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$$
\begin{tikzcd}
\mathrm{Hom}(A,A) \arrow[r, "{\mathrm{Hom}(A,f)}"] \arrow[d, "\Phi_A"'] & \mathrm{Hom}(A,X) \arrow[d, "\Phi_X"] \\
F(A) \arrow[r, "Ff"'] & F(X)
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathrm{id}_A \arrow[r, mapsto] \arrow[d, mapsto] & f \arrow[d, mapsto] \\
u \arrow[r, mapsto] & (Ff)u=\Phi_X(f)
\end{tikzcd}
$$

What you needa know about Yoneda

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Motivation

- A common sentiment in many cultures is the idea that people are defined by how they interact with their surroundings.
- "*Tell me your company, and I will tell you what you are.*"[1]
- The Yoneda Lemma is the result of applying this way of thinking to mathematical objects within the extremely general setting of *Category Theory*.

[1] Quoted as a proverb in *Don Quixote*

Categories

A *category* is defined as a class of *objects* together with a class of *morphisms* (sometimes called *arrows*)

- Each morphism $f : A \to B$ relates some *source* object A to some *target* object B.
- Two morphisms $f : A \to B$ and $g : B \to C$ can be composed into a morphism $g \circ f : A \to C$
- $h \circ (g \circ f) = (h \circ g) \circ f$ (Composition is associative)

$$
\begin{tikzcd}
A \arrow[r, "f"] \arrow[dr, "g\circ f"'] \arrow[loop left, "1_A"] & B \arrow[d, "g"] \arrow[loop right, "1_B"] \\
& C \arrow[loop right, "1_C"]
\end{tikzcd}
$$

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