE 30-/118-/200-bus test systems demonstrate that our approach generates feasible CC-ACOPF solutions with minor optimality loss while achieving a two-order-of-magnitude computational speedup compared to state-of-the-art iterative methods [33].

While our framework shares conceptual elements with existing *data-driven* tuning approaches [19, 31]. It offers several distinct advantages: applicability across diverse uncertainties, theoretical guarantees, and improved computational efficiency.

## 2 RELATED WORK

**Power system security** ensures reliable grid operation despite disturbances. While traditionally designed with deterministic N-1 security criteria [6], modern grids face evolving challenges on multiple fronts. Cyberattacks represent a growing threat, with the potential to manipulate measurements and trigger cascading failures across the network [14, 15, 27, 43]. Simultaneously, the rapid integration of renewable energy introduces significant generation variability and uncertainty [21, 30, 47], while evolving load patterns from electrification and demand response programs create additional operational complexities [35, 44, 45]. To address stochastic uncertainties effectively, CC-ACOPF models have emerged as powerful tools that enable operators to optimize economic objectives while maintaining robust system performance [30, 39].

**CC-ACOPF:** Current methods for solving CC-ACOPF fall into two primary categories: *deterministic approximation* and *data-driven methods*. Deterministic approximation techniques aim to convert probabilistic constraints into a tractable closed-form formulation using linearization [3, 11], partial linearization [39], polynomial chaos expansion [32], convex approximation [18], convex relaxation [42], and robust formulation [29]. However, this inherent simplification introduces model inaccuracies, potentially leading to sub-optimal or infeasible solutions. Alternatively, data-driven methods iteratively refine the problem formulation of standard ACOPF, either by incorporating critical scenarios [31] or by tightening inequality constraint bounds [19]. This scheme, while preserving a high model accuracy, suffers from a significant computational burden due to the need to solve a refined AC-OPF problem repeatedly.

**Learning-based OPF:** Existing learning-based methods for solving AC-OPF can be divided into two main categories: *learning-aided methods* and *end-to-end methods*. Learning-aided methods accelerate existing iterative solvers through various techniques, such as warm start prediction [4], and active constraint prediction [12, 28]. While effective, these methods offer limited speedup due to their dependence on iterative algorithms. Within the end-to-end scheme, solutions are directly outputted by an NN trained in different ways, such as supervised [23, 36, 37, 49] and unsupervised approaches [13, 20, 38]. To improve the feasibility of NN prediction with respect to equality/inequality constraint, various techniques have been proposed [24, 25, 41, 48]. Learning-based approaches have also been developed for solving stochastic AC-OPF or CC-ACOPF, such as adversarial learning [1], NN-based policies [17], or embedded NN scheme [9, 40]. However, these approaches suffer from model inaccuracy or lack a theoretical performance analysis.

To date, solving CC-ACOPF is marked by a dilemma: balancing model accuracy and computation burden. This dilemma poses a significant challenge to the real-time and secure operations of power systems. In this work, we propose a novel machine-learning scheme for solving CC-ACOPF with a probabilistic feasibility guarantee, minor optimality loss, and low run-time complexity.

**Table 1: Selected controllable and state variables [13, 36, 46].**

| | PV bus ($\mathcal{N}_{pv}$) | PQ bus ($\mathcal{N}_{pq}$) | Slack bus ($\mathcal{N}_s$) |
|---|---|---|---|
| $u$ | $P_i^g, V_i, i \in \mathcal{N}_{pv}$ | – | $\theta_i, V_i, i \in \mathcal{N}_s$ |
| $s$ | $\theta_i, Q_i^g, i \in \mathcal{N}_{pv}$ | $\theta_i, V_i, i \in \mathcal{N}_{pq}$ | $P_i^g, Q_i^g, i \in \mathcal{N}_s$ |

## 3 CHANCE-CONSTRAINED AC-OPF

We consider the following preventive[1] CC-ACOPF problem [39]:

$$\min_{x(\hat{\xi}),\hat{\xi}\in\Omega} \quad \mathbb{E}_\xi\left[f(x(\xi))\right] \tag{1}$$

$$\text{s.t.} \quad h(x(\hat{\xi}),\hat{\xi}) = 0,\ \forall \hat{\xi} \in \Omega \tag{2}$$

$$\mathbb{P}(g(x(\xi),\xi) \le 0) \ge 1-\delta. \tag{3}$$

To model uncertainty, we represent load demand or renewable generation as random variables $\xi \sim P_\omega$, where $P_\omega$ is a probability distribution with parameters $\omega$ and support $\Omega$. We denote a realization of $\xi$ as $\hat{\xi} = [P^d, Q^d] \in \Omega$.

The corresponding decision variable under a realization is $x(\hat{\xi})$. Specifically, the decision variable encompasses the active power generation $P^g$, reactive power generation $Q^g$, voltage magnitude $V$, and voltage phase angle $\theta$. For clarity and for use in subsequent sections, we decouple it as $x(\hat{\xi}) = [u, s(\hat{\xi})]$, where $u$ denotes the control variables and $s(\cdot)$ is the state variables, as enumerated in Table 1. Given control variables $u$ and uncertainty realization $\hat{\xi}$, the state variables $s(\hat{\xi})$ are determined by the power flow equations [32, 36], which we denote as $s(\hat{\xi}) = \mathrm{PF}(u, \hat{\xi})$.

The objective function $f(\cdot)$ in Eq. (1) represents the quadratic cost function for active power generation. CC-ACOPF minimizes the expected cost under uncertainty.

The equality constraints $h(\cdot) = 0$ in Eq. (2) represent the AC power flow equations:

$$P_i^g - P_i^d = \sum_{(i,j)\in\mathcal{L}} P_{ij},\ \ Q_i^g - Q_i^d = \sum_{(i,j)\in\mathcal{L}} Q_{ij},\ \ i \in \mathcal{N}, \tag{4}$$

$$P_{ij} = G_{ij}V_i^2 - V_iV_j(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}),\ \ (i,j)\in\mathcal{L}, \tag{5}$$

$$Q_{ij} = -B_{ij}V_i^2 - V_iV_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}),\ \ (i,j)\in\mathcal{L}, \tag{6}$$

where $\mathcal{N}$ and $\mathcal{L}$ represent the set of buses and transmission lines, respectively. $P^d$ and $Q^d$ denote the active and reactive load at each bus, respectively. $G_{ij}$ and $B_{ij}$ are the real and imaginary parts of the admittance for line $(i,j)$, respectively. $P_{ij}$ and $Q_{ij}$ are the active and reactive branch flow at line $(i,j)$, respectively.

The inequality constraints $g(\cdot) \le 0$ in Eq. (3) are defined as:

$$\underline{P_i^g} \le P_i^g \le \overline{P_i^g},\ \ \underline{Q_i^g} \le Q_i^g \le \overline{Q_i^g},\ \ \underline{V_i} \le V_i \le \overline{V_i},\ \ i \in \mathcal{N}, \tag{7}$$

$$\underline{\theta_{ij}} \le \theta_i - \theta_j \le \overline{\theta_{ij}},\ \ (P_{ij})^2 + (Q_{ij})^2 \le (\overline{S_{ij}})^2,\ \ (i,j)\in\mathcal{L}, \tag{8}$$

[1]OPF models under uncertainty employ either preventive [5] or corrective [7] approaches. Preventive approaches maintain fixed controllable variables while adjusting state variables for power balance, prioritizing system robustness through proactive control. Corrective approaches allow controllable and state variable adjustments within specified bounds, offering greater operational flexibility through real-time adaptation. While we consider preventive formulation in this work, we remark that the proposed framework can also be applied to other formulations.

where Eq. (7) defines the upper and lower bounds for the active generation $P_i^g$, reactive generation $Q_i^g$, and voltage magnitude $V_i$ at each bus $i \in \mathcal{N}$. Eq. (8) defines the bounds for the branch angle difference $\theta_{ij}$ and the branch power flow $S_{ij} = |P_{ij} + \mathrm{i} \cdot Q_{ij}|$ for each branch $(i, j) \in \mathcal{L}$.

The *joint chance constraint* $\mathbb{P}(\cdot) \geq 1 - \delta$ in Eq. (3) requires that all constraints be simultaneously satisfied with a probability of at least $1 - \delta$, where $\delta \in [0, 1)$ represents the acceptable risk level. We denote the feasible region defined by this chance constraint as $\mathcal{X}_\delta$.

**Research gap:** While CC-ACOPF effectively manages load and generation uncertainties in power systems, existing approaches either sacrifice solution quality for speed or require excessive computational resources that prevent practical implementation. To date, solving CC-ACOPF with a balanced approach that maintains both solution feasibility and quality in real-time remains an unresolved challenge.

## 4 APPROXIMATE-THEN-PROJECT

We propose an efficient NN-based scheme for solving CC-ACOPF as shown in Fig. 1. It first leverages existing NN-based methods to solve deterministic AC-OPF problem to obtain an approximated solution to CC-ACOPF in Sec. 4.1; We then apply our recent bisection-based projection [22] to restore feasibility with respect to the CC-ACOPF constraints with pre-specified confidence level in Sec. 4.2.

### 4.1 NN Approximated CC-ACOPF Solution

To get an approximated solution to CC-ACOPF efficiently, we employ existing NN-based approaches to learn the input-to-solution mapping of a deterministic AC-OPF problem, in which the net-load is set as the *mean value* of the uncertainty distribution, denoted as $\bar{\xi}$. The training dataset $\{\bar{\xi}_i, x_i^*\}_{i=1}^N$ is prepared by Matpower [51] through solving problems with randomly sampled loads. We then train a neural network, $\phi(\cdot)$, by minimizing the following loss function:

$$\min_{\phi} \quad \mathcal{L}_\phi = \frac{1}{N} \sum_{i=1}^{N} \left[ \left\| u_i' - u_i^* \right\| + \lambda \left\| [g(x_i', \hat{\xi}_i)]^+ \right\| \right], \tag{9}$$

where $u_i^*$ represents the control variables in training data, and $u_i' = \phi(\bar{\xi}_i)$ is the NN predicted solution. We then calculate the full set of variables under a sampled scenario $\hat{\xi}_i$ as $x_i' = [u_i', \mathrm{PF}(u_i', \hat{\xi}_i)]$. For inequality constraints, we evaluate constraint violations as $\|[g(x_i', \hat{\xi}_i)]^+\|$. The two loss terms are balanced by a positive coefficient $\lambda > 0$.

**Remark.** The NN $\phi$ aims to predict solutions close to the ones in the training data (by minimizing the first loss term), while reducing potential constraint violations under uncertainty (by minimizing the second loss term). We select AC-OPF solutions as training data since they can be solved efficiently by existing methods [36] and empirically yield lower generation costs compared to CC-ACOPF [39]. Moreover, alternative approaches like unsupervised learning schemes [17, 20] can also provide initial approximations, as discussed and compared in Sec. 5.

**Algorithm 1** Bisection for CC-ACOPF Solution Feasibility

**Input:** Uncertainty parameter $\omega$, predicted (infeasible) solution $x' = [u', s']$, risk level $\delta$, and number of samples $K$.
Predict interior point by IPNN: $u^\circ = \psi(\omega)$.
Set total iterations $B$, $\alpha_l = 0$, and $\alpha_u = 1$.
**for** $b = 1$ **to** $B$ **do**
  Bisection: $\alpha_m = (\alpha_l + \alpha_u)/2$.
  Candidate: $\hat{u} = (1 - \alpha_m)\, u^\circ + \alpha_m u'$.
  Sampling: a *new* set of i.i.d. samples $\{\hat{\xi}_k\}_{k=1}^K$.
  Evaluation: $\hat{x}_k = \left[\hat{u},\ \mathrm{PF}\left(\hat{u},\ \hat{\xi}_k\right)\right]$, for $k = 1, \ldots, K$.
  **if** $\left\{\frac{1}{K}\sum_{k=1}^{K} \mathbb{I}\left(\left\|[g\left(\hat{x}_k, \hat{\xi}_k\right)]^+\right\| > 0\right)\right\} \leq \delta$ **then**
    Increase lower bound: $\alpha_l \leftarrow \alpha_m$.
  **else**
    Decrease upper bound: $\alpha_u \leftarrow \alpha_m$.
  **end if**
**end for**
**Output:** Projected solution: $\hat{u} = (1 - \alpha_l)u^\circ + \alpha_l u'$.

### 4.2 Bisection-based Projection for Feasibility

Due to the first-phase approximation errors, the predicted solution typically violates the probabilistic constraints in CC-ACOPF. Existing feasibility recovery approaches, such as orthogonal projection [8], iterative constraint tightening [39], and adding critical scenarios [31], are computationally intensive and thus unsuitable for real-time operations.

Motivated by recent works on NN solution feasibility on ball-homeomorphic sets [24, 25], we recently develop a more general bisection-based projection approach that efficiently restores solution feasibility over non-convex chance constraints (potentially non-ball-homeomorphic) with bounded optimality loss [22]. The key idea is to train another NN to generate interior points of the constraint set under varying input parameters through robust training (Sec.4.2.1). For an infeasible solution, the generated interior point enables efficient projection onto the feasible region via simple bisection (Sec.4.2.2).

*4.2.1 Interior-Point Neural Network (IPNN).* We employ another regular NN, denoted as $\psi$, to predict interior points for the chance constraint set $\mathcal{X}_\delta$. To train the IPNN, we randomly sample different distribution parameters $\{\omega_i\}_{i=1}^M$, e.g., different means and variances for load demand considering Gaussian distributions. We then train $\psi$ following a robust loss function:

$$\min_{\psi} \quad \mathcal{L}_\psi = \frac{1}{M} \sum_{i=1}^{M} \left\| \left[ g(\tilde{x}_i, \hat{\xi}_i) \right]^+ \right\|. \tag{10}$$

For a uncertainty parameter $\omega_i$, IPNN first predicts control variables $u_i^\circ = \psi(\omega_i)$. We then randomly perturb the prediction as $\tilde{u}_i = u_i^\circ + \gamma \cdot z_i$, where $z_i$ is randomly sampled from a unit ball as $\|z_i\| \leq 1$ and $\gamma > 0$ is a positive scalar to adjust the magnitude of the random perturbation. Next, we calculate the full set of variables under a sampled scenario $\hat{\xi}_i$ as $\tilde{x}_i = [\tilde{u}_i, \mathrm{PF}(\tilde{u}_i, \hat{\xi}_i)]$. The constraint violation of perturbed predictions is treated as the loss function in (10).

**Remark.** Intuitively, the loss function derives the IPNN to predict central interior points $u^\circ$ of chance constraint set $\mathcal{X}_\delta$, such

**Table 2: Comparison of different approaches for solving CC-ACOPF on various networks with a risk level of $\delta = 5\%$.**

| Methods | 30-bus | | | 118-bus | | | 200-bus | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\eta_{\text{fea}}$ (%) | $\eta_{\text{cost}}$ (\$) | $\eta_{\text{spd}}$ (×) | $\eta_{\text{fea}}$ (%) | $\eta_{\text{cost}}$ (\$) | $\eta_{\text{spd}}$ (×) | $\eta_{\text{fea}}$ (%) | $\eta_{\text{cost}}$ (\$) | $\eta_{\text{spd}}$ (×) |
| Deterministic AC-OPF | 21.83 | 1.5654 | 242 | 0.05 | 5.8150 | 363 | 16.52 | 1.3778 | 199 |
| Iterative constraint tightening | 66.83 | 1.6156 | 29 | 2.26 | 5.8420 | 39 | 16.76 | 1.3778 | 23 |
| Scenario-based CC-DCOPF | 0.00 | 1.7358 | 22 | 0.00 | 5.8955 | 84 | 0.00 | 1.3785 | 15 |
| Scenario-based CC-ACOPF | 98.83 | 1.7358 | 3 | 85.24 | 5.8291 | 1 | 89.62 | 1.3779 | 1 |
| Data-driven CC-ACOPF | 96.04 | 1.7358 | 1 | 95.83 | 5.8335 | 1 | 97.73 | 1.3779 | 1 |
| Unsupervised NN solution | 77.25 | 1.6731 | $10^4$ | 16.18 | 5.8380 | $10^4$ | 87.79 | 1.3793 | $10^5$ |
| Approximate-w/o-project | 69.43 | 1.5952 | $10^4$ | 49.05 | 5.8175 | $10^4$ | 91.53 | 1.3781 | $10^5$ |
| **Approximate-then-project** | 95.03 | 1.6519 | 212 | 95.27 | 5.8374 | 304 | 97.68 | 1.3788 | 29 |

that the constraint violation of randomly perturbed point $\tilde{x}_i$ can be minimized. We remark that the noise $z$ plays a crucial role in IPNN training as (i) it provides a robust margin to the constraint boundary that enhances prior feasibility [26]; and (ii) by maintaining distance from the boundary, it will reduce optimality loss induced by projection in the next section [22].

*4.2.2 Projection through Bisection.* Under new uncertainty $\xi \sim P_\omega$, we first generate an NN-approximate solution $u' = \phi(\bar{\xi})$ following Sec. 4.1. We assess its feasibility with respect to the chance constraint as $\left\{\frac{1}{K}\sum_{k=1}^{K}\mathbb{I}\left(\left\|\left[g\left(\left[u', \mathrm{PF}(u', \hat{\xi}_k)\right], \hat{\xi}_k\right)\right]^+\right\| > 0\right)\right\} \leq \delta$, where $\{\hat{\xi}_k\}_{k=1}^{K} \sim P_\omega$ are i.i.d. samples for feasibility evaluation, and $K$ depends on a pre-specified confidence level. If the solution fails such a *posterior feasibility test*, we employ the trained IPNN to generate an interior point and execute the following projection process in Alg. 1 to restore feasibility; Otherwise, the predicted solution already meets the probabilistic constraints with a confidence level.

**Remark.** We remark that this algorithm shares the same spirit as the iterative tuning approach in [19, 39]. However, our bisection algorithm is more efficient since it only needs feasibility evaluation without solving a series of refined AC-OPF problems repeatedly [19, 31]. We also provide performance analysis for solution feasibility with a pre-specified confidence level in the full version of this paper.

# 5 NUMERICAL EXPERIMENTS

We conduct extensive simulations to demonstrate our framework’s efficiency, robustness, and adaptivity. Detailed experimental setup and baseline descriptions are provided in our technical report [26].

**Baselines:** We compare our approach against state-of-the-art (SOTA) methods for solving CC-ACOPF, including *iterative solver* based approaches: (1) deterministic AC-OPF; (2) scenario-based CC-DCOPF (randomly sampled) [3]; (3) scenario-based CC-ACOPF (randomly sampled) [34]; (4) data-driven CC-ACOPF [31]; (5) iterative constraint tightening [19, 39]; *machine learning* based methods: (6) unsupervised NN [17]; (7) approximate-w/o-projection in Sec. 4.1; (8) approximate-then-project in Sec. 4.2. We remark that iterative solver-based methods are executed in MATLAB using MIPS [51], while NN-based methods are executed in Python over GPU, ensuring a fair comparison of both frameworks.

**Evaluation metrics:** We consider the following evaluation metrics. (i) **feasibility** $\eta_{\text{fea}}$: the *out-of-sample* (OOS) feasibility rate under unseen 1,000 i.i.d. scenarios and averaged over 1,024 instances; (ii) **optimality (**$\eta_{\text{cost}}$**)**: average generation cost per bus (normalized by base MVA). (iii) **speedup (**$\eta_{\text{spd}}$**)**: run-time speedup relative to the SOTA data-driven CC-ACOPF methods, since there is no exact algorithm to solve CC-ACOPF optimally [16].

## 5.1 Baselines Comparison over Various Systems

We demonstrate the effectiveness of our proposed approach through simulations on PGLIB test cases [2], including 30/118/200-bus networks. Simulation results in Table 2 show that our method consistently achieves the desired OOS feasibility rates (larger than 95%) while maintaining comparable generation costs (e.g., within 0.1% in 118-bus system) to the SOTA data-driven CC-ACOPF baseline, and demonstrating a speedup of up to 304 times. NN-based methods without the project module always fail the posterior feasibility test (at least 95%). Constraint-tightening methods rely on the linearization of non-convex constraints, leading to inaccuracy for different systems. Further, for large-scale test cases, conventional scenario/data-driven methods are limited by the computational capacity of MIPS solvers [51], allowing computation with only a few scenarios (e.g., 50 for 200-bus), thus failing the OOS feasibility tests. Our bisection method, requiring only feasibility evaluation without iterative AC-OPF solving, proves more efficient for large systems.

## 5.2 Sensitivity Analysis of Our Framework

We then conduct sensitivity analysis to evaluate the adaptability and robustness of our framework.

**Diverse uncertainties.** To demonstrate the adaptability of our framework to different operating conditions and reliability requirements, we evaluate our approach across various uncertainty distributions and risk levels. As shown in Fig. 2, our framework maintains OOS feasibility close to the desired reliability targets across all tested distributions, with deviations less than 2%. The bisection-based projection module effectively balances reliability and cost, achieving tighter risk levels (e.g., from 10% to 5%) with only minor increases in generation costs (less than 0.5% increase).

**Different hyper-parameters.** To assess our framework’s robustness, we analyze the sensitivity to three key parameters: (i) initial solutions $\tilde{x}$, (ii) bisection steps $B$, and (iii) evaluation scenarios $K$. As shown in Fig. 3, our projection module consistently achieves desired OOS feasibility rates with deviations less than 1% across initial solutions of varying quality, while maintaining

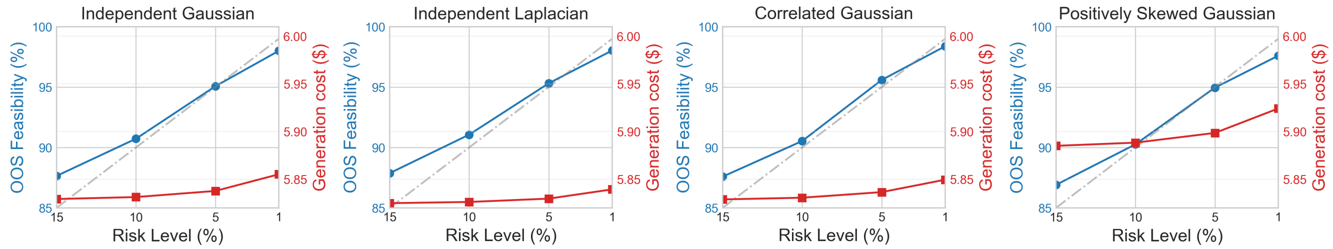


**Figure 2: Performance of our framework under different uncertainties and risk levels on 118-bus system.**

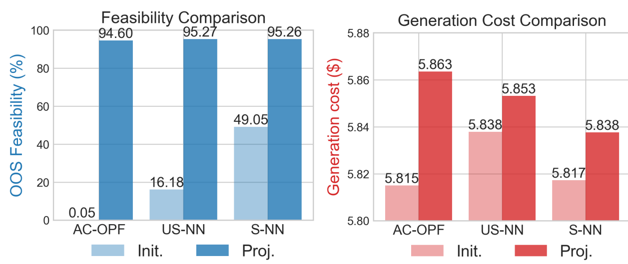


**Figure 3: Sensitivity analysis of our framework under different initial approximations and their projected solutions.**

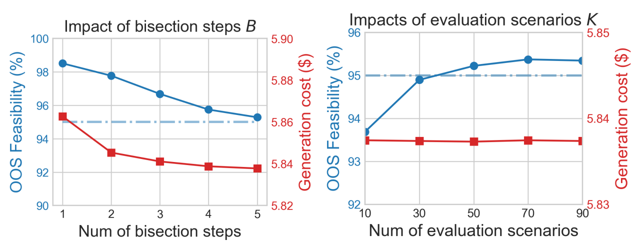


**Figure 4: Sensitivity analysis of our framework for the number of bisection steps ($B$) and evaluation scenarios ($K$).**

minimal increases in generation costs (less than 1%), demonstrating the robustness of the bisection-based projection module. Fig. 4 reveals the parameter selection trade-offs: increasing bisection steps $B$ improves convergence to the desired feasibility rate and reduces operation costs, while larger scenario sets $K$ strengthen the probabilistic guarantee for OOS feasibility at the expense of computational time.

Regarding computational complexity, The cost scales linearly with the number of bisection steps due to the sequential nature of Alg. 1 and sub-linearly with the number of scenarios due to efficient batch processing for feasibility evaluation as demonstrated in Fig. 5 (e.g., evaluating a solution for the 200-bus system under 1,000 scenarios take only about 0.6 second), showing the efficiency of our framework.

## 6 CONCLUSION AND DISCUSSION

In this work, we propose an efficient NN-based **approximate-then-project** framework for CC-ACOPF that demonstrates superior probabilistic feasibility guarantees and computational efficiency compared to state-of-the-art methods while maintaining competitive

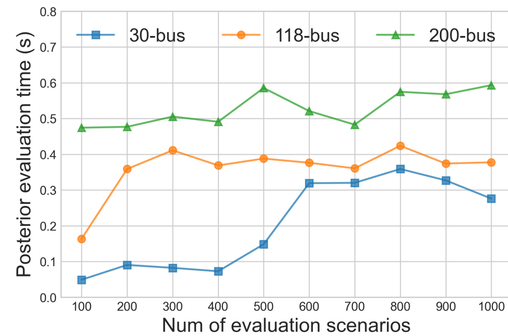


**Figure 5: Running time for feasibility checking (including power flow equation solving and inequality residual calculation) under a different number of evaluation scenarios $K$ and different systems.**

generation costs. Our approach enables power system operators to rapidly respond to potential variations and ensures reliable grid operation under uncertainty.

There are a couple of limitations and promising directions for extending our framework: (i) Uncertainty modeling: Our current framework assumes known distributional parameters for uncertainty modeling. However, many real-world power systems operate with limited historical data without a known distribution. Developing generative models for scenario generation and incorporating them with our framework is crucial for such a data-scarce setting [10]. (ii) Topology variation: Incorporating topology changes and network reconfiguration into the current framework will further strengthen the security operation of modern power grids under both renewable integration and potential cyber-physical attacks. By modeling the admittance matrix as input variables [50], our framework can seamlessly incorporate topology uncertainty.

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