



# MISSING COVARIATE IMPUTATION FOR INLA

*using*

# MEASUREMENT ERROR MODELS

FURTHER DETAILS,  
EXAMPLES and more!



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**MISSING COVARIATES** cannot be directly imputed in INLA, since the covariate with missingness is part of the latent field, and INLA does not allow missing values in the latent field.

By viewing the missingness as an extreme case of **MEASUREMENT ERROR**, we can directly use existing measurement error models for INLA to impute the missing values.

The measurement error model used is a **JOINT BAYESIAN MODEL**. It is adaptable to a wide variety of different situations and can be used to account for both missing data and **MEASUREMENT ERROR**.

## MODEL OF INTEREST

$$\eta = \beta_0 \mathbf{1} + \beta_x \mathbf{x} + \mathbf{Z} \boldsymbol{\beta}_z$$

$\eta$  is the linear predictor in a generalized linear model (GLM), given the true covariate values for  $x$ , as well as other covariates  $Z$ , which are observed without error.

## ERROR MODEL

$$\mathbf{w} = \mathbf{x} + \mathbf{u}_c, \quad \mathbf{u}_c \sim \mathcal{N}(0, \tau_{u_c} \mathbf{D}_{u_c})$$

$u_c$  is the error in the observed variable  $w$ .

## IMPUTATION MODEL

$$\mathbf{x} = \alpha_0 + \mathbf{Z} \boldsymbol{\alpha}_z + \boldsymbol{\varepsilon}_x, \quad \boldsymbol{\varepsilon}_x \sim \mathcal{N}(0, \tau_{\varepsilon_x} \mathbf{D}_{\varepsilon_x})$$

Describes the true covariate  $x$ , which possibly depends on the correctly observed covariates  $Z$ .

# A joint bayesian framework for MISSING data and measurement in INLA

## A JOINT BAYESIAN FRAMEWORK FOR MEASUREMENT ERROR AND MISSING DATA

**Reformulation to a measurement error problem allows us to impute missing covariates using INLA**

### REFERENCES

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