Modelling the economy and climate together: a two-region model

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Abstract

This paper presents a stock-flow consistent coupled macroeconomic and climate model of two distinct economic regions within a single country. The dynamics of the two-region model are explored and compared to a one-region model from the literature. The effects of climate change damages and different carbon pricing policies are examined. Sensitivity analysis is conducted on the model's most important parameters and on the climate damage curve used. I find that government intervention can limit the effect of global warming, but that no plausible intervention will keep the world to within its +2°C Paris Agreement goal.

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Contents

1 Introduction

As scientists have been telling us for many decades, anthropogenic climate change threatens to permanently alter life on our planet. In Canada, we are already feeling the effects of climate change through more forest fires, flooding, and other extreme weather events, and experts predict this trend will continue, as a recent government report has made clear (Natural Resources Canada, 2019)[1]. There is also increased recognition that climate change will affect more than just our environment, but will also affect economic outcomes and even has the potential to cause a financial crisis (Carrington, 2018)[2]. As policy makers decide the best way to combat climate change, they need models that integrate economic and climate outcomes in order to select the best policy.

In economics, this type of combined economic and climate model is called an Integrated Assessment Model (IAM). The most famous IAM is William Nordhaus's DICE model (Nordhaus, 1994)[3]. Nordhaus recently received the Nobel Memorial Prize in Economic Sciences for his work in this area. These models are influential, and are used to make projections by the Intergovernmental Panel on Climate Change (IPCC, 2018)[4], among others. However, most IAMs include as their economic model a general equilibrium model or another optimal behaviour model, which presupposes that economic crises can only occur as a result of some external shock. General equilibrium models are also not particularly well-suited to be combined with a climate module, as many assumptions underlying the economic models (such as perfect foresight of agents and sustained economic growth) do not make sense in the context of a possible climate catastrophe (see Stern (2016)[5]).

Another problem with IAMs is that their predictions about whether or not the government should intervene to prevent climate change are highly dependent on the 'discount rate', an arbitrary weighing of how much we value the present of the future. If authors put a greater weight on the future, the recommendation is that the government should intervene more aggressively against climate change, and if they weigh the present more, they find little need for government intervention (Schoder, 2017)[6]. This means debates in the field can boil down to arguments over how to value the future (for example, see Nordhaus' criticism of Stern (2007)[7]). Efforts have been made to formalize the choice of discount rate, but there is a clear need for a model that eliminates this concept altogether.

General equilibrium economic models are not just the subject of criticism when it comes to climate economics, but also in general, especially since the 2007-08 financial crisis. Many critics of general equilibrium models have looked to the work of American economist Hyman Minsky. In his famous "Financial Instability Hypothesis," Minsky argued that the financial system was inherently unstable, so economic crises should be an expected outcome of our economic system (Minsky, 1986)[8]. Given that both the interconnectedness and complexity of global financial markets have increased markedly in the last few decades, if anything we should expect the economy to be even more unstable than he postulated.

In 1995, economist Steve Keen published a non-linear dynamical system model that quantified Minsky's ideas (Keen, 1995)[9]. The model was based on economist Richard Goodwin's Lotka-Volterra inspired growth model, first proposed in 1967 (Goodwin, 1967)[10]. The model described the evolution of the wage share, the employment rate, and private debt in a closed economy. Keen's work represents an alternative to traditional economic models, and since its publication many have built upon it. One extension is the creation of full stock-flow consistent models that are based on this framweork, such as Godley and Lavoie (2012)[11] and Dafermos et al. (2017)[12].

Gaël Giraud, Florent Mc Isaac, Emmanuel Bovari and others from the *Agence française de développement* (AFD) (Bovari et al., 2018a, 2018b)[13][14] have developed a one country stock-flow consistent model that, like the DICE model, also includes the climate. They use this model to explore the intertwined outcomes of the economy and the environment. The economic and climate parts of the model are linked through emissions, as production causes CO_2 emissions which cause temperatures to increase, and through a 'damage curve'. The damage curve quantifies the effect of global warming on the economy, and increased temperatures lead to greater economic penalties.

Their model is a 16 dimensional deterministic nonlinear dynamical system. They show that, assuming no inflation and no emissions, the system has two equilibria: zero and a positive equilibrium. This positive equilibrium, where output continue to grow along a balanced path, is stable as long as the wage share decreases and debt increases, or vice versa. They also show that the inclusion of the climate aspect of the model can knock the model away from the steady state. If the damage curve is convex enough, the economy is knocked off its steady growth path and collapses to zero.

For this reason, the convexity of the damage function has a significant effect on the outcome of the model. The inverse quadratic damage curve in the DICE model generates only modest damages, and has been criticized as unrealistic (see Weitzman (2012)[15] and Dietz and Stern (2015)[16]). Both Weitzman and Dietz and Stern suggest alternative, more convex specifications. These damages can either be assigned to output or to capital (the latter is a more significant form of damage as production depends on the capital stock). Alternatively, following Burke et al. (2015)[17], one can assign damages directly to productivity. Burke et al. found a quadratic relationship between climate change and labour productivity, so a greater deviation in temperature make labour less effective. Bovari et al. (2018a)[13] compare results across damage specifications, and show how the outcome depends on the choice of damage curve. They find that if damages are assigned to capital, the initial conditions given by today's world economy are outside the basin of attraction of the good economic steady state unless there is public policy intervention.

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In terms of public policy, Bovari et al. (2018b)[14] explore the effects of both a carbon tax alone and a carbon tax combined with subsidies to reduce the cost of climate change abatement. They find that neither public policy keeps global warming to with the $+2°$C Paris Agreement goal. Without public policy, temperatures increase by $3.96°$C, but this increase can be reduced to $2.37°$C when the government intervenes with both a carbon tax and subsidy. Relative to the business-as-usual policy scenario, this government intervention reduces output by about 1% in 2050 but actually leads to slightly greater output by 2100.

Below, I describe a model that is similar to that of Bovari et al., but has two economic regions. This model allows a wider range of policy scenarios to be considered. For example, one can imagine a rich, industrialized country that is reducing emissions, and a developing country with greater emissions. Will damages caused by climate change driven by the developing country be enough to create an economic crisis in the developed country, and if so, how can the developed country prevent this? This is the type of question a two-region stock flow consistent macro-climate model can answer.

2 Model

In this paper, I present a two region coupled climate and stock-flow consistent macroeconomic model. The model is based on the one country model of Gaël Giraud, Florent Mc Isaac, Emmanuel Bovari and others from AFD (see Bovari et al. (2018a)[13] and Bovari et al. (2018b)[14]). However, this model has two distinct regions, which I call "North" (N) and "South" (S). In both regions, there are households, firms, and banks. There is one government for the whole country. For simplicity I assume that the government's role in the model is only to impose the carbon tax and to provide revenues for green technology subsidies, so the government does not make other taxing and spending decisions.

The main difference between the two regions is that they have separate capital stocks and different labour forces. In the description of the model below, variables and parameters with the subscript i may be different for each region, whereas variables and parameters without subscript are unique.

2.1 Stock-flow consistent accounting framework

As described above, these models are based on the stock-flow consistent framework. Table 1 shows the balance sheet, transaction items, and flow of funds in the economy. The table is based on the one presented in Bovari et al. (2018b)[14], but expanded to include the two regions in the household, firm, and banking sectors. The government sector remains unified as I assume there is one government in the country. Following the standard approach in the literature, I include all of the economy in the stock-flow table for consistency, though some of these items are set to zero in the model I present below.

The balance sheet items are stock items measured in money. Households can hold bank deposits,

	Households		Firms				Banks		Public Sector	Sum
Balance Sheet	South	North	South		North		South	North		
Capital Stock			pK_S		pK_N					$pK_S + pK_N$
Deposits	M_S^h	M_N^h	M_S^f		M_N^f		$-M_S$	$-M_N$		
Loans			$-L_S^f$		$-L_N^f$		L_S^f	L_N^f		
Bonds	B_S	B_N							$-B_S - B_N$	
Equities	E_S	E_N	$-E_S$		$-E_N$					
Sum (net worth)	S_S^h	S_N^h	$X_S^f = 0$		$X_N^f = 0$		$X_S^b = 0$	$X_N^b = 0$	$-B_S - B_N$	X
Transactions			current		capital					
	South	North	South	North	South	North	South	North		
Consumption	$-pC_S$	$-pC_N$	pC_S	pC_N						
Investment			pI_S	pI_N	$-pI_S$	$-pI_N$				
Gov. Spend.			pG_S	pG_N					$-pG_S - pG_N$	
Gov. Spend.			$-pT_S$	$-pT_N$					$p(T_S + T_N) = p(G_S + G_N)$	
Wages	$w_S L_S$	$w_N L_N$	$-w_S L_S$	$w_N L_N$						
Capital depr.			$-p\delta_{\mathbf{D}_S^K} K_S$	$-p\delta_{\mathbf{D}_N^K} K_N$	$p\delta_{\mathbf{D}_S^K} K_S$	$p\delta_{\mathbf{D}_N^K} K_N$				
Carbon taxes			$-pT_S^f$	$-pT_N^f$					$pT_S^f + pT_N^f$	
Abatement subsidies			pS_S^f	pS_N^f					$-pS_S^f - pS_N^f$	
Int. on loans			$-r^c L_S^c$	$-r^c L_N^c$			$r^c L_S^c$	$r^c L_N^c$		
Int. on deposits	$r^M M_S^h$	$r^M M_N^h$	$r^M M_S^f$	$r^M M_S^f$			$-r^M M_S$	$-r^M M_N$		
Int. on bonds	$r^B B_S^b$	$r^B B_N^b$							$-r^B B_S - r^B B_N$	
Banks' dividends	Π_S^b	Π_N^b					$-\Pi_S^b$	Π_N^b		
Firms' dividends	Π_S^d	Π_N^d	$-\Pi_S^d$	Π_N^d						
Sum (balance)	S_S^h	S_N^h	Π_S^r	Π_N^r	$-pI_S + p\delta_{\mathbf{D}_S^K} K_S$	$-pI_S + p\delta_{\mathbf{D}_N^K} K_N$	S_S^b	S_N^b	$S^g = 0$	
Flow of Funds	South	North	South		North		South	North		
Change in capital stock			$p\dot{K}_S$		$p\dot{K}_N$					$p\dot{K}_S + p\dot{K}_N$
Change in deposits	$\dot{M}_S^h$	$\dot{M}_N^h$	$\dot{M}_S^f$		$\dot{M}_S^f$		$-\dot{M}_S$	$-\dot{M}_N$		
Change in loans			$-\dot{L}_S^c$		$-\dot{L}_S^c$		$\dot{L}_S^c$	$\dot{L}_S^c$		
Change in bills	$\dot{B}_S^b$	$\dot{B}_N^b$							$-\dot{B}_S^b - \dot{B}_N$	
Column sum (savings)	S_S^h	S_N^h	Π_S^r		Π_N^r		S_S^b	S_N^b	S^g	
Change in firm equity	$\dot{E}_S^f$	$\dot{E}_N^f$	$-\Pi_S^r - \dot{p}K_S$		$-\Pi_N^r - \dot{p}K_N$					
Change in bank equity	$\dot{E}_S^b$	$\dot{E}_N^b$	$-\Pi_S^r - \dot{p}K_S$		$-\Pi_N^r - \dot{p}K_N$					
Change in net worth	$S_S^h + \dot{E}_S$	$S_N^h + \dot{E}_N$	0		0		0	0	S^g	$\dot{p}(K_S + K_N) +$ $p(\dot{K}_S + \dot{K}_N)$

Table 1: Balance sheet, transactions, and flow of funds in the economy with two regions, North and South

M^h, shares in firms, E, or government bonds, B. Firms can hold bank deposits, M^f, and have the shares held by households, E, and loans from banks, L^f, as liabilities. Banks' liabilities are deposits $M = M^h + M^f$ and their assets are the loans made to firms, L^f. Finally, the public sector can issue bonds B, a liability. As I assume that the government's role is only to impose the climate change policy (carbon taxes and/or subsidies), so in my model other government taxes, other government spending, and government borrowing (bonds) are all zero. For simplicity, I assume that the net worth of firms and banks is zero.

The transactions and flow of funds are measured in money per unit of time, and are presented below in the detailed description of the model.

2.2 Macroeconomic module

Each region produces output Y_i^0 using capital K_i:

$$Y_i^0 := \frac{K_i}{\nu_i} \tag{1}$$

where $1/\nu_i$ is the productivity of capital. However, damages to output caused by climate change, $\mathbf{D}_i^Y$, and losses resulting from climate change abatement activities, A_i (defined in section 2.4), reduce actual output to

$$Y_i := (1 - \mathbf{D}_i^Y)(1 - A_i)Y_i^0 \tag{2}$$

Labour is hired depending on the original output, Y_i^0, and labour productivity, a_i:

$$L_i := \frac{Y_i^0}{a_i} \tag{3}$$

The productivity rate grows exponentially according to

$$\dot{a}_i := \alpha_i \, a_i \tag{4}$$

The workforce in each region, N_i, is assumed to grow logistically according to

$$\dot{N}_i := \delta_{N_i} N_i \left(1 - \frac{N_i}{\overline{N}_i}\right), \tag{5}$$

where δ_{N_i} is the growth rate of each region's workforce and $\overline{N_i}$ represents the upper limit of each region's workforce.

The employment rate in each region is then computed as expected,

$$\lambda_i := \frac{L_i}{N_i} \tag{6}$$

Wages grow according to the Phillips curve relationship, represented by $\varphi_i(\cdot)$: higher employment leads to higher wage changes, as employees bargain for better wages when they have less competition. I implement two different Phillips curve specifications: a linear one as in Goodwin's original paper (Goodwin, 1967)[10], and a rational function that goes to infinity as the employment rate approaches one, originally used by Keen (1995)[9].

$$\dot{w}_i := w_i \, \varphi_i(\lambda_i) \tag{7}$$

Inflation dynamics depend on the wages in each region. To avoid discussion of exchnage rates, it is assumed that there is a unique price in the country. As in Bovari et al. (2018b)[14], inflation depends on the total wage share in the country.

$$c := \frac{w_N L_N + w_S L_S}{Y_N + Y_S} \tag{8}$$

$$\dot{p} := \eta \left(\mu(c + \iota) - p \right) \tag{9}$$

Inflation also depends on the parameters η, μ, and ι, which are the relaxation parameter, the markup price, and a calibration parameter, respectively.

Finally, the wage share in each region is defined to be

$$\omega_i := \frac{w_i L_i}{p Y_i} \tag{10}$$

2.3 Profits and investment

Following Bovari et al. (2018b)[14], firms' nominal profit in each region, Π_i, is defined to be revenue from production minus payments to labour, capital depreciation, and payments to service debt, plus any transfers from the government

$$\Pi_i := pY_i - w_i L_i - \delta_{\mathbf{D}_i^K} pK_i - r_i D_i + p\Upsilon_i. \tag{11}$$

Capital depreciation is denoted by $\delta_{\mathbf{D}_i^K}$ which is the standard depreciation rate plus any climate-related damages to capital. r_i denotes the nominal interest rate in the region, and D_i is total nominal debt in the region. The nominal interest rate is determined according to Taylor's rule by $r_i = \max\{0, r_i^* + i_i + \phi_i(i_i - i_i^*)\}$, where r_i^* is the long-term real interest rate, i_i^* is the target inflation rate determined by the central bank, and ϕ_i is a parameter that represents the magnitude of the central bank's response to inflation (Taylor, 1993)[18]. Υ_i is net public transfers in the region (defined in section 2.5).

The debt and profit to output ratios are given by:

$$d_i := \frac{D_i}{pY_i} \tag{12}$$

$$\pi_i := \frac{\Pi_i}{pY_i} \tag{13}$$

Firms either pay their profits to households as dividends, denoted Π_{d_i}, or retain them, denoted Π_{r_i}. As in Bovari et al. (2018b)[14], it is assumed that firms pay out dividends according to an increasing linear function function $\Delta(\cdot) \in [0, 1]$ of the return on assets $\pi_{K_i} := \Pi_i/pK_i$. Dividends and retained earnings are thus given by:

$$\Pi_{d_i} := \Delta(\pi_{K_i})pK_i \tag{14}$$

$$\Pi_{r_i} := \Pi_i - \Pi_{d_i} \tag{15}$$

Similarly, there is a linear investment function denoted $\kappa(\cdot) \in [0, 0.3]$ that determines the firms' demand for investment depending on the profit ratio. Investment demand also depends on the investment multiplier, denoted ρ_i:

$$I_i^d := \rho_i\, \kappa(\pi_i)Y_i \tag{16}$$

Following Bovari et al. (2018b)[14] and Dafermos et al. (2017)[12], it is assumed that firms in each region demand a certain amount of credit, denoted D_i^d, from the banking sector. This aggregate credit demand is defined by

$$D_i^d := pI_i^d + s_{rep_i} D_i - \delta_{\mathbf{D}_i^K} pK_i - \Pi_{r_i} \tag{17}$$

where s_{rep_i} is a parameter determining how much of the debt firms must pay back each time step.

The banks in each region satisfy this demand for credit according to a credit rationing function, given by

$$CR_i := \tau_i \left(\frac{D_i}{pK_i} \right), \tag{18}$$

where $\tau_i(\cdot) \in [0,1]$ is an increasing linear function of the leverage ratio. This credit rationing function designates an upper limit, set by the banking sector in each region, on the supply of credit in the market. Then the nominal debt dynamics in each region are given by

$$\dot{D}_i := (1 - CR_i)D_i^d - s_{rep_i} D_i. \tag{19}$$

Finally real investment, I_i, and the rate of change of capital, $\dot{K}_i$, in each region are given by:

$$I_i := CR_i \left(\frac{\Pi_{r_i}}{p} + \delta_{\mathbf{D}_i^K} K_i - \frac{s_{rep_i} D_i}{p} \right) + (1 - CR_i) I_i^d \tag{20}$$

$$\dot{K}_i := I_i - \delta_{\mathbf{D}_i^K} K_i. \tag{21}$$

2.4 Emissions and climate change abatement

A major linkage between the economic module and the climate module is through emissions. Here, as in Bovari et al. (2018b)[14] I use the climate framework laid out by Nordhaus in his DICE model (Nordhaus, 1994)[3]. It is assumed that industrial emissions, E_{ind_i}, in each region depend on total output, the region's carbon intensity σ_i, and its emissions reduction rate, n_i:

$$E_{ind_i} := \sigma_i (1 - n_i) Y_i^0 \tag{22}$$

Each economy's carbon intensity decreases according to the coupled equations

$$\dot{\sigma_i} := g_{\sigma_i} \sigma_i \tag{23}$$

$$\dot{g_{\sigma_i}} := \delta_{g_{\sigma_i}} g_{\sigma_i} \tag{24}$$

where $\delta_{g_{\sigma_i}} < 0$.

Land-use emissions in the entire world are assumed to be exogenous and decrease at the rate

$\delta_{E_{land}} < 0$:

$$\dot{E}_{land} := \delta_{E_{land}} E_{land} \tag{25}$$

Global CO_2 emissions are assumed to be the sum of industrial emissions in each region and land-use emissions:

$$E_T := E_{land} + E_{ind_N} + E_{ind_S} \tag{26}$$

As in Nordhaus (2018)[19], I assume there exists a green 'backstop' technology, that firms may switch to at some cost. The price of the backstop technology decreases exogenously over time according to

$$\dot{p_{BS}} := \delta_{p_{BS}} p_{BS}, \tag{27}$$

where $\delta_{p_{BS}} < 0$.

The previously mentioned abatement cost, A_i, in each region depends on the price of the backstop technology, p_{BS}, the region's carbon intensity, σ_i, and the region's emissions reduction rate, n_i. Following Bovari et al. (2018b)[14], let

$$A_i := \frac{\sigma_i \, p_{BS}}{\theta} \, n_i^{\theta}, \tag{28}$$

where the parameter $\theta > 0$ controls the convexity of the cost.

2.5 Public sector

The government in each region sets a price on carbon, p_{C_i}. The initial price of carbon and the path it follows over time are assumed to be set exogenously by the government. Due to the carbon tax, the government collects tax revenues

$$T_i^C := p_{C_i} E_{ind_i} \tag{29}$$

The government may also subsidizes a fraction s_{A_i} of the firms' abatement cost in each region. In this case the government incurs a cost of

$$S_i^C := s_{A_i} A_i Y_i^0. \tag{30}$$

Therefore, net transfers from the public sector to the private sector in each region are given by

$$\Upsilon_i := S_i^C - T_i^C. \tag{31}$$

Finally, as in Bovari et al. (2018b)[14], the emissions reduction rate n_i in each region results from an arbitrage between the carbon price, p_{C_i}, and the cost of backstop technology, p_{BS} (possibly subsidized by the government):

$$n_i = \min\left\{ \left(\frac{p_{C_i}}{(1 - s_{A_i}) p_{BS_i}} \right)^{\frac{1}{\theta - 1}}, 1 \right\} \tag{32}$$

2.6 Climate Module

The carbon cycle is modeled in three layers. CO_2 emissions can accumulate in the atmosphere (CO_2^{AT}), the upper ocean and biosphere (CO_2^{UP}), or the lower ocean (CO_2^{LO}). CO_2 accumulation evolves according to:

$$\begin{pmatrix} \dot{CO_2}^{AT} \\ \dot{CO_2}^{UP} \\ \dot{CO_2}^{LO} \end{pmatrix} = \begin{pmatrix} E_T \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -\phi_{12} & \phi_{12} C_{UP}^{AT} & 0 \\ \phi_{12} & -\phi_{12} C_{UP}^{AT} - \phi_{23} & \phi_{23} C_{LO}^{UP} \\ 0 & \phi_{23} & -\phi_{23} C_{LO}^{UP} \end{pmatrix} \begin{pmatrix} CO_2^{AT} \\ CO_2^{UP} \\ CO_2^{LO} \end{pmatrix} \tag{33}$$

where $C_i^j = \frac{C_{j_{preind}}}{C_{i_{preind}}}$ for $i, j \in \{AT, UP, LP\}$ and ϕ_{12}, ϕ_{23} are parameters.

The accumulation of CO_2 also increases radiative forcing, F, of CO_2. This is modeled as follows, where F_{dbl} is an exogenous parameter that represents the effect on forcing of a doubling of pre-industrial CO_2 levels, and F_{exo} increases exogenously over time.

$$F_{ind} := \frac{F_{dbl}}{\log(2)} \log\left(\frac{CO_2^{AT}}{C_{AT_{preind}}} \right) \tag{34}$$

$$F := F_{ind} + F_{exo} \tag{35}$$

Finally, radiative forcing, F, affects temperature, T. Mean temperature is divided into two layers, the atmosphere, land, and upper ocean layer, T, and the lower ocean layer, T_{LO}. C and C_{LO} are the heat capacities of each layer, while γ^* is a parameter representing the heat exchange between layers, and S is the equilibrium climate sensitivity parameter. Temperature evolves according to the following two equations:

$$\dot{T} = \frac{F - \frac{F_{dbl}}{S} T - \gamma^*(T - T_{LO})}{C} \tag{36}$$

$$\dot{T_{LO}} = \frac{\gamma^*(T - T_{LO})}{C_{LO}} \tag{37}$$

2.7 Damages

2.7.1 Damages to Output and Capital

As in Bovari et al. (2018a)[13], it is assumed that climate change can affect the economy by reducing output and/or capital. Climate change damages are based on the mean temperature deviation, T. Following Dietz and Stern (2015)[16], the damage function for region i is defined to be

$$\mathbf{D}_i = 1 - \frac{1}{1 + \xi_{1_i} T + \xi_{2_i} T^2 + \xi_{3_i} T^{\zeta_i}} \tag{38}$$

where ξ_{1_i}, ξ_{2_i}, ξ_{3_i}, ζ_i are given parameters. The total damages are then allocated between output and capital according to a fraction $f_{k_i} \in [0, 1]$:

$$\mathbf{D}_i^K = f_{k_i} \mathbf{D} \tag{39}$$

$$\mathbf{D}_i^Y = (1 - f_{k_i}) \mathbf{D} \tag{40}$$

2.7.2 Damages to Labour Productivity

An alternate specification of climate change damages assigns them to labour productivity. Burke et al. (2015)[17] measured the effect of temperature on labour productivity, using data from 166 countries from 1960-2010. They found a quadratic relationship between temperature and labour productivity, with a maximum at 13°C. As in Bovari et al. (2018a)[13], I adopt this effect of temperature on the economy as a damage specification in some runs. In this case, equation (4) is redefined as:

$$\frac{\dot{a}}{a} = \alpha_1 (T_{preind} + T) + \alpha_2 (T_{preind} + T)^2 \tag{41}$$

3 Results

3.1 Dynamics of the two region model

The model boils down to a system of 24 differential equations: eight equations that exist for both the North and the South, and an additional eight equations for other variables. If the two regions are identical, the model is the same as the one country model described in the literature. However, when the regions are differentiated, different dynamics are possible. The two region model does not appear to approach an equilibrium point, as is shown below, and economic collapse appears to be more likely.

3.1.1 Two identical regions (one region)

As Bovari et al. (2018a)[13] show, the one-country model can be solved analytically if one assumes zero inflation and no climate module. They show that in this case there is a single positive equilibrium, stable as long as the wage share decreases and debt decreases, or vice versa, and the zero equilibrium.

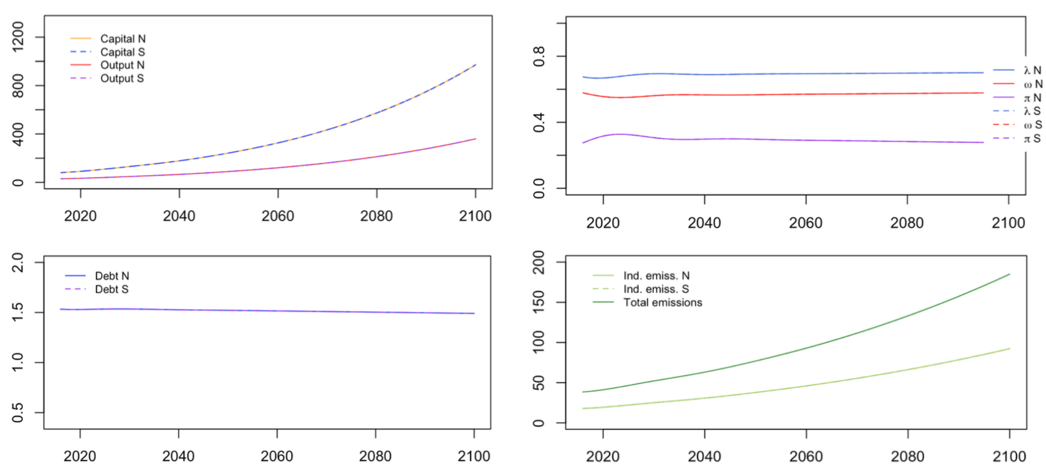


Figure 1: Results for two regions with the same initial conditions and the same parameters. There are no damages and no climate policy. Capital and output grow smoothly. Key economic variables, employment, λ, wage share, ω, and profit, π converge to the positive equilibrium. The debt level in the economy is decreasing slightly over time. Emissions grow smoothly.

When the model is simulated with one region (or equivalently two identical regions), the dynamics are those described in Bovari et al. (2018b)[14]. Without climate damages, the economy converges to the positive equilibrium, as is shown in Figure 1. This means capital and output grow smoothly, and therefore so do emissions.

However this rosy economic picture omits the effect on the global temperature. Continued economic growth creates continued growth in emissions, which causes greater global warming. As shown in Figure 2, temperature continues to grow smoothly, reaching nearly +5°C by 2100. Without government intervention, the world blows past its Paris Agreement goals: the +2°C limit is surpassed in 2047.

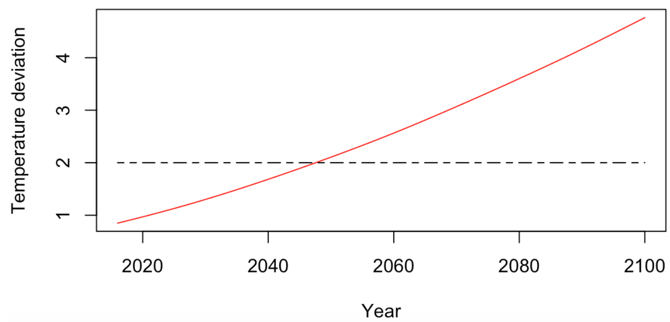


Figure 2: Deviation in temperature from per-industrial levels over time, measured °C. The dashed line marks the +2°C goal of the Paris Agreement. Without intervention, the world passes this threshold in 2047.

3.1.2 Two differentiated regions

The dynamics are more complex once the second region is introduced. In Figure 3, I simulate the model with two different regions: North and South. The idea is that the model represents a ‘Global North’ region of industrialized, developed countries like Canada, the United States, and Europe, and a ‘Global South’ region of developing countries. I assume that the North is economically stronger than the South, with a higher capital stock and higher employment rate. The North also starts with a higher emissions reduction rate, but with more debt. The initial conditions and parameters are derived from Bovari et al. (2018b)[14], modified as appropriate for the two regions. The exact initial conditions and parameters are given in the Appendix.

As is shown in Figure 3, the two regions feed off each other. A downturn in one region benefits the other region, and vice versa. This interplay can be seen in the paths of key economic variables employment, λ, wage share, ω, and profit, π, which oscillate. This oscillation affects the paths of the other state variables as well. Capital and output grow in general, with dips when employment drops in the region. The debt levels also oscillate in opposite directions. Emissions grow as a result of the growth in output. The oscillations have no impact on temperature, however. Without government intervention, temperatures continue to grow, as is shown in Figure 2.

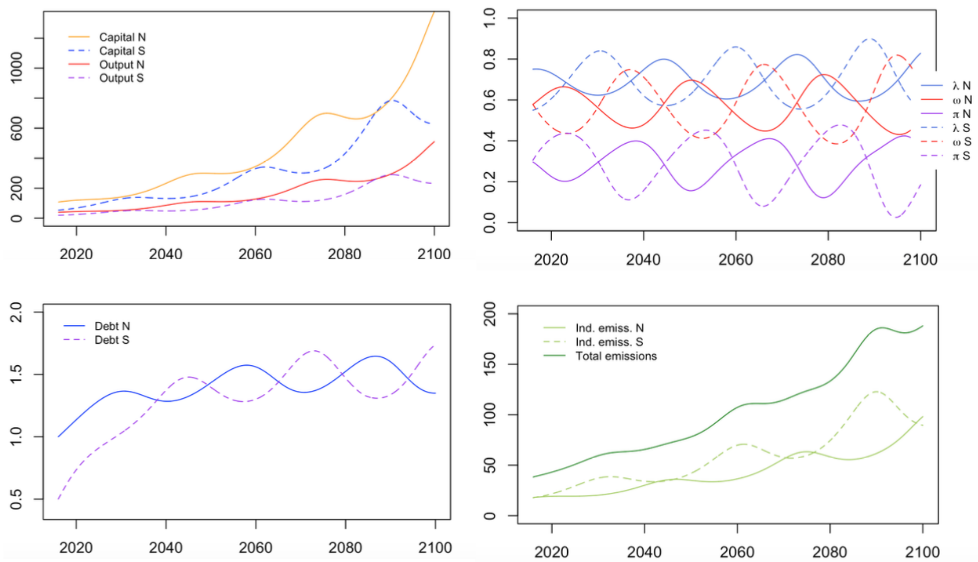


Figure 3: Results for two regions with differences in initial conditions and parameters. There are no damages and no climate policy. Key economic variables employment, λ, wage share, ω, and profit, π, oscillate. The two regions play off each other, as a downturn in one benefits the other. The amplitude of the oscillations grows over time. The oscillations affect the other state variables. Capital and output grow in general, with dips when employment drops in the region. The debt levels also oscillate in opposite directions. Emissions grow as a result of the growth in output.

3.1.3 Changing the Phillips curve

One feature of the model is that the amplitude of the oscillations of the key economic variables grows over time. This is not very noticeable when the simulation ends in 2100, but creates a problem when the simulation runs for longer. Given that employment and wage share are given as a fraction, its is not sensible for them to exceed one or be negative. Figure 4 presents the paths of the key economic variables employment, λ, wage share, ω, and profit, π, until 2400. Over time, the employment rate and the wage share begin to exceed one (which is demarcated with a dashed line). This does not occur until 2200, but the behaviour of increasing oscillation in economic variables is not desirable.

For this reason, I chose to implement an alternative Phillips curve specification, given in Keen (1995)[9]. The functional form of this Phillips curve is:

$$\varphi_i(\lambda_i) := \frac{\varrho_1}{1-\lambda^2} - \varrho_2 \tag{42}$$

I also use the values of ϱ_1 and ϱ_2 given by Keen. The key feature of this specification is that the employment rate cannot surpass one.

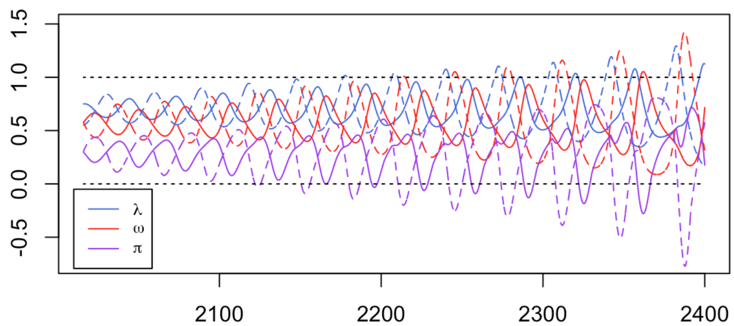


Figure 4: Paths of key economic variables employment rate, λ, wage share, ω, and profit, π, until 2400 in the two differentiated region scenario. The initial conditions and parameters are the same as in Figure 3. The amplitude of the oscillations continues to grow over time.

In the one region model, this specification does not impact the long term dynamics of the system. However, it does have an effect on the behaviour of the two-region model. Figure 5 shows the results when the linear Phillips curve is replaced with the above specification, using the same initial conditions and parameters from the two region run shown in Figure 3. The oscillations of the economic variables can no longer increase in amplitude due to the upper bound on the employment rate, λ. This also constrains the oscillation in the paths of the other variables, including debt, output, capital, and emissions. When the simulation is run for a long time, the oscillations seem to settle at a constant level, as is shown in Figure 6. It appears that the model may be reaching a limit cycle, something that could be investigated further.

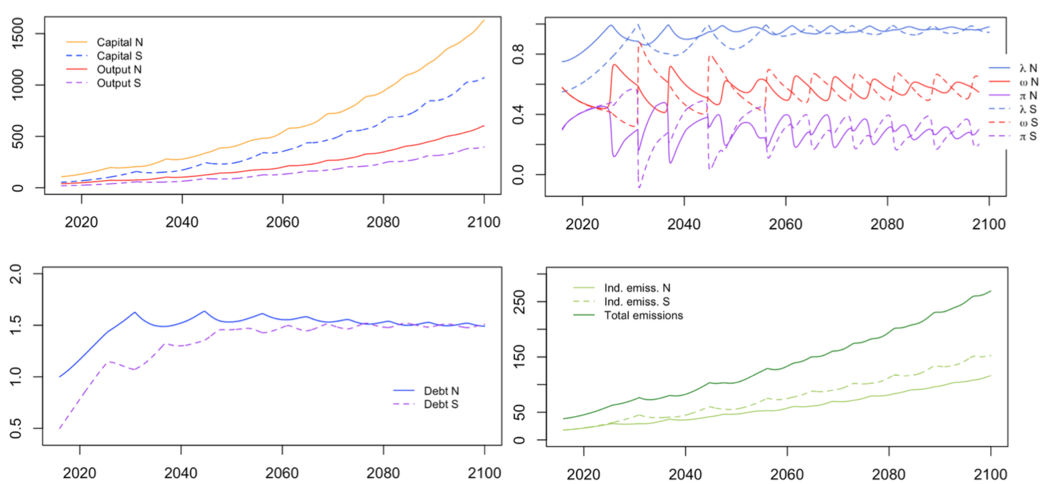


Figure 5: Results for two regions with differences in initial conditions and parameters, no damages and no climate policy, but the Keen Phillips curve specification. The amplitude in the oscillation of the employment rate, λ, wage share, ω, and profit, π, is constrained. Oscillation in the other economic variables is therefore less pronounced. The debt levels appear to converge.

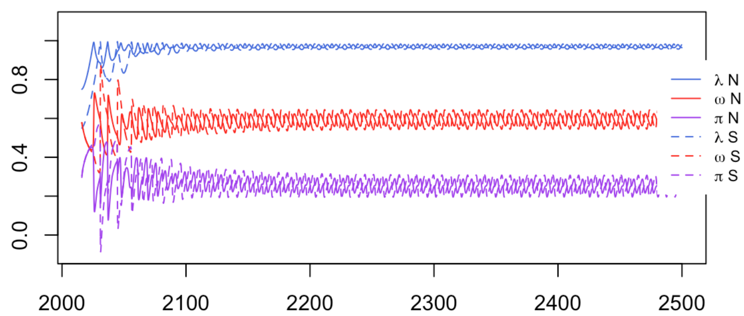


Figure 6: Paths of key economic variables employment rate, λ, wage share, ω, and profit, π, until 2500 in the two differentiated region scenario with the Keen Phillips curve. The amplitude of the oscillations over time appears to converge.

3.2 Sensitivity analysis

I follow Bovari et al. (2018b)[14] and perform sensitivity analysis on three uncertain parameters: the growth rate of labour productivity, α_i, the climate sensitivity level, S, and pre-industrial concentration of CO_2 in the biosphere and upper ocean layer, C^{UP} (also called the size of the intermediate reservoir layer). These parameters are identified by Nordhaus (2018)[19] as particularly uncertain. The parameters for labour productivity growth, the climate sensitivity parameter, and the size of the biosphere and upper ocean layer are drawn randomly from the following probability density functions, given in Nordhaus (2018)[19]: labour productivity growth is drawn from a normal distribution with mean 2.06% and a standard deviation of 1.12%; equilibrium climate sensitivity is drawn from a log-normal distribution with mean 1.107 and standard deviation 0.264 (on the log scale); and the size of the biosphere and upper ocean layer is drawn from a log-normal distribution with mean 5.8856 and standard deviation 0.2513 (on the log scale).

The results for two identical regions are shown in Figure 7, while those for two differentiated regions are shown in Figure 8. In the latter case, labour productivity in the North and South are different draws from the distribution described above. In both cases, there are no climate damages, no climate policy, and the linear Phillips curve specification is used. The figures show the mean and the 95% confidence interval for the paths of debt in both regions, the wage share in both regions, output in both regions, inflation, emissions, and temperature. When the initial conditions are the same, the model converges to the positive equilibrium. When the regions are different, we again see oscillation in the economic variables, so the existence of this oscillation is not sensitive to changes in these three parameters. In both cases, the wage share is between zero and one, and emissions and output are non-negative, as they should be. Without climate damages or climate policy, the temperature exceeds the 2°C threshold by 2080 in all cases.

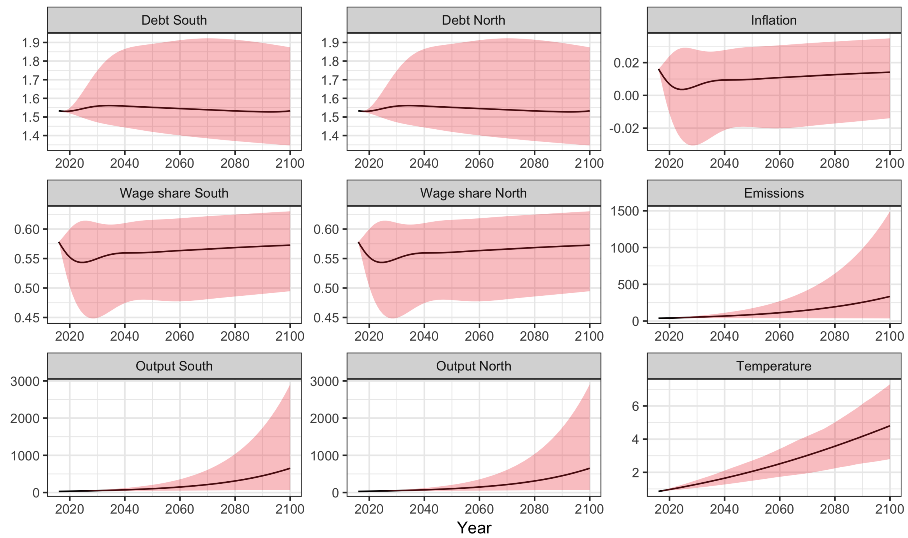


Figure 7: Monte Carlo simulation with 200 runs. The same initial conditions and parameters are used for both regions, and there is no damage and no climate policy. Plots show the mean and the 95% confidence interval. All runs give positive output and in all cases the 2°C threshold is exceeded.

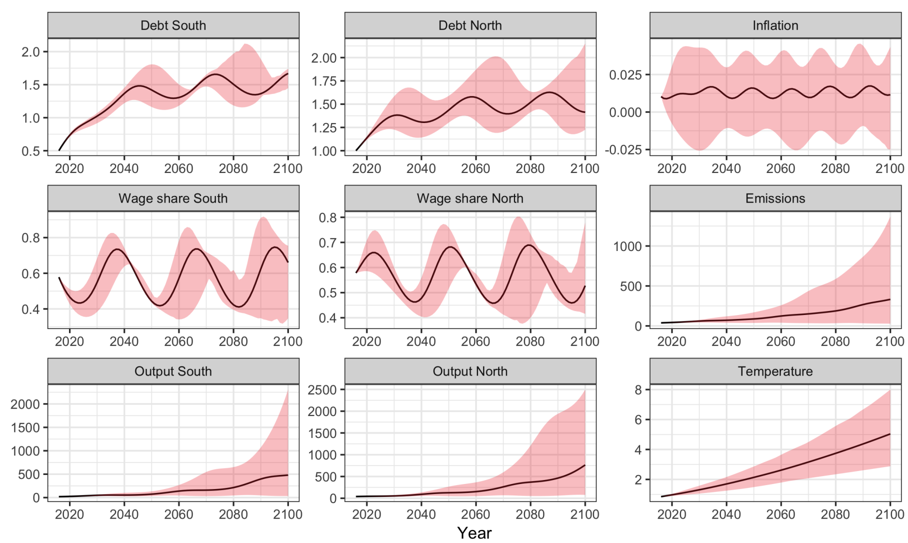


Figure 8: Monte Carlo simulation of the differentiated regions scenario, 200 runs. There are no damages and no climate policy. Plots show the mean and the 95% confidence interval. All runs give positive output and in all cases the 2°C threshold is exceeded. The oscillation in the economic variables is not sensitive to the changes in parameters.

3.3 Quantifying climate change damages

We know that increasing global warming will have an increasingly negative effect on the economy, as there will be an increased incidence of major storms, flooding, forest fires, and the like. How exactly to quantify the effect of global warming on the economy is an area of active research. As described above, many prominent economists have suggested different damage curve specifications, and the choice of damage curve has a significant effect on the outcome of the model.

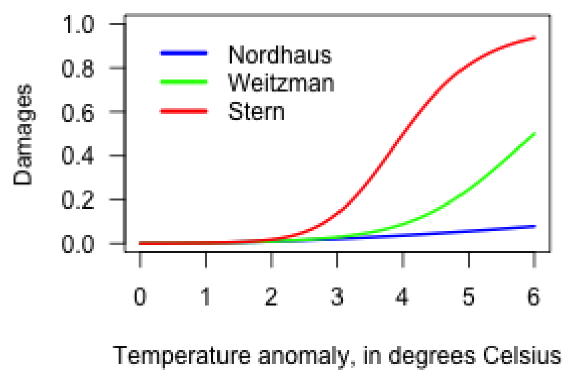


Figure 9: Comparison of the Nordhaus, Weitzman, and Dietz and Stern damage curve specifications. Damages are given as a percentage of output.

To explore the different effects of the damage curves, I implemented four different specifications of the damage curve. In order of convexity, the damage curves are that of Nordhaus (2018)[19], Weitzman (2012)[15], Dietz and Stern (2015)[16]. The fourth specification is that of Burke et al. (2015)[17] which affects labour productivity instead of output. Figure 9 shows the first three damage curves. All damage curves predict only 1% damage in a +2°C world. The Nordhaus curve predicts low impacts of global warming on output, even if the world were to warm to a catastrophic +5°C. In contrast, the Stern curve predicts 20% damages at +3°C. The curve suggested by Weitzman falls in the middle. It is worth noting that Nordhaus's specification, which is widely used in economics as part of his DICE model, predicts only 2% damages at +3°C, while experts suggest climate change of that magnitude would lead to flooding of major population centres, massive food shortages, and major weather events.

As is shown in Figure 10, the choice of damage curve changes the outcome of the model. The Nordhaus damage curve has very little effect on key economic variables before 2100, with output, employment, and the wage share only slightly reduced in 2100, and debt only slightly higher. The intermediate Weitzman curve predicts a levelling off of output by 2100, with a lower employment rate and wage share and higher debt. The most convex curve, the Stern specification, leads to

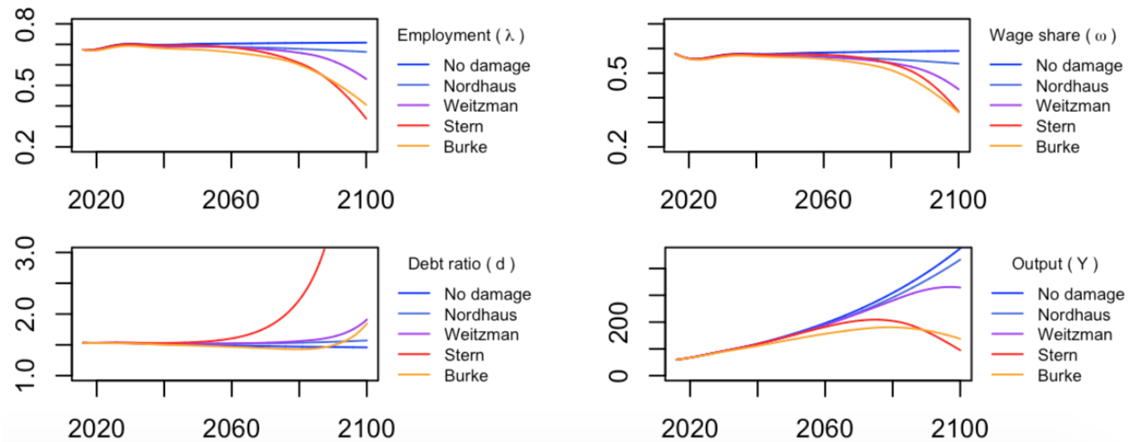


Figure 10: Simulation of the one country (two identical regions) model with different damage specifications. Initial conditions are from Bovari et al. (2018b). The outcome of the model depends on the choice of damage function, as the use of less convex Nordhaus specification leads to predicted continued economic growth, whereas the more convex Stern specification leads to decreasing output after 2080.

economic collapse driven by high debt. Employment and the wage share drop significantly by 2100. The alternative Burke specification also leads to economic collapse, but in this case it is not caused by increased debt. This is because damages no longer impact output or capital, causing increased borrowing, but labour productivity.

Figure 11 shows the results of running a Monte Carlo simulation, altering the same three parameters as before, with different damage specifications. Here I use the differentiated regions scenario, but assume the same damage specification for both North and South. The Phillips curve is assumed to be linear, and there is no climate policy. As above, the average and the 95% confidence intervals are plotted. The least convex curve, labelled 'Nordhaus' (in orange), leads to continued economic growth, similar to the no damage scenario. The most convex curve, 'Stern', can lead to a debt fuelled collapse in the two regions (as seen by the blue spikes in the debt plots), and the possibility of a zero wage share, negative output, and negative emissions. In this model, negative output and emissions should be read as a complete economic collapse. Economic collapse before 2100 is also a possible outcome with the intermediate 'Weitzman' curve (shown in magenta). In this model, negative output and emissions should be read as a complete economic collapse.

The most convex damage specifications highlight a problem with the model specification, because several economic variables including output and emissions can become negative when they ought to be constrained to be non-negative. This is due to the way credit rationing is defined, since investment can become arbitrarily negative, pushing variables such as capital, K, output, Y, and employment, λ below zero. This is not a sensible prediction of the model, and suggests that the specification should be changed in future research. However, in practical terms, whether the model goes to zero or becomes negative, there is clearly an economic collapse.

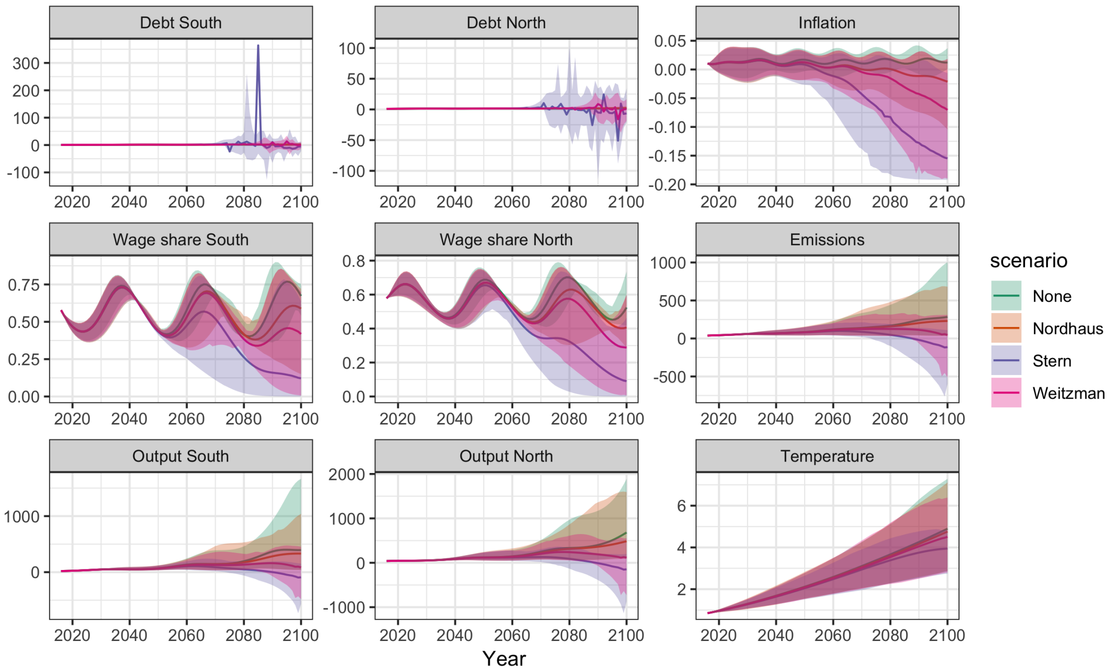


Figure 11: Monte Carlo simulation of the differentiated two region model with different damage curve specifications. The average and the 95% confidence intervals are plotted. The same damage curve is used both regions. The outcome of the model depends on the choice of damage function. The least convex specification, 'Nordhaus', leads to continued economic growth, while the most convex specification, 'Stern', can lead to a debt-fuelled economic collapse. No run keeps the globe under the 2°C goal.

In sum, the choice of damage curve specification has an enormous impact on the outcome of the model. Given that the choice of damage curve can knock an economy from a balanced growth path towards economic collapse, the estimation of the damage curve is very important. Further exploration of the damage curve, especially estimate of damage curves for different regions, is left for further research.

3.4 Government policy: Carbon taxes and green technology subsidies

In 2017 the High-Level Commission on Carbon Prices, a panel of experts assembled by the World Bank, released a report recommending carbon pricing paths aligned with the Paris Agreement climate targets (High-Level Commission on Carbon Prices, 2017)[20]. Their report recommend a carbon price of between \$40 and \$80 per tonne in 2020, and between \$50 and \$100 per tonne in 2030 (all prices in USD). In their paper, Bovari et al. (2018b)[14] considered carbon pricing scenarios corresponding to to the upper range of their recommendations. I choose to model several different carbon pricing scenarios. I create 'Low' and 'High' carbon pricing scenarios based on the upper and lower bounds of the recommendations of the High-Level Commission. I also create a scenario based on the Canadian government's recently implemented federal backstop carbon pricing plan (Environment and Climate Change Canada, 2018)[21]. The Canadian plan serves as an intermediate scheme. It starts off at a lower price than recommended, due to its late implementation, but ramps up relatively quickly. I assume initially that the price of carbon levels off at its 2030 level. I also allow for government subsidization of abatement costs. In scenarios with this subsidy, it is assumed that the government covers 50% of the costs, as in Bovari et al. (2018b)[14].

Scenario	2020	2030	Source
Low	\$40	\$50	High-Level Commission on Carbon Prices
Canada	\$22	\$90	Government of Canada
High	\$80	\$100	High-Level Commission on Carbon Prices

Table 2: Different carbon pricing scenarios

I show the deterministic results in Figure 12. The results show that, in terms of limiting global warming, any carbon price is better than none, that a higher carbon price is better than a lower one, and that combining a carbon tax with government subsidization of abatement is the most effective policy. However, these results are bad news for the planet: the most aggressive policy, high carbon prices with a subsidy (the purple line) is not nearly enough to stay under the 2°C threshold, as temperatures exceed 3°C by 2100.

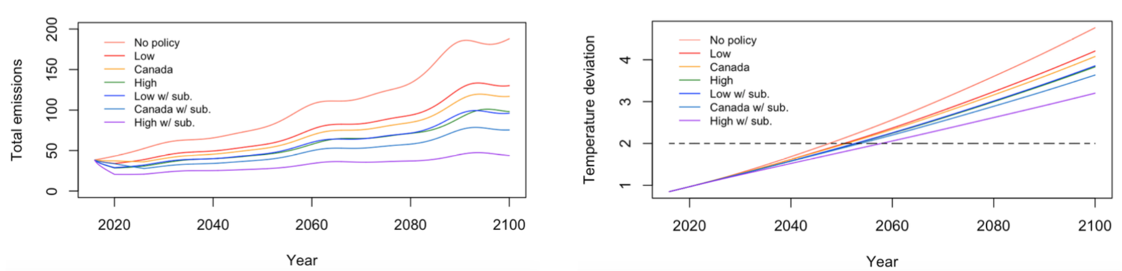


Figure 12: Emissions and temperature deviation over time with different carbon pricing and subsidy schemes, assuming a fixed carbon price after 2030. No policy keeps temperature to within 2, or even 3°C of warming.

This raises the question of what would happen if the carbon price continued to grow over time. Bovari et al. (2018b)[14] assume that the carbon price will continue to grow at the rate it grew at between 2020 and 2030 in their scenarios. I choose, however, to assume lower growth rates over the rest of the century. This is because the initial growth rate of the carbon price could be high as a country adjusts to the optimal carbon price level. For example, Canada will increase its carbon price relatively rapidly because it starts at a low price, but it is unknown what the growth rate will be once it reaches \$50/tonne. I do not think it is reasonable to assume that the carbon price will continue to grow at such a high rate (in fact, depending on political considerations, it may not continue to grow or even remain in force at all). Given this, I choose to set the final price of carbon to \$120, \$150, and \$200 for the ‘Low’, ‘Canada’, and ‘High’ specifications respectively. This means that the price of carbon continues to grow in all scenarios until 2100.

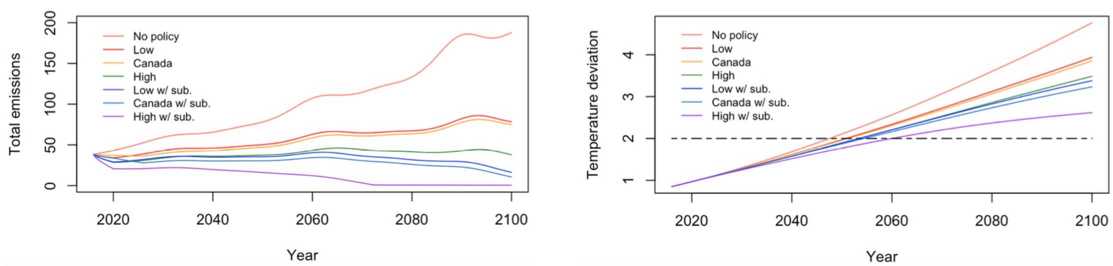


Figure 13: Emissions and temperature deviation over time with different carbon pricing and subsidy schemes, assuming continued growth in carbon price levels until 2100. No policy keeps temperature to within 2°C of warming, but the most aggressive policy, a high price with a subsidy, limits warming to 2.6°C in 2100. This policy causes a complete switch to green technology.

Figure 13 shows the results of the same runs with the new carbon price specifications. Again no scenario keeps the world under 2°C of warming, but higher carbon prices and subsidies improve the outcome. The best outcome in terms of temperature is given by the high carbon price combined with government subsidy, which limits the temperature increase to +2.6°C by the end of the century. In this scenario there is a complete transition to green energy by 2072.

The results demonstrate a need for immediate and aggressive government policy, but the 2°C threshold of the Paris Agreement appears beyond reach. In fact, in order for the model to predict global temperature rise within the 2°C threshold in 2100, there has to be a complete transition to green energy by 2023, which is clearly outside the realm of what is possible.

4 Extension: two temperature regions

I explored an extension to the two region model where there is not just two economic regions, but also two distinct atmospheric temperature regions[1]. All of the economic dynamics, emissions dynamics, and the climate cycle are modeled as described above. Creating two climate regions only requires changes to the temperature dynamics, as there are now two atmospheric temperature regions, representing the North and South, as well as the temperature region representing the ocean.

[1] This extension was suggested to me by Etienne Espagne of the French Development Agency (AFD).

4.1 Changes to temperature dynamics

The two region temperature dynamics are adapted from Cai et al. (2018)[22]. They created a discrete time IAM, based on the DICE model, with two distinct temperature regions. Their model allows for temperature transfers between the North ad South atmospheric temperature regions. They also explored additional damages due to sea level rise and permafrost melt.

I adapted their model to the continuous time framework as follows:

$$\begin{pmatrix} \dot{T_N^{AT}} \\ \dot{T_S^{AT}} \\ \dot{T^{LO}} \end{pmatrix} = \begin{pmatrix} F/C \\ F/C \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -\varepsilon_2 - \varepsilon_4 - \varepsilon_6 & \varepsilon_4 + \varepsilon_5 & \varepsilon_2 \\ \varepsilon_4 & -\varepsilon_2 - \varepsilon_4 - \varepsilon_5 - \varepsilon_6 & \varepsilon_2 \\ \varepsilon_3 & \varepsilon_3 & -2\varepsilon_3 \end{pmatrix} \begin{pmatrix} T_N^{AT} \\ T_S^{AT} \\ T^{OC} \end{pmatrix} \tag{43}$$

where the parameters are

- ε_1: the increase in temperature for each one unit increase in radiative forcing
- ε_2: transfer received by ocean layer from the atmospheric layers
- ε_3: transfer received by atmospheric layers from the ocean layer
- ε_4: transfer from Northern atmospheric layer to Southern atmospheric layer
- ε_5: transfer from Southern atmospheric layer to Northern atmospheric layer
- γ: scaling factor that adapts the discrete parameters to the continuous setting.

It may be possible to formalize the meaning of the parameter γ using heat capacities. This possibility for future research.

4.2 Results

To isolate the climate module, it is assumed that both output, Y, and the emissions reduction rate, n, are linearly increasing for both regions, following paths similar to when they are endogenously determined in the full model. I use the initial conditions from Bovari et al. (2018b)[14]. The results are given in figure 14. The paths of the atmospheric temperature in North and South in the continuous model follow closely the results of the discrete model in Cai et al. (2018)[22].

Average global temperature is calculated for the purpose of comparison with the one region model, and assumes both regions are weighted equally. Figure 15 compares the results of two region and one region model for temperature. The temperature in the oceans is not increasing as rapidly in the two region model as in the one region model. However, since ocean temperature is not a

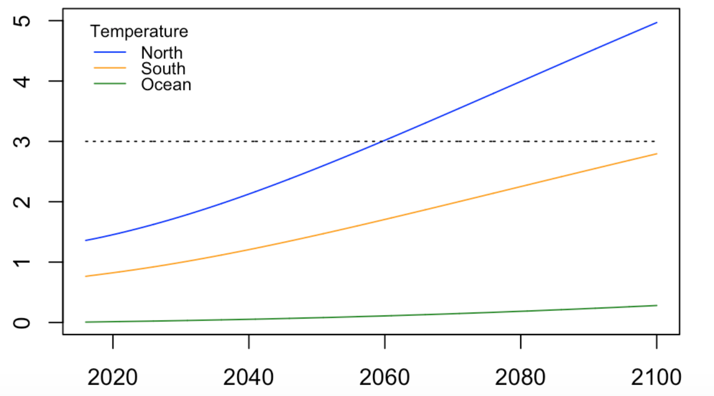
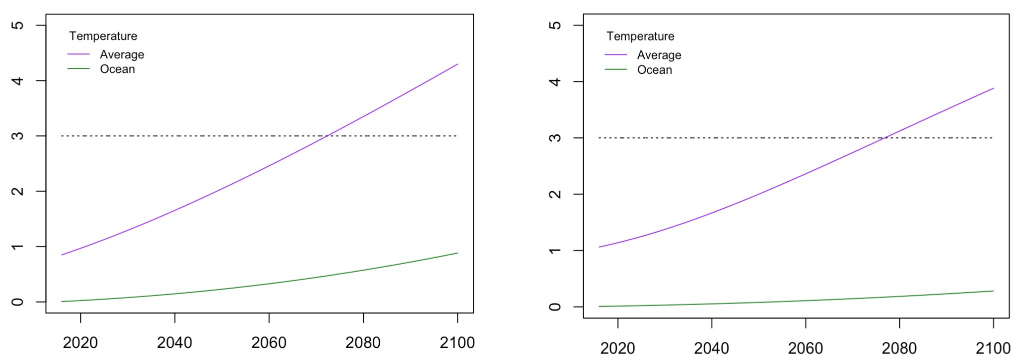


Figure 14: Change in temperature (in degrees Celsius from pre-industrial levels) over time in the three different temperature regions of the two temperature region model.

variable of interest, this discrepancy would not affect the outcome of the model.



(a) One region

(b) Two regions

Figure 15: Change in temperature in the one region and two region climate models.

The two region temperature module could be integrated into the general two region model, allowing for different changes in temperature in the North and South. Given that we know that temperatures in the North are in fact increasing much faster than the global average (Natural Resources Canada, 2019)[1], this is a tempting direction. However, increased temperatures in the North could more easily be modeled using one global atmospheric temperature and then assigning different damage curves to the North and South. I chose to pursue the latter option, but this extension remains a possible are for future research.

5 Conclusion

I have shown that the one country model from the literature can be extended to a model of two regions within a single country. If there are more differences between the two regions initially, more oscillation is seen as the two regions play off each other. These results are robust to changing the growth rate of labour productivity, the climate sensitivity level, and the size of the intermediate reservoir layer, parameters which are identified as uncertain in the literature. However, changing the damage curve has a significant effect on the results of the model, with more convex damage curves leading to economic collapse in both regions.

I explored several carbon tax and green subsidy policies, and found that a high carbon tax combined with government subsidization of abatement costs was the most effective policy in terms of reducing emissions and mitigating temperature increases. However, no policy resulted in the global average temperature increasing by less that 2°C, the goal of the Paris Agreement. The model predicts that this goal cannot be reached without a full transition to green technology within the next five years.

In terms of avenues for future research, a first step would be to further separate the North and South, moving towards a model with two separate countries. The governments in each region could also be modeled explicitly, allowing them to tax, spend, and borrow. There are also several additional green policies that could be considered, including private or public green loans, and a 'Green New Deal' where the government undertakes deficit spending to fund the transition to green technology. Given the dependence of the outcome of the model on the choice of damage curve, the preferred damage curve specification should be identified. One could also look at estimating a damage curve, though this would be a difficult task given the uncertainty surrounding the effects of significant global climate change.

To extend the Global North and South scenario, initial conditions and parameters should be estimated. One could also estimate separate damage curves for the Global North and South. There is then the possibility of examining further government policies, including different carbon taxes in the North and South, investment by the North in green technology in the South, or green loans from the North to the South. Exploring some of these extensions would allow for more conclusive policy recommendations.

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6 Appendix

6.1 Initial conditions of the model

Symbol	Initial condition	Variable description
North		
Y	40	GDP, in trillions USD
d	1	Private debt ratio
λ	0.75	Employment rate
ω	0.5782	Wage share in the economy
N	2.4127	Workforce, in billions
E_{ind}	17.925	Industrial CO_2-e emissions, in Gt C
n	0.04	Emissions reduction rate
g_σ	-0.0105	Growth rate of the emissions intensity of the economy
South		
Y	20	GDP, in trillions USD
d	0.5	Private debt ratio
λ	0.55	Employment rate
ω	0.5782	Wage share in the economy
N	2.4127	Workforce, in billions
E_{ind}	17.925	Industrial CO_2-e emissions, in Gt C
n	0.02	Emissions reduction rate
g_σ	-0.0105	Growth rate of the emissions intensity of the economy
Entire country		
p	1	Composite good price level
p_{BS}	547.22	Price level of backstop technology
E_{land}	2.6	Exogenous land use CO_2-e emissions, in Gt C
CO_2^{AT}	851	CO_2-e concentration in the atmosphere layer, in Gt C
CO_2^{UP}	460	CO_2-e concentration in the biosphere and upper ocean layer, in Gt C
CO_2^{LO}	1740	CO_2-e concentration in the lower ocean layer, in Gt C
T	0.85	Temperature anomaly, in degrees Celsius
T_{LO}	0.0068	Temperature anomaly in lower ocean layer, in degrees Celsius

6.2 Model Parameters

Symbol	Value	Parameter description
α	0.02	Productivity growth rate
α_1	0.0072	Productivity growth rate function parameter
α_2	-0.0004	Productivity growth rate function parameter
δ	0.04	Depreciation rate of capital
ν	2.7	Capital to output ratio
δ_N^N	0.01	Growth rate of workforce in the North
δ_S^N	0.02	Growth rate of workforce in the North
$\overline{N}$	7.056	Ceiling on global workforce, in billions
φ_0	-0.292	Phillips curve constant - linear specification
φ_1	0.469	Phillips curve slope - linear specification
φ_3	6.4103e-05	Phillips curve denominator - nonlinear specification
φ_3	-0.040064	Phillips curve constant - nonlinear specification
Δ_0	0.0513	Dividend function constant
Δ_1	0.4729	Dividend function slope
Δ_{min}	0	Dividend function minimum
Δ_{max}	1	Dividend function maximum
κ_0	0.03178	Investment function constant
κ_1	0.5753	Investment function slope
κ_{min}	0	Investment function minimum
κ_{max}	0.3	Investment function maximum
ϕ_i	0.5	Taylor interest rate parameter
r_i^*	0.01	Long term interest rate
i^*	0.02	Inflation target
η	0.3	Inflation relaxation parameter
μ	1.2	Price markup
ι	0.3	Inflation calibration parameter
ρ	1.25	Investment multiplier
C_{preind}^{AT}	588	Preind. concentration of CO_2 in the atmosphere layer, in Gt C
C_{preind}^{UP}	360	Preind. concentration of CO_2 in the biosphere/upper ocean layer, in Gt C
C_{preind}^{LO}	1720	Preind. concentration of CO_2 in the lower ocean layer, in Gt C
ϕ_{12}	0.0239069	Transfer coefficient for carbon from AT to UP
ϕ_{23}	0.0013409	Transfer coefficient for carbon from UP to LO
δ_{g_σ}	-0.001	Variation rate of the growth of emission intensity
$\delta_{E_{land}}$	-0.022	Growth rate of land use change CO2-e emissions
F_{dbl}	3.6813	Change in radiative forcing from a doubling of preind CO_2, in W/m^2
F_{exo}^{start}	0.5	Initial value of exogenous radiative forcing
F_{exo}^{end}	1	Initial value of exogenous radiative forcing
T_{preind}	13.74	Preindustrial temperature, in degrees Celsius
C_{init}	49.761	Heat capacity of AT and UP, in SI
C_{LO}	3.52	Heat capacity of the lower ocean layer
γ^*	0.0176	Heat exchange coefficient between temperature layers, in SI
S	3.1	Equilibrium climate sensitivity, in degrees Celsius
ξ_1	0	Damage function parameter
ξ_2	0.00236	Damage function parameter
ξ_3	0.00000507	Damage function parameter - Weitzman
ξ_3	0.0000819	Damage function parameter - Dietz and Stern
θ	2.6	Abatement cost function parameter
$g_{p_{BS}}$	-0.0051	Growth rate of the price of backstop technology