Time Series Analysis and Models Homework 2 Report

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Problem 1

We are given $y(t) = [3, 9, 27, 81, 243]$. We need to compute $\hat{R}_y(\tau)$ for $\tau = 0, 1, 2, 3, 4$.

$$\hat{R}_y(\tau) = \frac{\sum_{t=\tau+1}^{T}(y_t - \bar{y})(y_{t-\tau} - \bar{y})}{\sum_{t=1}^{T}(y_t - \bar{y})^2} \tag{1}$$

1. $\hat{R}_y(0)$:

$$\begin{aligned}\hat{R}_y(0) &= \frac{\sum_{t=1}^{T}(y_t - \bar{y})(y_t - \bar{y})}{\sum_{t=1}^{T}(y_t - \bar{y})^2} \\ &= \frac{\sum_{t=1}^{T}(y_t - \bar{y})^2}{\sum_{t=1}^{T}(y_t - \bar{y})^2} \\ &= 1\end{aligned} \tag{2}$$

For the rest, notice that y_t is a geometric sequence: $y_1 = 3$ and $y_{t+1} = 3y_t$, and $y_t = 3^{\tau}\, y_{t-\tau}$. Then, let's compute the mean:

$$\begin{aligned}\bar{y} &= \frac{\sum_{t=1}^{T} y_t}{T} \\ &= \frac{3(1 - 3^T)}{1 - 3}\frac{1}{T} \\ &= \frac{3(1 - 3^5)}{-2}\frac{1}{5} \\ &= 72.6\end{aligned} \tag{3}$$

First, compute the denominator:

$$\begin{aligned}\sum_{t=1}^{T}(y_t - \bar{y})^2 &= \sum_{t=1}^{T}(y_t^2 - 2y_t\bar{y} + \bar{y}^2) \\ &= \sum_{t=1}^{T}(3^2 \times 3^{2t-2}) - 2\bar{y}\sum_{t=1}^{T} y_t + T\bar{y}^2 \\ &= \sum_{t=1}^{T} 9^t - 2\bar{y}\sum_{t=1}^{T} y_t + T\bar{y}^2 \\ &= 66429 - 2 \times 72.6 \times 363 + 5 \times 72.6^2 \\ &= 40075.2\end{aligned} \tag{4}$$

Now, let's compute the numerator, and let's call it $A(\tau)$.

(a) $A(4)$:

$$A(4) = (y_5 - \bar{y})(y_1 - \bar{y}) = (243 - 72.6)(3 - 72.6) = -11859.8406 \tag{5}$$

(b) $A(3)$:

$$A(3) = (y_4 - \bar{y})(y_1 - \bar{y}) + (y_5 - \bar{y})(y_2 - \bar{y}) = -11422.08026 \tag{6}$$

(c) $A(2)$:

$$\begin{aligned} A(2) &= (y_3 - \bar{y})(y_1 - \bar{y}) + (y_4 - \bar{y})(y_2 - \bar{y}) + (y_5 - \bar{y})(y_3 - \bar{y}) \\ &= -5130.71971 \end{aligned} \tag{7}$$

(d) $A(1)$:

$$\begin{aligned} A(1) &= (y_2 - \bar{y})(y_1 - \bar{y}) + (y_3 - \bar{y})(y_2 - \bar{y}) \\ &+ (y_4 - \bar{y})(y_3 - \bar{y}) + (y_5 - \bar{y})(y_4 - \bar{y}) \\ &= 8375.0406 \end{aligned} \tag{8}$$

Now, we are ready to compute everything:

2. $\hat{R}_y(1) = \frac{A(1)}{40075.2} = 0.209$
3. $\hat{R}_y(2) = \frac{A(2)}{40075.2} = -0.128$
4. $\hat{R}_y(3) = \frac{A(3)}{40075.2} = -0.285$
5. $\hat{R}_y(4) = \frac{A(3)}{40075.2} = -0.296$

Problem 2

Figure 1 shows the Histogram. The sample mean and standard deviation calculated are:

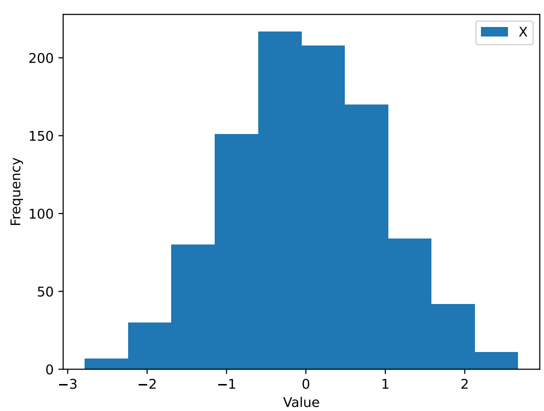

Figure 1: Histogram of Data Generated from a Normal Distribution with $\mu = 0$ and $\sigma = 1$

- The sample mean is -0.002
- The sample standard deviation is 0.957

Problem 3

Part (a)

Figure 2 shows the Auto-correlation Function (ACF) of the white noise generated earlier. This plot

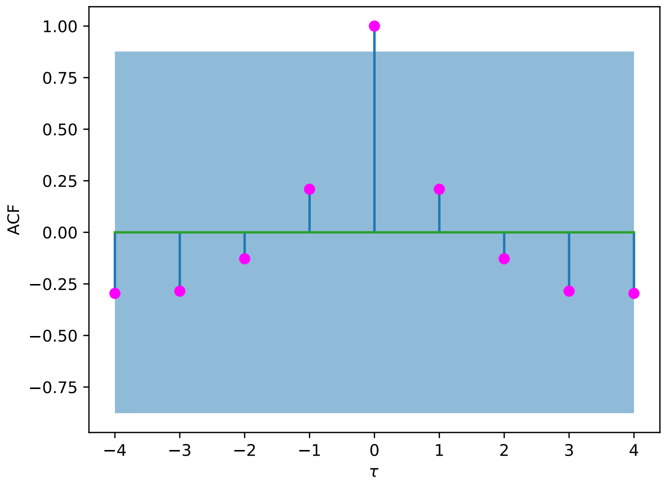

Figure 2: ACF of the made-up data in Problem 1

agrees with our calculations in Problem 1.

Part (b)

Figure 3 shows the Auto-correlation Function (ACF) of the white noise generated earlier.

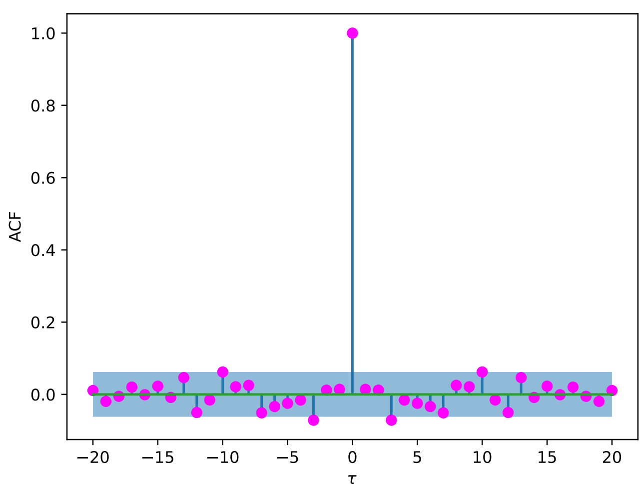

Figure 3: ACF of Data Generated from a Normal Distribution with $\mu = 0$ and $\sigma = 1$ (WN)

Part (c)

For a stationary dataset, the ACF plot quickly drops to zero because the observations far in the future are not strongly related on the current ones. For example, the ACF plot in Figure 3 is a very close to a white noise (practically a white noise). There, already at $\tau = 1$, the plot drops to zero, showing that the time-series is stationary. For the made-up time-series does not look stationary since it seems like the ACF has not asymptoted to zero. This makes sense because the made-up dataset is a geometric series. So, all the data must be correlated.

Problem 4

Part (a)

Figure 4 shows the time series of closing values of six different stocks indicated on the plots.

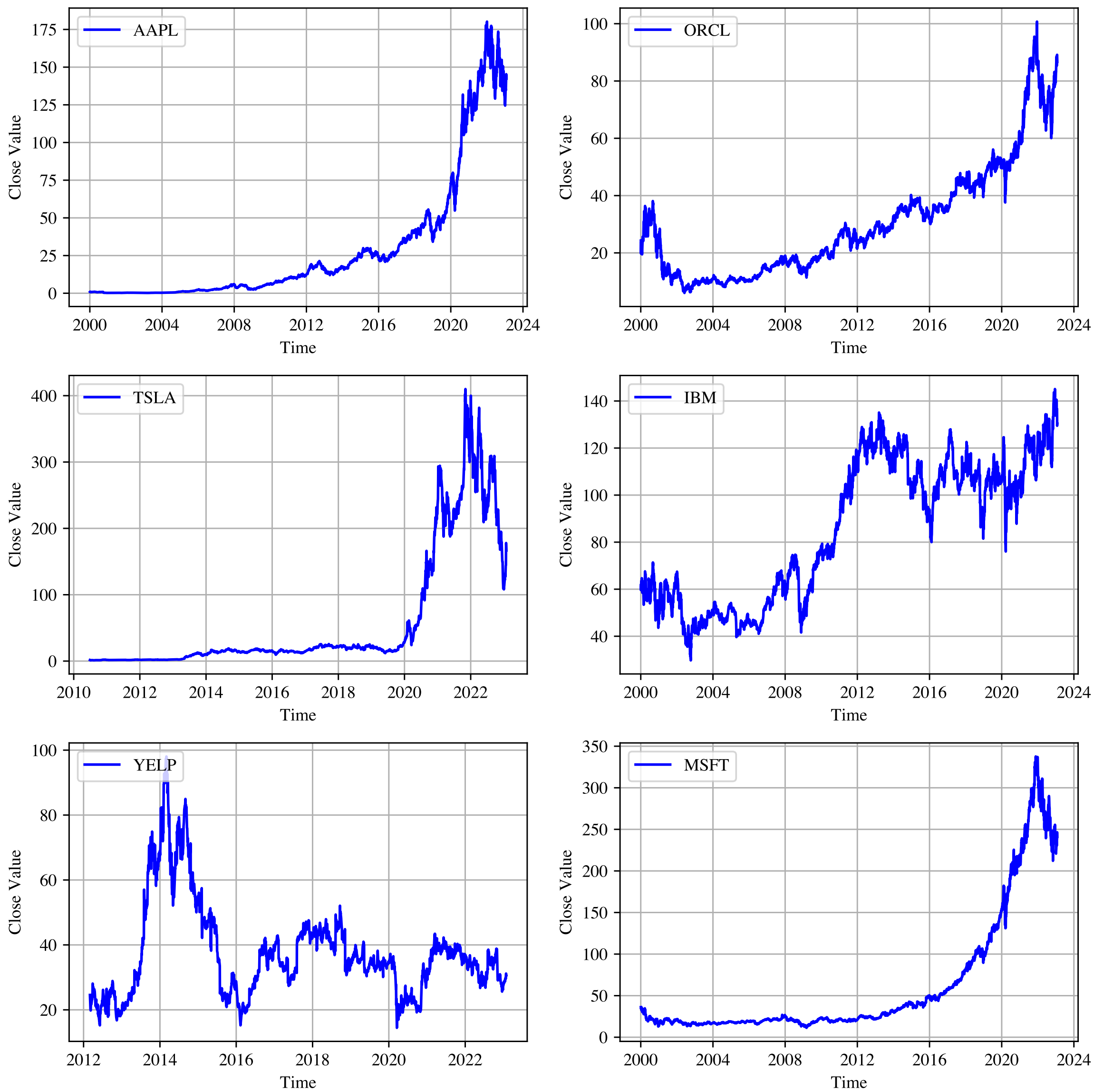

Figure 4: Close values of six different stock closing values between Jan 1st, 2000 and Jan. 31st, 2023

Part (b)

Figure 5 shows the ACF plots of the six different stocks indicated in the plot.

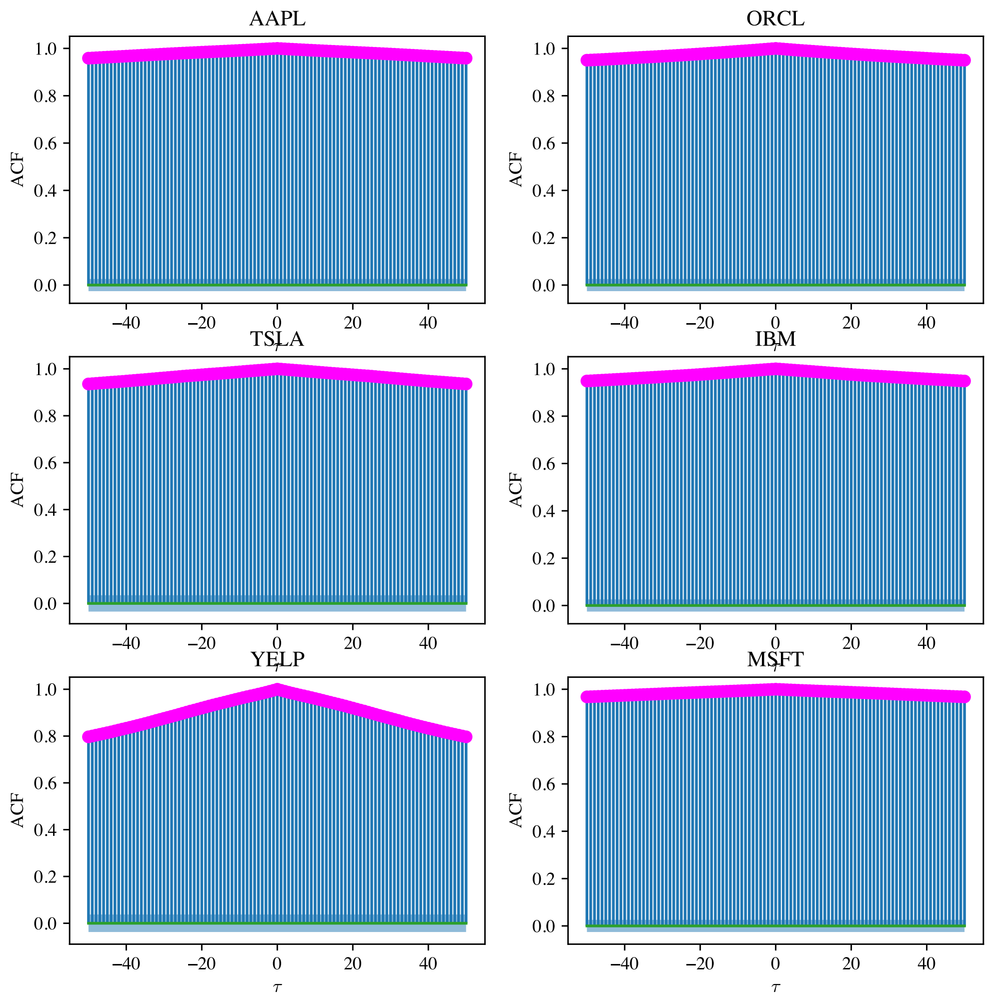

Figure 5: ACF Plots of six different stocks closing values between Jan 1st, 2000 and Jan. 31st, 2023

Problem 5

Figure 5 shows for all the six stocks, the auto-correlation slowly decreases (in a straightline) fashion as we go away from $\tau = 0$. This is a feature of non-stationary time series. In non-stationary time-series, the auto-correlation slowly decreases while in stationary time-series, it decreases fast to zero.