



# WE SHOW HOW TO HANDLE MISSING DATA in INLA\* by GENERALIZING EXISTING MEASUREMENT ERROR MODELS

FURTHER DETAILS,  
EXAMPLES and more!



\*INLA = integrated nested Laplace approximations, see Rue et al. (2009)

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**MISSING COVARIATES** cannot be directly imputed in INLA, since the covariate with missingness is part of the latent field, and INLA does not allow missing values in the latent field.

By viewing the missingness as an extreme case of **MEASUREMENT ERROR**, we can directly use existing measurement error models for INLA (Muff et al., 2015) to impute the missing values.

The measurement error model used is a **JOINT BAYESIAN MODEL**. It is adaptable to a wide variety of different situations and can be used to account for both missing data and **MEASUREMENT ERROR**.

## MODEL OF INTEREST

$$\eta = \beta_0 \mathbf{1} + \beta_x \mathbf{x} + \mathbf{Z} \boldsymbol{\beta}_z$$

$\eta$  is the linear predictor in a generalized linear model (GLM), given the true covariate values for  $x$ , as well as other covariates  $Z$ , which are observed without error.

## ERROR MODEL

$$\mathbf{w} = \mathbf{x} + \mathbf{u}, \quad \mathbf{u} \sim \mathcal{N}(0, \sigma_u^2 \mathbf{I})$$

$\mathbf{u}$  is the error in the observed variable  $w$ .

## IMPUTATION MODEL

$$\mathbf{x} = \alpha_0 + \mathbf{Z} \boldsymbol{\alpha}_z + \boldsymbol{\varepsilon}_x, \quad \boldsymbol{\varepsilon}_x \sim \mathcal{N}(0, \sigma_{\varepsilon_x}^2 \mathbf{I})$$

Describes the true covariate  $x$ , which possibly depends on the correctly observed covariates  $Z$ .



FIGURE 1: The continuum of measurement error, with observation-level priors illustrated in the top row. Inspired by Blackwell et al. (2017)

## REFERENCES

- Muff, S., Riebler, A., Held, L., Rue, H., and Saner, P. (2015). Bayesian analysis of measurement error models using integrated nested Laplace approximations. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 64(2):231–252.
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