Stock version of the Keen model with prices

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1 Model

We follow the setup given in [1], based on the Keen model [2]. We assume that real output, Y, depends on the capital stock, K, according to a constant capital-to-output ratio ν:

$$Y = \frac{K}{\nu} \tag{1}$$

Capital itself evolves according to the investment of firms, I, and depreciation, δ:

$$\dot{K} = I - \delta K \tag{2}$$

Labour is hired depending on the original output, Y, and labour productivity, a:

$$L = \frac{Y}{a} \tag{3}$$

The productivity rate grows exponentially according to

$$\dot{a} = \alpha\, a \tag{4}$$

The workforce, N, is assumed to grow exponentially (this can be relaxed to allow for logistic growth, as in [3]):

$$\dot{N} = \beta N \tag{5}$$

The employment rate in each region is then computed as

$$\lambda = \frac{L}{N} \tag{6}$$

Firms' profits are nominal output minus payments to workers and payments to service debt:

$$\Pi = pY - wL - rD \tag{7}$$

where p denotes the price level and D denotes debt.

We assume that firms' investment decisions depend on their profit share, $\pi = \Pi/pY$, according to the function $\kappa(\cdot)$:

$$I = \kappa(\pi)Y \tag{8}$$

We assume the change in debt is difference between investment and profits:

$$\dot{D} = pI - \Pi \tag{9}$$

The wage share and debt share are defined to be

$$\omega = \frac{wL}{pY} \text{ and } d = \frac{D}{pY} \tag{10}$$

which immediately gives the relationship

$$\pi = 1 - \omega - rd \tag{11}$$

Let i be the inflation rate. We assume wages grow according to the Phillips curve relationship, represented by $\varphi(\cdot)$: higher employment leads to higher wage changes, as employees bargain for better wages when they have less competition. This curve is assumed to be either linear, as in Goodwin's original paper [4], or rational function that goes to infinity as the employment rate approaches one, as in [2]. Following [1], we also assume wages depend on inflation according to the parameter $0 \leq \gamma \leq 1$:

$$\dot{w} = w\,\varphi(\lambda) + \gamma i \tag{12}$$

We assume inflation evolves according to

$$i = \frac{\dot{p}}{p} = \eta(\xi\omega - 1) \tag{13}$$

where $\eta > 0$ and $\xi \geq 1$.

1.1 Differential System

Grasselli and Nguyen Huu [1] use the following three dimensional differential system:

$$\dot{\lambda} = \lambda[g(\pi) - \alpha - \beta] \tag{14a}$$

$$\dot{\omega} = \omega[\varphi(\lambda) - \alpha - (1-\gamma)i(\omega)] \tag{14b}$$

$$\dot{d} = \kappa(\pi) - \pi - d[i(\omega) + g(\pi)] \tag{14c}$$

where $g(\pi) = \frac{\kappa(\pi)}{\nu} - \delta$.

We are instead using the following system:

$$\dot{K} = I - \delta K \tag{15a}$$

$$\dot{w} = w\varphi(\lambda) + \gamma i \tag{15b}$$

$$\dot{p} = ip \tag{15c}$$

$$\dot{D} = pI - \Pi \tag{15d}$$

$$\dot{a} = \alpha a \tag{15e}$$

$$\dot{N} = \beta N \tag{15f}$$

2 Code

```
# Packages
library("deSolve")
library("RColorBrewer")

# Colour palettes for graphs
greens <- brewer.pal(n = 6, name = "Greens")
colourful <- brewer.pal(n = 3, name = "Set1")
colour2 <- brewer.pal(n = 3, name = "Set2")

#===============================================================================
### Functions
# Phillips curve
phill <- function(lambda, pars){
  Phill <- pars[['phi1']] / ((1 - lambda)^2) - pars[['phi0']]
  return(Phill)
}

# Investment function
inv <- function(profit_share, pars) {
  Inv <- pars[['kappa0']] + exp(pars[['kappa1']] +
                                  pars[['kappa2']] * profit_share)
  return(Inv)
}

# Inflation
infl <- function(omega, pars){
```

```
  Infl <-  pars[['eta_p']] * (pars[['markup']] * omega - 1)
  return(Infl)
}

# ============================================================================
simulation <- function(time, init_state, options = Options, parms,
                       method = 'lsoda') {
  # timer
  start_time <- proc.time()
  # time sequence for ode
  time_seq <- seq(0, time[['end']] - time[['start']], by = time[['step']])
  # intial conditions
  init_vars <- c(init_state,count=0)

  #============================================================================
  ## DIFFERENTIAL SYSTEM
  # Function for deSolve:
  Keen_w_prices <- function(t, state, parms) {
    with(as.list(c(parms,state)), {

      ### Auxiliary equations
      # Output (eq. 1)
      Y <- K / nu
      # Labour (eq. 2)
      L <- Y / a
      # Profits (eq. 7)
      Profits <- p * Y - w * L - r * debt
      # profit share
      profit_share <- Profits / (p * Y)
      # wage share (eq. 10)
      omega <- w * L / (p * Y)
      # employment rate (eq. 6)
      lambda <- L / pop
      # inflation (eq. 13)
      i  <- infl(omega,parms)
      # investment (eq. 8)
      investmt <- inv(profit_share,parms) * Y
      # Phillips curve function
      phill <- phill(lambda,parms)
```

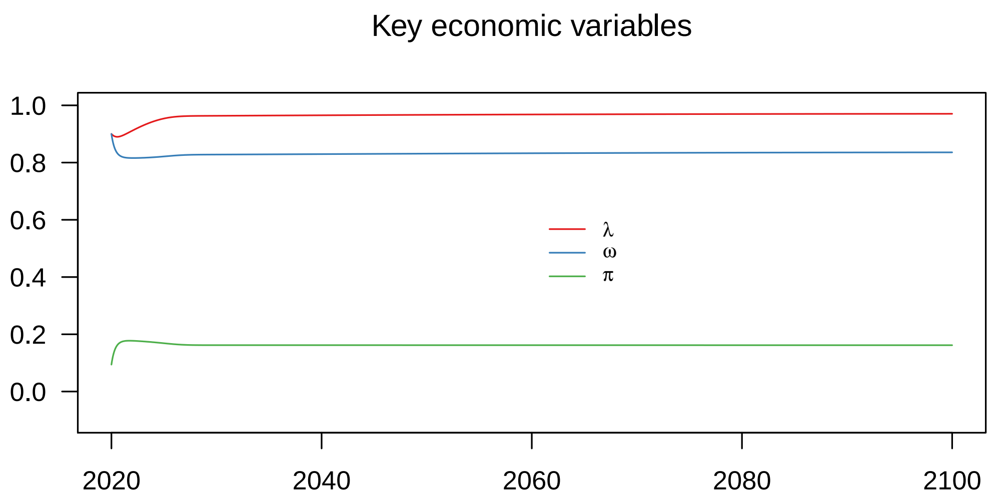
```r
    ### Differential system
    # capital (eq. 2)
    d_K <- investmt - delta * K
    # wages (eq. 12)
    d_w <- w * phill + gamma * i
    # prices (eq. 13)
    d_p <- i * p
    # debt (eq. 9)
    d_debt <- p * investmt - Profits
    # labour productivity (eq. 4)
    d_a <- alpha * a
    # population (eq. 5)
    d_pop <- beta * pop
    # counter
    d_count <- 1

    # ============================================================================
    ## Return the list of gradients
    return(list(c(
      d_K      =  d_K ,
      d_w      =  d_w,
      d_p      =  d_p,
      d_debt   =  d_debt,
      d_a      =  d_a,
      d_pop    =  d_pop,
      d_count  =  d_count)))
  })
}

#==============================================================================
## SIMULATION
simulation <- as.data.frame(ode(func = Keen_w_prices,
                                method = method,
                                y = init_vars,
                                parms = c(parms,time),
                                times = time_seq))

# ============================================================================
# REMAINING VARIABLES COMPUTATION
```

```r
compute.remaining <- function(sim = simulation, params = c(parms,time)){
  with(as.list(c(sim,params)), {
    ### Auxiliary equations
    # Output (eq. 1)
    Y <- K / nu
    # Labour (eq. 2)
    L <- Y / a
    # Profits (eq. 7)
    Profits <- p * Y - w * L - r * debt
    # profit share
    profit_share <- Profits / (p * Y)
    # debt share (eq. 10)
    debt_share <- debt / (p * Y)
    # wage share (eq. 10)
    omega <- w * L / (p * Y)
    # employment rate (eq. 6)
    lambda <- L / pop
    # inflation (eq. 13)
    i  <- infl(omega,parms)
    # investment (eq. 8)
    investmt <- inv(profit_share,parms) * Y
    # Phillips curve function
    phill <- phill(lambda,parms)

    # return results as a data frame
    sim_output <- data.frame(
      year     = start + time,
      count   = count,
      K = K,
      w = w,
      p = p,
      debt = debt,
      a = a,
      pop = pop,
      lambda   = lambda,
      omega    = omega,
      debt_share = debt_share,
      profit_share = profit_share,
      i        = i)
```

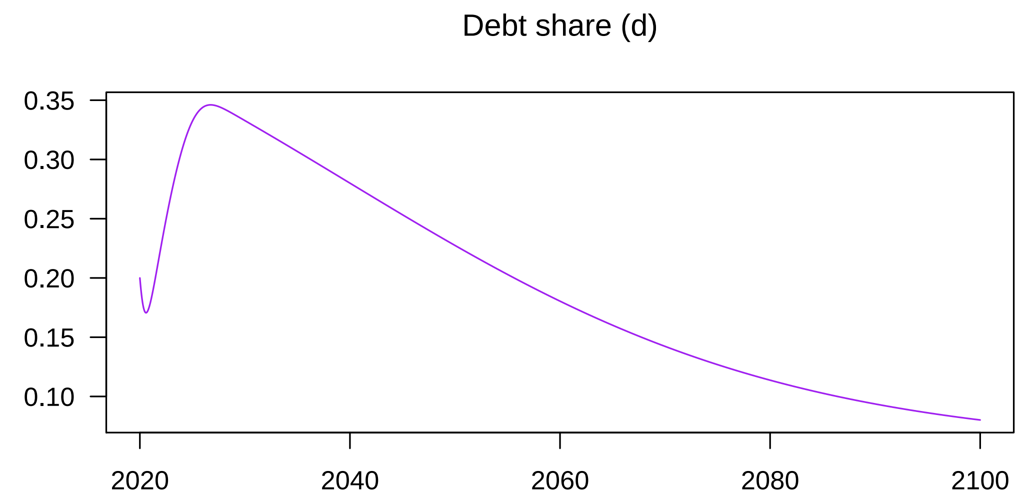
```
      return(sim_output)
    })
  }

  # Return data frame
  simulation_output <- compute.remaining()
  return(simulation_output)
}
#==============================================================================
Parms <- c(
  alpha = 0.025,     # productivity growth rate
  beta = 0.02,       # population growth rate
  delta = 0.01,      # depreciation rate
  nu = 3,            # capital to output ratio
  r = 0.03,          # interest rate on loans

  eta_p = 3,         # adjustment speed for prices
  markup = 1.2,      # mark-up value
  gamma = 0.8,       # inflation sensitivity in the bargaining equation

  phi0 = 0.0401,     # Phillips curve parameter
  phi1 = 6.41e-05,   # Phillips curve parameter

  kappa0 = -0.0065,  # constant parameter in investment function
  kappa1 = -5,       # affine parameter in exponent of investment function
  kappa2 = 20        # coefficient in the exponent of investment function
)

#==============================================================================
### Set initial conditions
IC <- c(
  K  =  27,
  w   =  9,
  p   =  1,
  debt = 27/15,
  a = 10,
  pop = 1
)
```

```r
omega <- IC[['w']]/ (IC[['p']] * IC[['a']])
cat('initial omega is', omega, '\n')

## initial omega is 0.9

lambda <- IC[['K']]/ (IC[['pop']] * IC[['a']] * Parms[['nu']])
cat('initial lambda is', lambda, '\n')

## initial lambda is 0.9

debt_share <- (IC[['debt']] * Parms[['nu']]) / (IC[['p']] * IC[['K']])
cat('initial debt share is', debt_share, '\n')

## initial debt share is 0.2

# Set up simulation
Time <- c(
  start         =     2020,
  end           =     2100,
  step          =     0.05
)

Options <- list(
  transform_vars  = TRUE  # T / F
)




## Run a scenario
Sim <- simulation(time              = Time,
                  init_state        = IC,
                  parms             = Parms,
                  method            = 'lsoda'
)



par(mfrow = c(1,1), las=1, xpd=T)
plot(x = Sim$year, y = Sim$lambda, type = 'l',
     col = colourful[1], xlab = '', ylab = '', ylim = c(-0.1,1),
     main=expression(paste("Key economic variables")))
lines(x = Sim$year, y = Sim$omega, type = 'l',
      col = colourful[2])
```

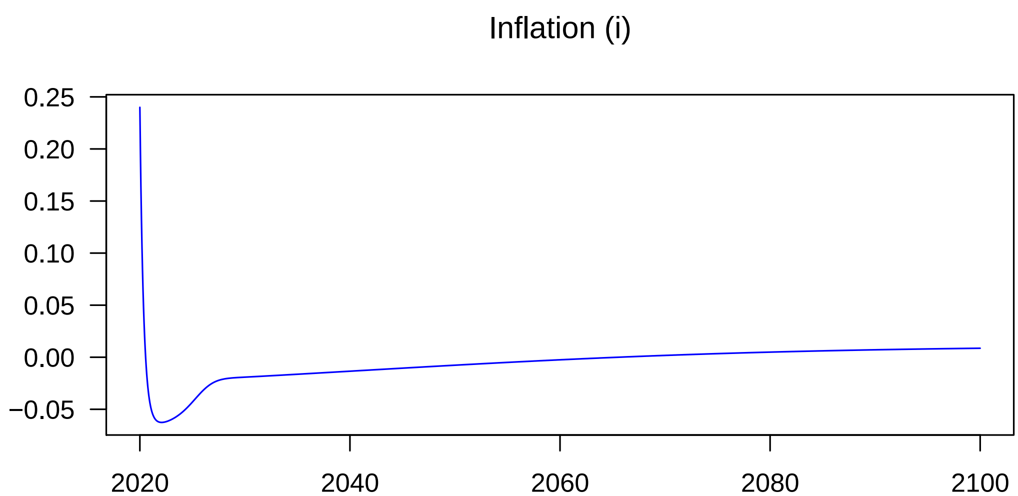
```r
lines(x = Sim$year, y = Sim$profit_share, type = 'l',
      col = colourful[3])
legend(x=2060, y=0.65, legend = c(expression(paste(lambda)),
                                  expression(paste(omega)),
                                  expression(paste(pi))),
       col=colourful, lty=1:1,box.lty=0, cex=0.75)
```




```r
plot(x = Sim$year, y = Sim$debt_share, type = 'l',
     col = 'purple', xlab = '', ylab = '', main=expression(paste("Debt share (d)")))
```




```
plot(x = Sim$year, y = Sim$i, type = 'l',
     col = 'blue', xlab = '', ylab = '', main=expression(paste("Inflation (i)")))
```



References

[1] Grasselli MR, Huu AN. Inflation and speculation in a dynamic macroeconomic model. Journal of Risk and Financial Management. 2015;8(3):285–310.

[2] Keen S. Finance and Economic Breakdown: Modeling Minsky's 'Financial Instability Hypothesis'. Journal of Post Keynesian Economics. 1995;17(4):607–35.

[3] Bovari E, Giraud G, Mc Isaac F. Coping With Collapse: A Stock-Flow Consistent Monetary Macrodynamics of Global Warming. Ecological Economics. 2018 May;147:383–398. Available from: http://www.sciencedirect.com/science/article/pii/S0921800916309569.

[4] Goodwin RM. A growth cycle. In: Feinstein CH, editor. Socialism, Capitalism and Economic Growth. Cambridge: Cambridge University Press; 1967. p. 54–58.