Problem 1. *The rapid MMV test gives a positive result:*

- *100% of the time for people with the virus*
- *5% of the time for people without the virus*

A certain population has a prevalence of 1%. You pick a person from this population at random, and test them, and the test is positive. What is the probability they have MMV? You learn that your friend has a positive rapid test for MMV. What do you tell them?

Solution. The probability an individual, who was picked at random and has tested positive, has MMV can be answered using conditional probability, specifically Bayes' Theorem, given by:

$$\mathrm{P}\left(\text{disease}|\text{positive}\right) = \frac{\mathrm{P}\left(\text{disease}\right)\mathrm{P}\left(\text{positive}|\text{disease}\right)}{\mathrm{P}\left(\text{positive}\right)}, \tag{1}$$

where $\mathrm{P}\left(\text{disease}|\text{positive}\right)$ is the probability an individual has MMV *given* they have tested positive. Since the population the individual was picked from has a prevalence of 1%, and the probability of a true positive from the rapid test is given, as the rapid test gives a positive result 100% of the time for people with the virus, it is known that

$$\mathrm{P}\left(\text{disease}\right) = 0.01,$$
$$\mathrm{P}\left(\text{positive}|\text{disease}\right) = 1.$$

Furthermore, the probability an individual will test positive is given by

$$\mathrm{P}\left(\text{positive}\right) = \mathrm{P}\left(\text{disease}\right)\mathrm{P}\left(\text{positive}|\text{disease}\right) + \mathrm{P}\left(\overline{\text{disease}}\right)\mathrm{P}\left(\text{positive}|\overline{\text{disease}}\right)$$

since an individual can test positive in two ways; with a false positive rapid test, or a true positive rapid test. Since $\mathrm{P}\left(\text{disease}\right)$ is known, it is known that

$$\mathrm{P}\left(\overline{\text{disease}}\right) = 1 - \mathrm{P}\left(\text{disease}\right) = 0.99.$$

It is also known that the probability of an individual having a false positive rapid test is given by

$$\mathrm{P}\left(\text{positive}|\overline{\text{disease}}\right) = 0.05,$$

as the rapid MMV test gives a positive result 5% of the time for people without the virus. So, the probability an individual, who was picked at random and has tested positive, has MMV is about 16.8%, as evidenced below:

$$\mathrm{P}\left(\text{disease}|\text{positive}\right) = \frac{(0.01)(1)}{(0.01)(1) + (0.99)(0.05)} \approx 0.168.$$

The posterior probability, the probability an individual has MMV, given the positive test, is only 0.168. Although there is an extremely high probability of getting a true positive

from this rapid test (1), there is a relatively low probability of actually having MMV (0.01) combined with a 0.05 probability of a false-positive rapid test, which results in the low posterior probability. However, if the probability of having MMV is now updated as the posterior probability (0.168), and the individual takes a second rapid test, and repeats Bayes' theorem iteratively, the new posterior probability would be given as

$$\mathrm{P}\left(\text{disease}|\text{positive}\right) = \frac{(0.168)\,(1)}{(0.168)\,(1) + (0.99)\,(0.05)} \approx 0.772.$$

Now the posterior probability better reflects the probability this individual has MMV, given they have tested positive. If this individual continues this iterative process, the most recent test will more accurately reflect the probability they have MMV. Furthermore, I would tell a friend who has a positive rapid test for MMV that they need to take multiple rapid tests to accurately guage whether they have MMV or not.