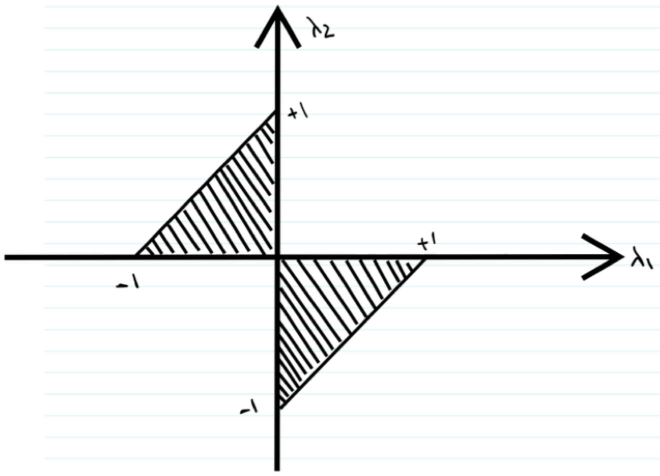
EDISON MURATI

1. Consider the following joint density function for the random variable X and Y with the following distribution:

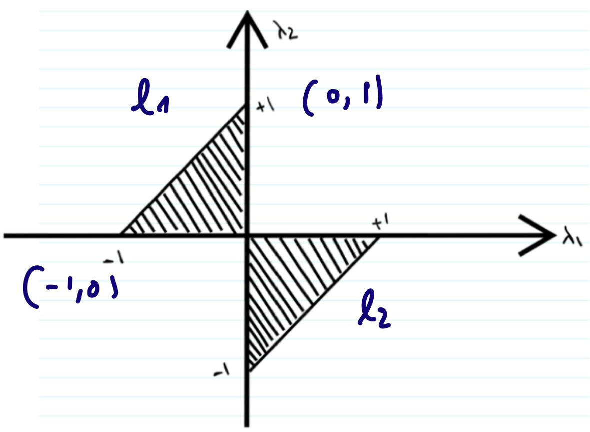
$$f_{X,Y}(\lambda_1, \lambda_2) = \begin{cases} c \; ; & -1 < \lambda_1 < 1 \; and \; -1 < \lambda_2 < 1 \\ 0 \; ; & Else \end{cases}$$

a. Find the constant c.
b. Find the marginal density $f_X(\lambda_1)$ and $f_Y(\lambda_2)$
c. Graph the marginal density function for random variable X and Y and show that the area under each curve is unity.
d. What is E[X]? what is E[Y]?
e. Find and graph $f_{X|Y}(\lambda_1|\lambda_2)$. What is $f_{X|Y}(\lambda_1|\lambda_2)$ when λ_2 = 0.5.
f. Find and graph $f_{Y|X}(\lambda_2|\lambda_1)$. What is $f_{Y|X}(\lambda_2|\lambda_1)$. when λ_1 = 0.5.
g. Are random variable X and Y independent?



2. Consider the following joint density function for the random variable X and Y with the

$$f_{X,Y}(\lambda_1, \lambda_2) = \begin{cases} c \; ; & -1 < \lambda_1 < 1 \; and \; -1 < \lambda_2 < 1 \\ 0 \; ; & Else \end{cases}$$



l_1 : $\lambda_2 - 1 = \frac{1-0}{0+1}(\lambda_1)$

$\lambda_2 = \lambda_1 + 1$

l_2 : $\lambda_2 = \lambda_1 - 1$ (shifted two units down)

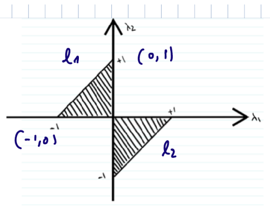
Part a

$$c \text{ Area} = 1$$

$$c\left(\frac{1}{2}+\frac{1}{2}\right) = 1$$

$$c = 1$$

Part b : Marginal densities



l_1 : $\lambda_2 - 1 = \frac{1-0}{0+1}(\lambda_1)$

$\lambda_2 = \lambda_1 + 1$

l_2 : $\lambda_2 = \lambda_1 - 1$ (shifted two units down)

- $$f_X(\lambda_1) = \int_{-\infty}^{+\infty} f_{X,Y}(\lambda_1, \lambda_2)\, d\lambda_2$$

* $\lambda_1 \in [-1, 0]$:

$$f_X(\lambda_1) = \int_0^{\lambda_1+1} d\lambda_2 = 1 + \lambda_1$$

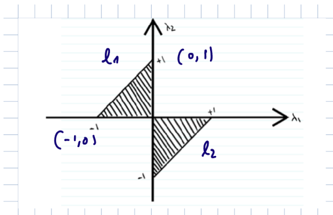
* $\lambda_1 \in [0, 1)$:

$$f_X(\lambda_1) = \int_{\lambda_1-1}^{0} d\lambda_2 = 1 - \lambda_1$$

* otherwise, $f_X(\lambda_1) = 0$

$$f_x(\lambda_1) = \begin{cases} 1 + \lambda_1 & \text{if } \lambda_1 \in (-1, 0] \\ 1 - \lambda_1 & \text{if } \lambda_1 \in [0, 1) \\ 0 & \text{else} \end{cases}$$

- $f_y(\lambda_2) = \int_{-\infty}^{+\infty} f_{x,y}(\lambda_1, \lambda_2) \, d\lambda_1$



ℓ_1 :

$$\lambda_2 - 1 = \frac{1-0}{0+1}(\lambda_1)$$

$$\lambda_2 = \lambda_1 + 1$$

ℓ_2 : $\lambda_2 = \lambda_1 - 1$ (shifted two units down)

ℓ_1: $\lambda_1 = \lambda_2 - 1$

ℓ_2: $\lambda_1 = \lambda_2 + 1$

* $\lambda_2 \in (-1, 0]$:

$$f_y(\lambda_2) = \int_0^{1+\lambda_2} d\lambda_1 = 1 + \lambda_2$$

* $\lambda_2 \in [0, 1)$:

$$f_y(\lambda_2) = \int_{\lambda_2 - 1}^{0} d\lambda_1 = 1 - \lambda_2$$

else zero

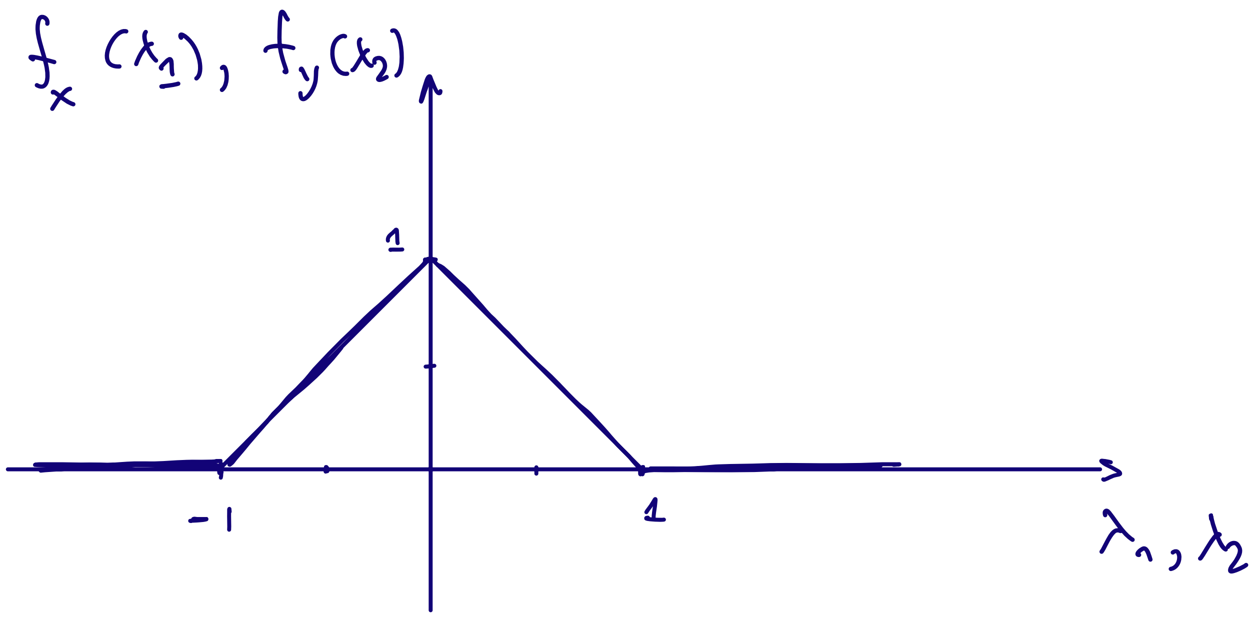
$\Rightarrow$

$$f_y(\lambda_2) = \begin{cases} 1 + \lambda_2 & \text{if } \lambda_2 \in (-1, 0] \\ 1 - \lambda_2 & \text{if } \lambda_2 \in [0, 1) \\ 0 & \text{otherwise} \end{cases}$$

b. Find the marginal density $f_X(\lambda_1)$ and $f_Y(\lambda_2)$

c. Graph the marginal density function for random variable X and Y and show that the area under each curve is unity.

I will draw the same plot for $f_x(\lambda_1)$ and $f_y(\lambda_2)$



They are both valid because

- $f_x(\lambda_1) \geq 0$ and $f_y(\lambda_2) \geq 0$
- $\displaystyle\int_{-\infty}^{+\infty} f_x(\lambda_1)\, d\lambda_1 = \int_{-\infty}^{+\infty} f_y(\lambda_2)\, d\lambda_2$

$$= \frac{1}{\cancel{2}} \times \cancel{2} \times 1 = 1 \quad //$$

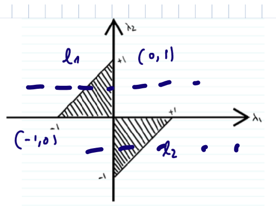
$$f_x(\lambda_1) = \begin{cases} \lambda_1 + 1 & \text{if } \lambda_1 \in [-1, 0] \\ -\lambda_1 + 1 & \text{if } \lambda_1 \in [0, 1] \\ 0 & \text{otherwise} \end{cases} \quad //$$

d. What is E[X]? what is E[Y]?

$$E[x] = \int_{-\infty}^{+\infty} \lambda_1 f(\lambda_1)\, d\lambda_1 = 0$$

Since $\lambda_1 f(\lambda_1)$ is an odd function, and the integral is over a symmetric interval around zero.

Similarly, $E[y] = 0$,



l_1 : $\lambda_2 - 1 = \frac{1-0}{0+1}(\lambda_1)$

$\lambda_2 = \lambda_1 + 1$ $\qquad \lambda_1 = \lambda_2 - 1$

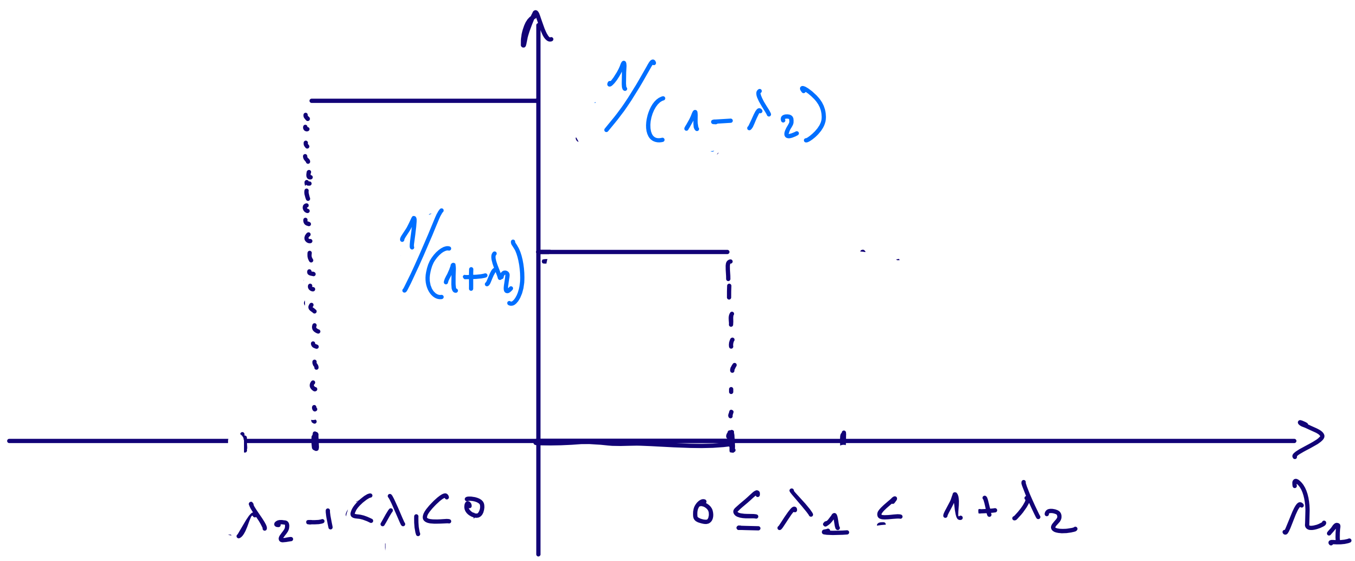
l_2 : $\lambda_2 = \lambda_1 - 1$ (shifted two units down) $\qquad \lambda_1 = \lambda_2 + 1$

e. Find and graph $f_{X|Y}(\lambda_1|\lambda_2)$. What is $f_{X|Y}(\lambda_1|\lambda_2)$ when $\lambda_2 = 0.5$.

$$f_{X|Y}(\lambda_1|\lambda_2) = \frac{f_{X,Y}(\lambda_1,\lambda_2)}{f_Y(\lambda_2)}$$

$$f_Y(\lambda_2) = \begin{cases} 1+\lambda_2 & \text{if} \quad \lambda_2 \in (-1,0] \\ 1-\lambda_2 & \text{if} \quad \lambda_2 \in [0,1) \\ 0 & \text{otherwise} \end{cases}$$

$$f_{X|Y}(\lambda_1|\lambda_2) = \begin{cases} \dfrac{1}{1+\lambda_2} & \text{if} \quad \lambda_2 \in (-1,0] \text{ and } 0 \leq \lambda_1 \leq \lambda_2+1 \\ \dfrac{1}{1-\lambda_2} & \text{if} \quad \lambda_2 \in [0,1) \text{ and } \lambda_2 - 1 \leq \lambda_1 \leq 0 \\ 0 & \text{otherwise} \end{cases}$$



if $\lambda_2 = 0.5$, $\quad f_{X|Y}(\lambda_1|\lambda_2) = \dfrac{1}{1-0.5} = 2$ //

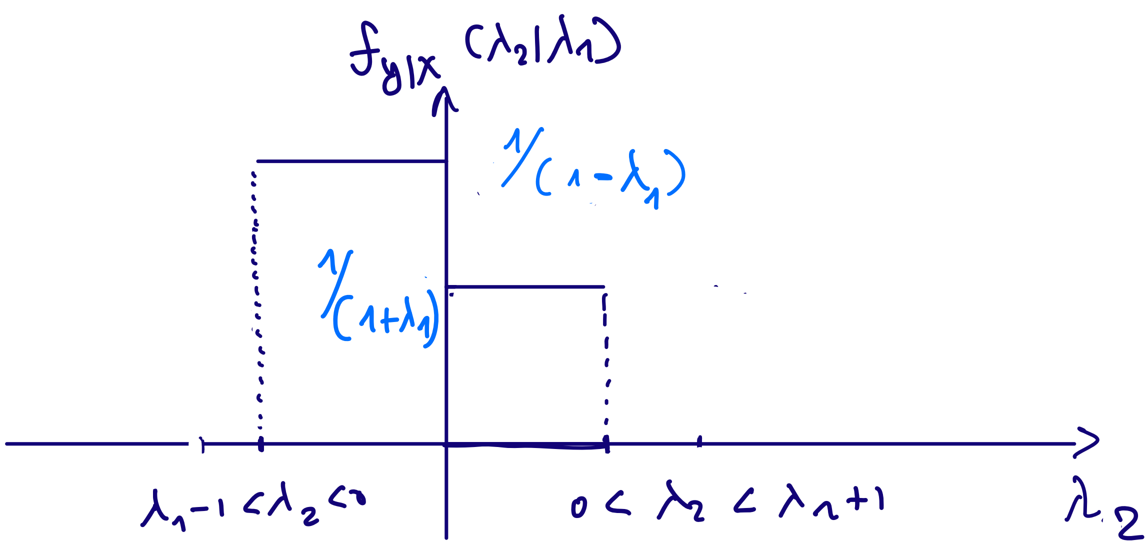
Part d

$$f_{y|x}(\lambda_2|\lambda_1) = \frac{f_{x,y}(\lambda_1, \lambda_2)}{f_x(\lambda_1)}$$

$$f_x(\lambda_1) = \begin{cases} 1 + \lambda_1 & \text{if } \lambda_1 \in (-1, 0] \\ 1 - \lambda_1 & \text{if } \lambda_1 \in [0, 1) \\ 0 & \text{else} \end{cases}$$

$f_{y|x}$ will have the same form as $f_{x|y}$

$$f_{y|x}(\lambda_2|\lambda_1) = \begin{cases} \dfrac{1}{1+\lambda_1} & \text{if } \lambda_1 \in (-1, 0] \text{ and } 0 \leq \lambda_2 \leq \lambda_1 + 1 \\ \dfrac{1}{1-\lambda_1} & \text{if } \lambda_1 \in [0, 1) \text{ and } \lambda_1 - 1 \leq \lambda_2 \leq 0 \\ 0 & \text{otherwise} \end{cases}$$



if $\lambda_1 = 0.5$, $\quad f_{y|x}(\lambda_2|\lambda_1) = \dfrac{1}{1-0.5} = 2$ //

Part e

Recall $f_x(\lambda_1) = \begin{cases} 1+\lambda_1 & \text{if } \lambda_1 \in (-1,0] \\ 1-\lambda_1 & \text{if } \lambda_1 \in [0,1) \\ 0 & \text{else} \end{cases}$

$$f_y(\lambda_2) = \begin{cases} 1+\lambda_2 & \text{if } \lambda_2 \in (-1,0] \\ 1-\lambda_2 & \text{if } \lambda_2 \in [0,1) \\ 0 & \text{otherwise} \end{cases}$$

Take $\lambda_1 = \lambda_2 = 0.5$

$$f_x(\lambda_1)\, f_y(\lambda_2) = 0.5 \times 0.5 = 0.25$$

$$f_{x,y}(\lambda_1, \lambda_2) = 1 \neq f_x(\lambda_1)\, f_y(\lambda_2)$$

The variables are not independent.

2. Consider the following joint density function for the random variable X and Y with the following distribution:

$$f_{X,Y}(\lambda_1,\lambda_2) = \begin{cases} c \; ; & -1 < \lambda_1 < 1 \; and \; -1 < \lambda_2 < 1 \\ 0 \; ; & Else \end{cases}$$

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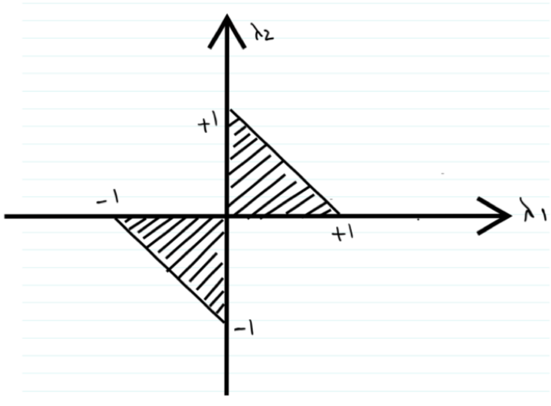
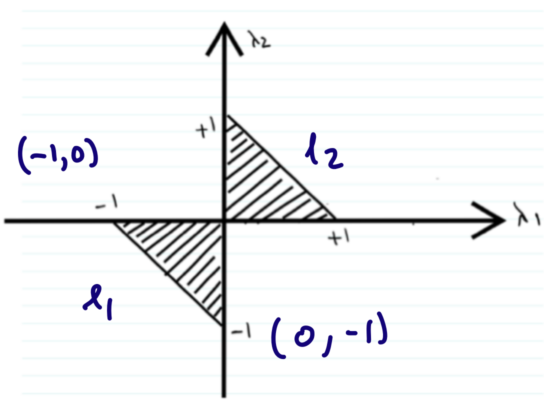
d. What is E[X]? what is E[Y]?
e. Find and graph $f_{X|Y}(\lambda_1|\lambda_2)$. What is $f_{X|Y}(\lambda_1|\lambda_2)$ when λ_2 = 0.5.
f. Find and graph $f_{Y|X}(\lambda_2|\lambda_1)$. What is $f_{Y|X}(\lambda_2|\lambda_1)$. when λ_1 = 0.5.
g. Are random variable X and Y independent?

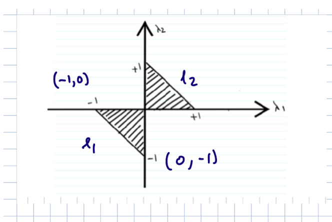




$l_1: \quad \lambda_2 = \frac{0+1}{-1-0}(\lambda_1+1)$

$\lambda_2 = -\lambda_1 - 1$

$l_2: \quad \lambda_2 = -\lambda_1 + 1$



$l_1: \quad \lambda_2 = \frac{0+1}{-1-0}(\lambda_1+1)$

$\lambda_2 = -\lambda_1 - 1$

$l_2: \quad \lambda_2 = -\lambda_1 + 1$

a) let $I = \iint f_{x,y}(\lambda_1, \lambda_2)\, d\lambda_1\, d\lambda_2$

$$= c \text{ Area}$$

$$= c\left(\frac{1}{2} + \frac{1}{2}\right) = c$$

$$\Rightarrow \quad c = 1$$

b) Marginal density :

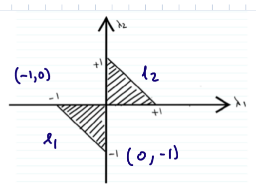
$$\bullet \; f_x(\lambda_1) = \int_{-\infty}^{+\infty} f_{x,y}(\lambda_1, \lambda_2)\, d\lambda_2$$

$$* \; \lambda_1 \in [-1, 0] : \; f_x(\lambda_1) = \int_{-\lambda_1 - 1}^{0} d\lambda_2 = \lambda_1 + 1$$

$$* \; \lambda_1 \in [0, 1] : \; f_x(\lambda_1) = \int_{0}^{-\lambda_1 + 1} d\lambda_2 = -\lambda_1 + 1$$

$$f_x(\lambda_1) = \begin{cases} 1 + \lambda_1 & \text{if } \lambda_1 \in [-1, 0] \\ 1 - \lambda_1 & \text{if } \lambda_1 \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

//



l_1: $\lambda_2 = \frac{0+1}{-1-0}(\lambda_1 + 1)$

$\lambda_2 = -\lambda_1 - 1$

l_2: $\lambda_2 = -\lambda_1 + 1$

$-\lambda_1 = \lambda_2 + 1 \Rightarrow \lambda_1 = -\lambda_2 - 1$

$-\lambda_1 = -1 + \lambda_2 \Rightarrow \lambda_1 = 1 - \lambda_2$

• $f_y(\lambda_2) = \int_{-\infty}^{+\infty} f_{X,Y}(\lambda_1, \lambda_2)\, d\lambda_1$

* $\lambda_2 \in [-1, 0]$: $f_y(\lambda_2) = \int_{-\lambda_2 - 1}^{0} d\lambda_1 = 1 + \lambda_2$

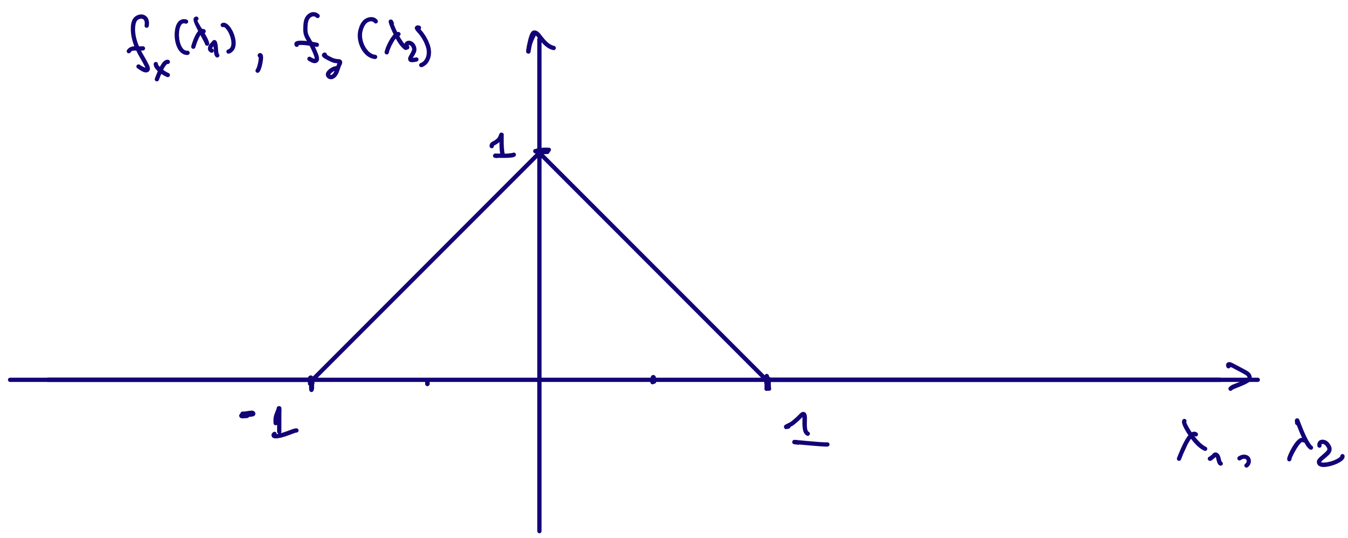
* $\lambda_2 \in [0, 1]$: $f_y(\lambda_2) = \int_{0}^{1-\lambda_2} d\lambda_1 = 1 - \lambda_2$

$\Rightarrow$

$$f_y(\lambda_2) = \begin{cases} 1 + \lambda_2 & \text{if } \lambda_2 \in [-1, 0] \\ 1 - \lambda_2 & \text{if } \lambda_2 \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

part c

I will plot $f_x(\lambda_1)$ and $f_y(\lambda_2)$ together since they have the same form.



Both $f_x(\lambda_1)$ and $f_y(\lambda_2)$ are valid because they are positive and $\int f_x(\lambda_1) = \int f_y(\lambda_2) = 1$

Since the area $= 1$.

Part d

$$E[x] = \int \lambda_1 f_1(\lambda_1)\, d\lambda_1 = 0$$

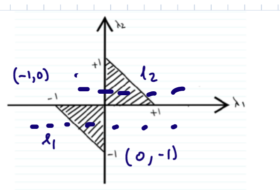
Since $\lambda_1 f_1(\lambda_1)$ is an odd function and the integral is over a symmetric interval around 0.

Similarly, $E[y] = 0$ //

Part(e)

$$f_{x|y}(\lambda_1|\lambda_2) = \frac{f_{x,y}(\lambda_1, \lambda_2)}{f_y(\lambda_2)}$$

$$f_y(\lambda_2) = \begin{cases} 1 + \lambda_2 & \text{if } \lambda_2 \in [-1, 0] \\ 1 - \lambda_2 & \text{if } \lambda_2 \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$



$l_1: \ \lambda_2 = \frac{0+1}{-1-0}(\lambda_1 + 1)$ $\qquad \lambda_1 = -\lambda_2 - 1$

$\lambda_2 = -\lambda_1 - 1$

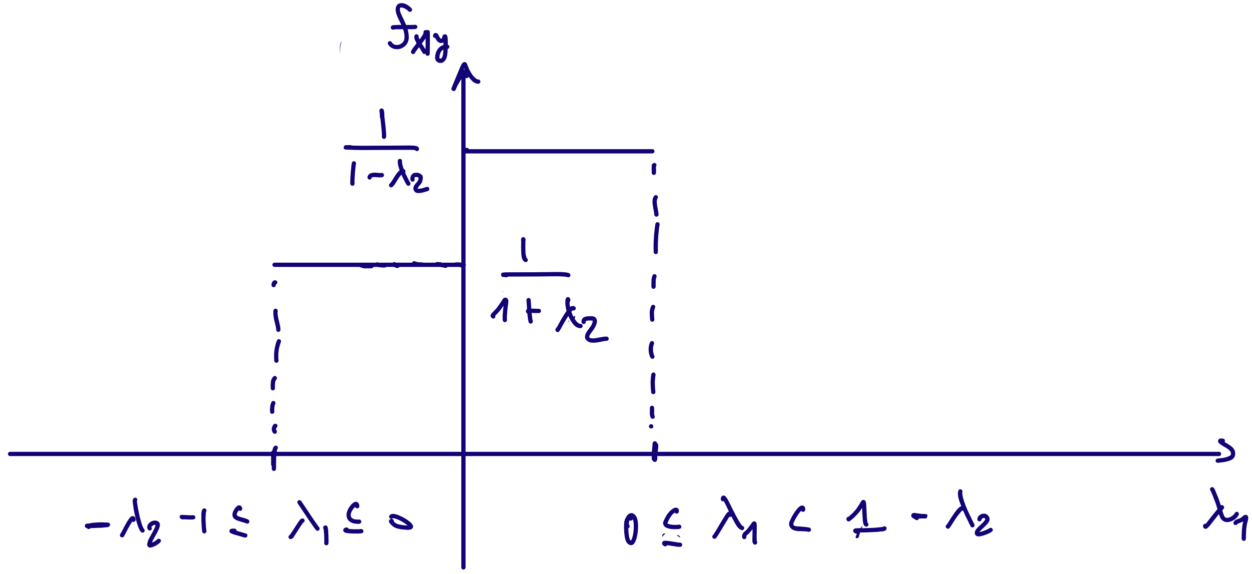
$l_2: \ \lambda_2 = -\lambda_1 + 1$ $\qquad \lambda_1 = 1 - \lambda_2$

- if $-1 < \lambda_2 < 0$: $\quad -\lambda_2 - 1 \le \lambda_1 \le 0$

$$f_{x|y}(\lambda_1|\lambda_2) = \frac{1}{1+\lambda_2} \quad //$$

- if $0 < \lambda_2 < 1$: $\quad 0 \le \lambda_1 \le 1 - \lambda_2$

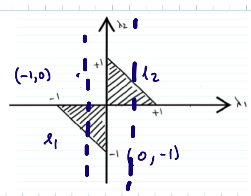
$$f_{x|y} = \frac{1}{1-\lambda_2} \quad //$$



if $\lambda_2 = 0.5$, $f_{x|y} = \frac{1}{1-0.5} = 2$ //

Part f

$$f_x(\lambda_1) = \begin{cases} 1 + \lambda_1 & \text{if } \lambda_1 \in [-1, 0] \\ 1 - \lambda_1 & \text{if } \lambda_1 \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$



l_1: $\lambda_2 = \frac{0+1}{-1-0}(\lambda_1 + 1)$

$\lambda_2 = -\lambda_1 - 1$

l_2: $\lambda_2 = -\lambda_1 + 1$

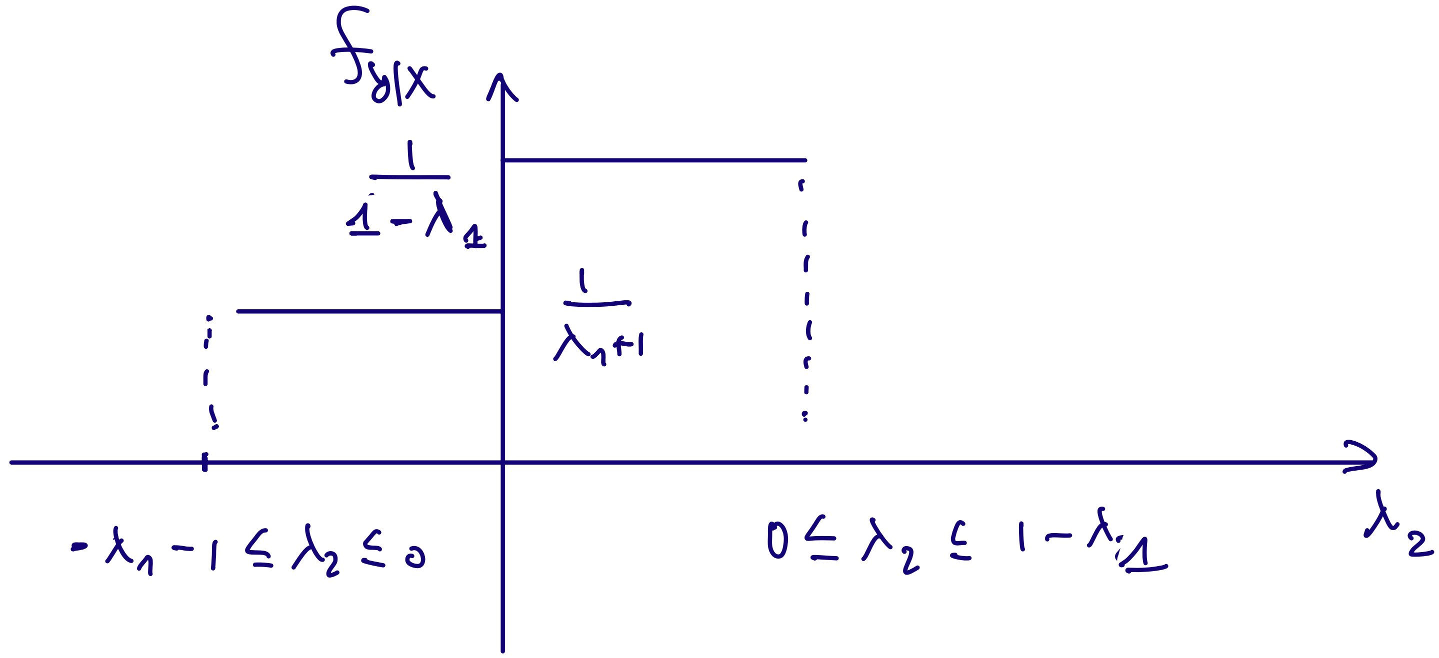
$$f_{y|x}(\lambda_1 | \lambda_2) = \frac{f_{x,y}(\lambda_1, \lambda_2)}{f_x(\lambda_1)}$$

- $-1 < \lambda_1 \leq 0$: $-\lambda_1 - 1 \leq \lambda_2 \leq 0$

$$f_{y|x}(\lambda_2 | \lambda_1) = \frac{1}{1 + \lambda_1}$$

- $0 \leq \lambda_1 < 1$: $0 \leq \lambda_2 \leq 1 - \lambda_2$

$$f_{y|x}(\lambda_2 | \lambda_1) = \frac{1}{1 - \lambda_1}$$



If $\lambda_1 = 0.5$, $\quad f_{y|x} = \frac{1}{1-0.5} = 2$ //

Part (g)

Recall $\quad f_y(\lambda_2) = \begin{cases} 1+\lambda_2 & \text{if } \lambda_2 \in (-1, 0] \\ 1-\lambda_2 & \text{if } \lambda_2 \in [0, 1) \\ 0 & \text{else} \end{cases}$

$$f_x(\lambda_1) = \begin{cases} 1+\lambda_1 & \text{if } \lambda_1 \in [-1, 0] \\ 1-\lambda_1 & \text{if } \lambda_1 \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

Take $\quad \lambda_1 = \lambda_2 = 0.5$

$$f_y(\lambda_2)\, f_x(\lambda_1) = 0.5 \times 0.5 = 0.25$$

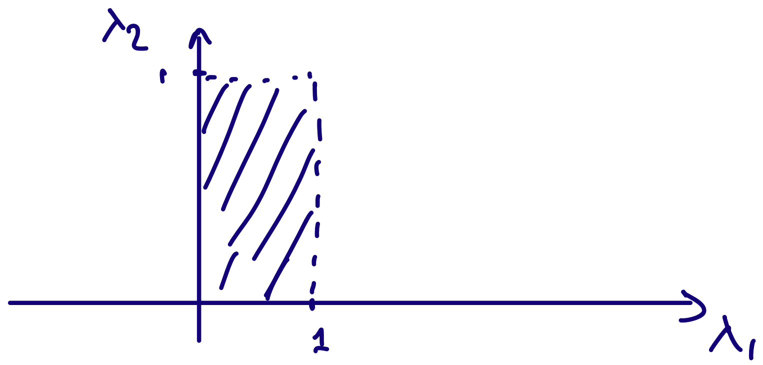
$$f_{x,y}(\lambda_1, \lambda_2) = 1 \neq 0.25$$

The variables are not independent //

3. Consider the following joint density function for the random variable X and Y uniformly distributed as:

$$f_{X,Y}(\lambda_1,\lambda_2) = \begin{cases} c\lambda_1^2 + \dfrac{\lambda_1\lambda_2}{3} & ;\ 0<\lambda_1<1 \text{ and } 0<\lambda_2<2 \\ 0 & ;\ Else \end{cases}$$

a. Find the constant c.
b. Find the marginal density $f_X(\lambda_1)$ and $f_Y(\lambda_2)$.
c. Graph the marginal density function for random variable X and Y and show that the area under each curve is unity.
d. Find $P(\lambda_1 + \lambda_2 > 1)$
e. Are random variable X and Y independent?



Part a

let $I = \iint f_{x,y}(\lambda_1,\lambda_2)\, d\lambda_1\, d\lambda_2$

$$I = \int_0^2\int_0^1 c\lambda_1^2\, d\lambda_1\, d\lambda_2 + \frac{1}{3}\int_0^2\int_0^1 \lambda_1\lambda_2\, d\lambda_1 d\lambda_2$$

$$= c\int_0^2 \frac{1}{3}\lambda_1^3\Big|_0^1 d\lambda_2 + \frac{1}{3}\int_0^2 \lambda_2\ \frac{1}{2}\lambda_1^2\Big|_0^1 d\lambda_2$$

$$= \frac{c}{3}\int_0^2 d\lambda_2 + \frac{1}{6}\int_0^2 \lambda_2\, d\lambda_2$$

$$= \frac{2c}{3} + \frac{1}{6}\ \frac{1}{2}\lambda_2^2\Big|_0^2 = \frac{2c}{3} + \frac{1}{6}\ \frac{1}{2}\ 4$$

$$= \frac{2c}{3} + \frac{1}{3}$$

$$\frac{2c}{3} + \frac{1}{3} = 1$$

$$2c + 1 = 3$$

$$2c = 2$$

$$c = 1 \quad //$$

$$f_{X,Y}(\lambda_1, \lambda_2) = \begin{cases} c\lambda_1^2 + \frac{\lambda_1\lambda_2}{3} & ; 0 < \lambda_1 < 1 \text{ and } 0 < \lambda_2 < 2 \\ 0 & ; \quad Else \end{cases}$$

Part b : Marginal densities :

$$* \quad f_X(\lambda_1) = \int_{-\infty}^{+\infty} f_{X,Y}(\lambda_1, \lambda_2)\, d\lambda_2$$

$$= \int_0^2 \left(\lambda_1^2 + \frac{\lambda_1}{3}\lambda_2\right) d\lambda_2$$

$$= 2\lambda_1^2 + \frac{\lambda_1}{3}\left(\frac{1}{2}\lambda_2^2\right)_0^2$$

$$= 2\lambda_1^2 + \frac{\lambda_1}{3}(2)$$

$$= 2\lambda_1^2 + \frac{2\lambda_1}{3} \quad \text{for} \quad 0 < \lambda_1 < 1 \quad //$$

Zero otherwise.

$$* \quad f_y(\lambda_2) = \int_{-\infty}^{+\infty} f_{x,y}(\lambda_1, \lambda_2)\, d\lambda_1$$

$$= \int_0^1 \left(\lambda_1^2 + \frac{\lambda_2}{3}\lambda_1\right) d\lambda_1$$

$$= \frac{1}{3} + \frac{\lambda_2}{3}\left(\frac{1}{2}\lambda_1^2\right)\Big|_0^1$$

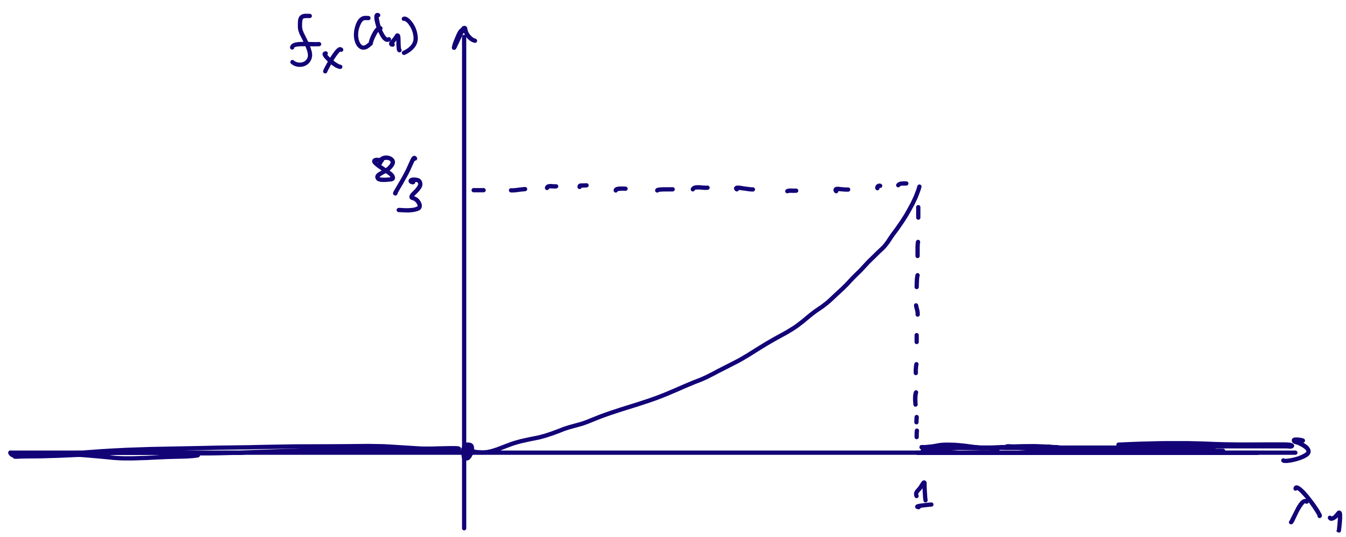
$$= \frac{1}{3} + \frac{\lambda_2}{6} \qquad \text{for } 0 < \lambda_2 < 2 \;//$$

Zero otherwise.

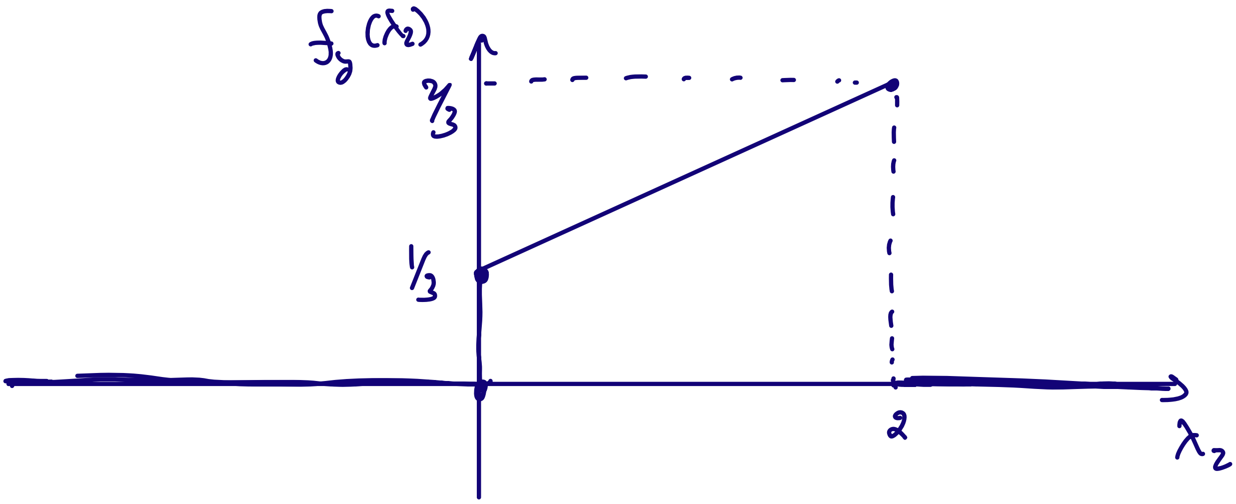
Part c

- $f_x(\lambda_1) = 2\lambda_1^2 + \frac{2\lambda_1}{3}$ for $0 < \lambda_1 < 1$, 0 otherwise.

(parabola starting at $f_x(0) = 0$ and ending at $f_x(1) = 8/3$, then zero everywhere).



- $f_y(\lambda_2) = \frac{1}{3} + \frac{\lambda_2}{6}$ for $0 < \lambda_2 < 2$ and zero otherwise



* The areas under the curves:

For f_x: Area $= \int_0^1 f_x(\lambda_1)\, d\lambda_1$

$$= \frac{2}{3}\lambda_1^3 \Big|_0^1 + \frac{2}{3}\frac{1}{2}\lambda_1^2 \Big|_0^1$$

$$= \frac{2}{3} + \frac{2}{3}\frac{1}{2}$$

$$= \frac{2}{3}\left(1 + \frac{1}{2}\right) = \frac{2}{3} \times \frac{3}{2} = \underline{1} \;//$$

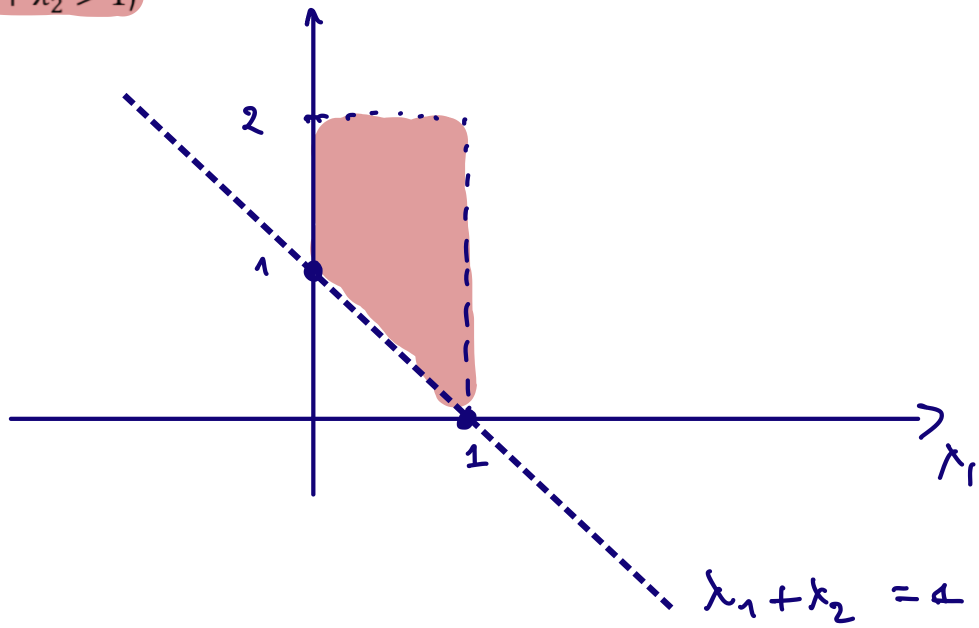
For f_y: The area is just a Trapezoid

$$\text{Area} = \frac{1}{\cancel{2}}\left(\frac{1}{3} + \frac{2}{3}\right) \times \cancel{2}$$

$$= \underline{1} \;//$$

$$f_{X,Y}(\lambda_1, \lambda_2) = \begin{cases} c\lambda_1^2 + \frac{\lambda_1 \lambda_2}{3} & ; 0 < \lambda_1 < 1 \text{ and } 0 < \lambda_2 < 2 \\ 0 & ; \quad Else \end{cases}$$

d. Find $P(\lambda_1 + \lambda_2 > 1)$



$$P(\lambda_1 + \lambda_2 > 1) = 1 - P(\lambda_1 + \lambda_2 \leq 1)$$

- $P(\lambda_1 + \lambda_2 \leq 1)$

$$= \int_0^1 \int_0^{1-\lambda_1} \left(\lambda_1^2 + \frac{\lambda_1 \lambda_2}{3} \right) d\lambda_2 \, d\lambda_1$$

$$= \int_0^1 \left(\lambda_1^2 \; \lambda_2 \Big|_0^{1-\lambda_1} + \frac{\lambda_1}{3} \; \frac{1}{2} \; \lambda_2^2 \Big|_0^{1-\lambda_1} \right) d\lambda_1$$

$$= \int_0^1 \left(\lambda_1^2 (1 - \lambda_1) + \frac{\lambda_1}{6} (1 - \lambda_1)^2 \right) d\lambda_1$$

$$= \int_0^1 \left(\lambda_1^2 - \lambda_1^3 + \frac{\lambda_1}{6} (1 - 2\lambda_1 + \lambda_1^2) \right) d\lambda_1$$

$$= \int_0^1 \left(\lambda_1^2 - \lambda_1^3 + \frac{\lambda_1}{6} - \frac{\lambda_1^2}{3} + \frac{\lambda_1^3}{6} \right) d\lambda_1$$

$$= \frac{1}{3} - \frac{1}{4} + \frac{1}{12} - \frac{1}{9} + \frac{1}{24}$$

$$= \frac{24 - 18 + 6 - 8 + 3}{72}$$

$$= \frac{7}{72}$$

Then, $P(\lambda_1 + \lambda_2 > 1) = 1 - P(\lambda_1 + \lambda_2 \leq 1)$

$$= 1 - \frac{7}{72}$$

$$= \frac{65}{72} \quad //$$

d. Find $P(x_1 + x_2 > 1)$

e. Are random variable X and Y independent?

$$f_{x|y}(\lambda_1|\lambda_2) = \frac{f_{x,y}(\lambda_1,\lambda_2)}{f_y(\lambda_2)}$$

$$= \frac{\lambda_1^2 + \frac{\lambda_1\lambda_2}{3}}{\frac{1}{3} + \frac{\lambda_2}{6}}$$

let $\lambda_2 = 0$, $f_{x|y}(\lambda_1|\lambda_2=0) = 3\lambda_1^2$

However $f_x(\lambda_1) = 2\lambda_1^2 + \frac{2\lambda_1}{3} \neq 3\lambda_1^2$

$$f_{x|y}(\lambda_1|\lambda_2) \neq f_x(\lambda_1)$$

Therefore x, y are not independent //