Equivalence of the two representations

- The Yoneda lemma tells us that for any functor F:

$$F(A) \simeq \mathbb{S}et^{\mathbb{C}}(\mathbb{C}(A, =), F)$$

Equivalence of the two representations

- The Yoneda lemma tells us that for any functor F:

$$F(A) \simeq \mathbb{S}et^{\mathbb{C}}(\mathbb{C}(A, =), F)$$

- If we remove the reference to any specific functor, we get:

$$-(A) \simeq \mathbb{S}et^{\mathbb{C}}(\mathbb{C}(A, =), -)$$

Equivalence of the two representations

- The Yoneda lemma tells us that for any functor F:

$$F(A) \simeq \mathbb{S}et^{\mathbb{C}}(\mathbb{C}(A, =), F)$$

- If we remove the reference to any specific functor, we get:

$$-(A) \simeq \mathbb{S}et^{\mathbb{C}}(\mathbb{C}(A, =), -)$$

- So we have:

$$\begin{aligned}
&\mathbb{A}daP((A, B), (S, T)) \\
= \;& \mathbb{S}et^{\mathbb{P}rof}(-(A, B), -(S, T)) \\
\simeq \;& \mathbb{S}et^{\mathbb{P}rof}(\mathbb{S}et^{\mathbb{A}da}(\mathbb{A}da((A, B), =), -), \mathbb{S}et^{\mathbb{A}da}(\mathbb{A}da((S, T), =), -))
\end{aligned}$$

Equivalence of the two representations

$$
\begin{aligned}
&\mathbb{A}daP((A,B),(S,T))\\
\simeq\ &\mathbb{S}et^{\mathbb{P}rof}(\mathbb{S}et^{\mathbb{A}da}(\mathbb{A}da((A,B),=),-),\mathbb{S}et^{\mathbb{A}da}(\mathbb{A}da((S,T),=),-))
\end{aligned}
$$

- Applying the Yoneda embedding once, we get an isomorphism between profunctor adapters and profunctors:

$$
\begin{aligned}
&\mathbb{S}et^{\mathbb{P}rof}(\mathbb{S}et^{\mathbb{A}da}(\mathbb{A}da((A,B),=),-),\mathbb{S}et^{\mathbb{A}da}(\mathbb{A}da((S,T),=),-))\\
\simeq\ &\mathbb{P}rof(\mathbb{A}da((A,B),=),\mathbb{A}da((S,T),=))
\end{aligned}
$$

- Applying the Yoneda embedding again gives an isomorphism between profunctors and adapters:

$$
\begin{aligned}
&\mathbb{P}rof(\mathbb{A}da((A,B),=),\mathbb{A}da((S,T),=))\\
=\ &\mathbb{S}et^{\mathbb{A}da}(\mathbb{A}da((A,B),=),\mathbb{A}da((S,T),=))\\
\simeq\ &\mathbb{A}da((A,B),(S,T))
\end{aligned}
$$