LAB REPORT: LAB 3

SPLINES AND SUBDIVISION

TNM079 - MODELING AND ANIMATION

Emma Forsling Parborg
emmfo731@student.liu.se

Monday 18$^{\text{th}}$ January, 2016

Abstract

This report describes the third out of six labs in the course TNM079 - Modeling and Animation given at Linköping University. This report covers the subject of splines and subdivision of two-dimensional curves and three-dimensional surfaces. In this report the half-edge mesh structure created in the first lab will be further expanded with the ability to refine it through subdivision using Loop's subdivision scheme. The results show that the implemented assignments are successful.

1 Introduction

The purpose of this lab is to demonstrate how splines and subdivision can be utilized and show the potential they may have when combining them with the half-edge mesh data structure created in previous labs. Splines are used to produce smooth curves and these can be approximated by representing the curves using parametric functions through basis splines and a set of points. Smooth curves can also be represented by subdivision curves, which iteratively is refined until it sufficiently approximates the smooth curve. For this lab, spline approximation through curve subdivision and mesh refinement using Loop's subdivision scheme is implemented.

2 Assignments

The individual assignments that were performed in the lab are presented in this section. Each subsection below represents one lab assignment.

2.1 Implement curve subdivision

A spline curve can be evaluated by successive applications of subdivision. With this method a line strip can be changed into a smooth curve, that is equal to the spline, by repositioning the points. When utilizing subdivision, all original points are looped through and each point has its original position altered according to Equation 1.

$$\begin{aligned} \mathbf{c}'_i &= \tfrac{1}{8}\left(1\mathbf{c}_{i-1} + 6\mathbf{c}_i + 1\mathbf{c}_{i+1}\right) \\ \mathbf{c}'_{i+\frac{1}{2}} &= \tfrac{1}{8}\left(4\mathbf{c}_i + 4\mathbf{c}_{i+1}\right) \end{aligned} \tag{1}$$

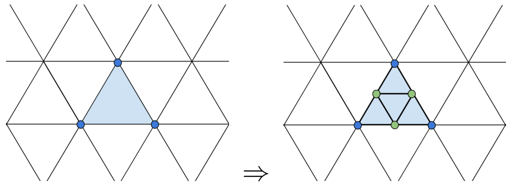
For the current point, $\mathbf{c}'_i$ and $\mathbf{c}_i$ denote the new position and the original position, respectively. The previous and next points for the original position is represented by $\mathbf{c}_{i-1}$ and $\mathbf{c}_{i+1}$, and $\mathbf{c}'_{i+\frac{1}{2}}$ denotes the position of the new point.

To derive a correct approximation of the spline, the positions of the start and end points of the line strip can not be altered, see Equation 2. This is implemented by adding these points before and after the loop.

$$\begin{aligned} \mathbf{c}'_0 &= \mathbf{c}_0 \\ \mathbf{c}'_{end} &= \mathbf{c}_{end} \end{aligned} \tag{2}$$

2.2 Implement mesh subdivision

Multiple subdivision algorithms exist and in this lab Loop's subdivision scheme was utilized, which is illustrated in Figure 1.



(a) Before subdivision (b) After subdivision

Figure 1: This image illustrates the triangle refinement with Loop's subdivision scheme. The blue points represent the vertices of the triangle before subdivision has been performed. The green points represent the vertices which have been inserted by the subdivision scheme.

This algorithm introduces new vertices and faces in order to make the mesh more smooth. To derive this, the method loops through all edges and adds new vertices along them. The position of a new vertex is determined as a weighted average of the old ones. If it is a boundary edge, the average will be determined by two points, and if it is an internal edge, it will be determined by four points, see Figure 2.

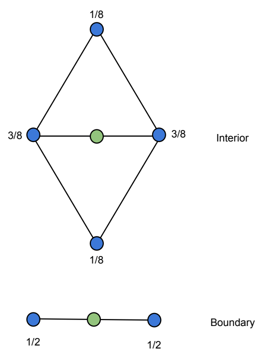


Figure 2: The weights for a new vertex position in Loop's subdivision scheme are illustrated in this image, for both interior and boundary cases.

After all new vertices have been created, the positions for the original vertices have to be updated. The new position of an old point is determined as a weighted average of the point itself and the new points surrounding it, see Figure 3.

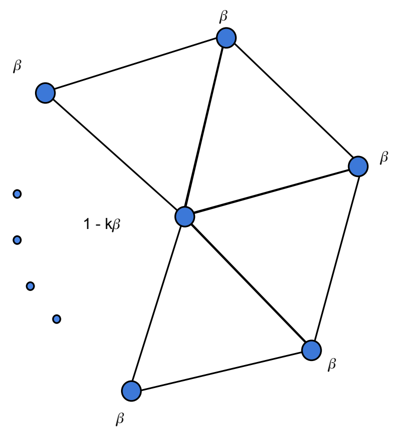


Figure 3: The weights for a new vertex position in Loop's subdivision scheme are illustrated in this image. The k and β represent the valence and the weight, respectively.

Here, k denotes the valence, i.e. the number of incident edges, and β represents the weight. To derive β, Equation 3 was utilized.

$$\beta = \begin{cases} \dfrac{3}{8k}, & k > 3 \\ \dfrac{3}{16}, & k = 3 \end{cases} \tag{3}$$

2.3 Localize mesh evaluation of the analytical spline

The analytical spline consists of a number of basis functions which have support only between two control points. These control points form an interval but the base code for the lab implements a simple evaluation of the spline which does not take this interval into account.

To remove unnecessary computations and thereby improve the performance, Equation 4 is implemented in order to localize the evaluation of the spline to these intervals, by introducing a lower and an upper bound.

$$\begin{aligned} b_l &= max(floor(t) - 1.0, 0.0) \\ b_u &= min(ceil(t) + 1.0, n) \end{aligned} \tag{4}$$

The equation introduces a lower bound b_l and an upper bound b_u for the loop that goes over the spline. The number of control points is represented by n and t is the parameterization variable.

2.4 Implement a scheme for adaptive mesh subdivision

When using the mesh subdivision previously described, all the faces for the mesh will be subdivided. The downside with this is that for a mesh containing a mixture of small and large faces, areas which may not need it will be subdivided, e.g. flat areas. This leads to unnecessary computation time being spent on areas that hardly improve the visual quality of the mesh.

In this assignment, an adaptive mesh subdivision scheme is implemented to avoid this. This method selects which faces that shall be subdivided by utilizing the curvature. The mean value of the mean curvature of the whole mesh is compared to the local mean curvature. If the mean value is smaller or greater than the local value, the face will be subdivided or unaltered, respectively.

3 Results

The important results for each of the assignments are presented in this section.

3.1 Curve subdivision

The subdivided curve is illustrated in Figure 4. In this image, three subdivisions are performed. The results show that the subdivided curve gets better after three iterations have been performed.

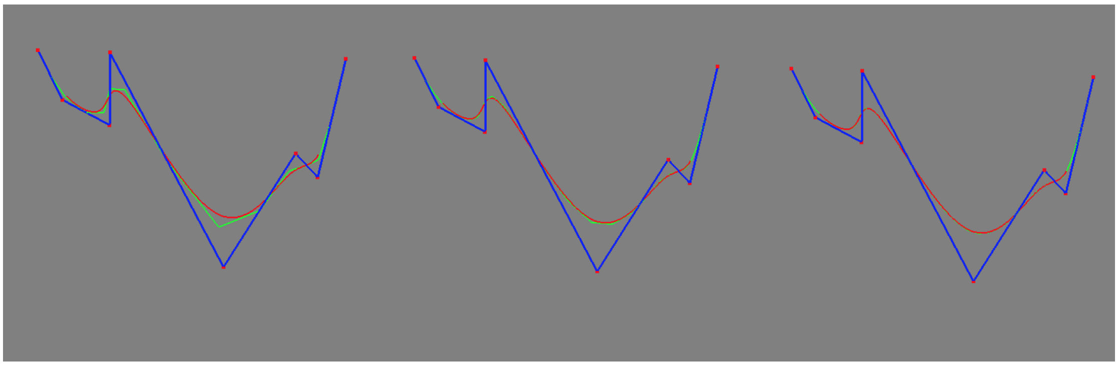


Figure 4: This figure illustrates a subdivision curve in different stages. The control polyline and the analytical spline is represented by a blue and a red curve, respectively. The green line is the subdivision curve. From left to right, the subdivision curve has been subdivided once, twice and thrice.

3.2 Mesh subdivision

In Figure 5, the results from performing mesh subdivision on the mesh *cow.obj* is visualized. In the image, the mesh in its original form is visualized to the left. In the middle and to the right, the mesh has been subdivided once and twice, respectively.

3.3 Localize evaluation of the analytical spline

The results from introducing the lower and upper bounds, for the evaluation of the analytical spline, decreased the function calls from 4000 to 1999.

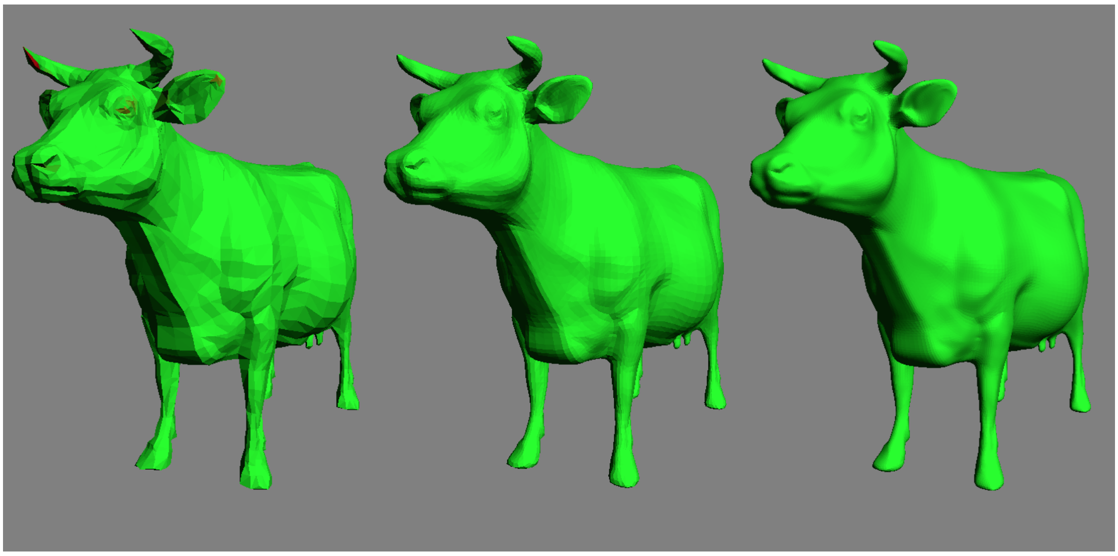


Figure 5: This figure illustrates mesh subdivision. From left to right: The original mesh, the mesh subdivided once and twice.

3.4 Adaptive mesh subdivision

In Figure 6, the results from performing the adaptive mesh subdivision is displayed.

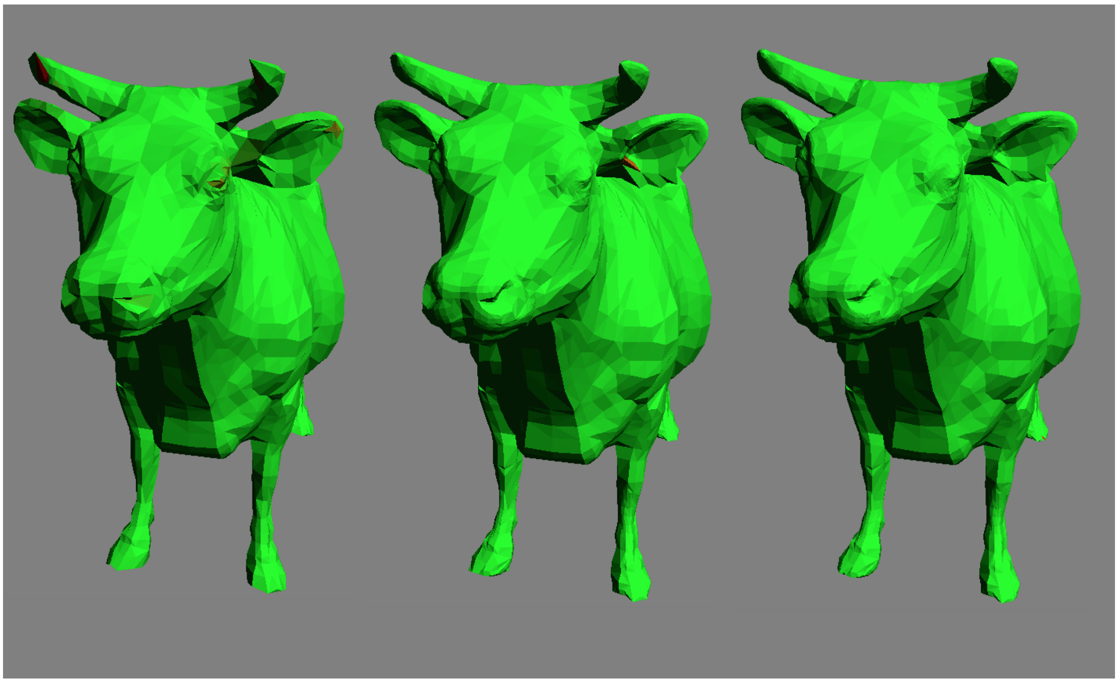


Figure 6: This figure illustrates mesh subdivision. From left to right: The original mesh, the mesh subdivided once and twice using the adaptive subdivision rule.

4 Conclusion

The results derived for this lab illustrate that the assignments have been performed successfully. The curve subdivision results in Figure 4 illustrate that the analytical spline can be accurately estimated using only few subdivisions. The mesh subdivisions results in Figure 5 illustrate that the implementation has been performed correctly and for each iteration the mesh gets smoother. The evaluation of the analytical spline also proved to be successful and the function calls decreased significantly. The adaptive mesh subdivision in Figure 6 also proved to be successful, in which rough/sharp areas have been subdivided, e.g. the ears and the horns, while flat areas have not been subdivided, e.g. the side of the cow.

5 Lab partner and grade

This lab was completed in collaboration with Martin Gråd, margr484@student.liu.se. Since I have completed all the assignments marked with (*) and (**), as well as one marked with (***), I should qualify for grade 5.