I. Introduction

As an engineer for the Apollo Program, I strive to help both NASA and America achieve the most that we can. In this mission, my team and I have been investigating the gravitational forces of the Earth and the Moon, as well as the capabilities and limitations of the Saturn V Rocket that will allow us to continue to progress into space.

We began our code by importing numpy, matplotlib.pyplot, and math. Having these functions imported into the code allows for easier computation, graphing, and analysis throughout the entire lab. Importing numpy as np allows significant numerical composition, matplotlib.pyplot as plt allows for plotting figures, and math allows for more complex functioning. We next set up my constants that we would be utilizing throughout the lab. Establishing these constants allowed for easier commands when completing computations.

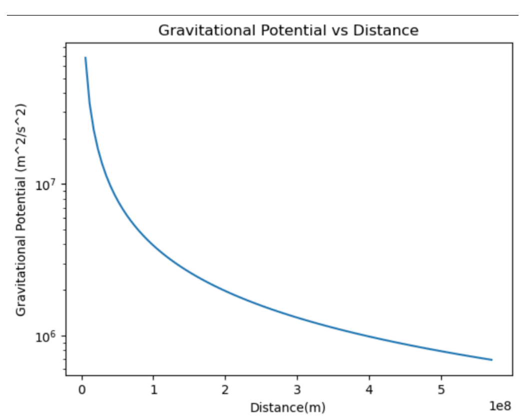
The constants are as follows:

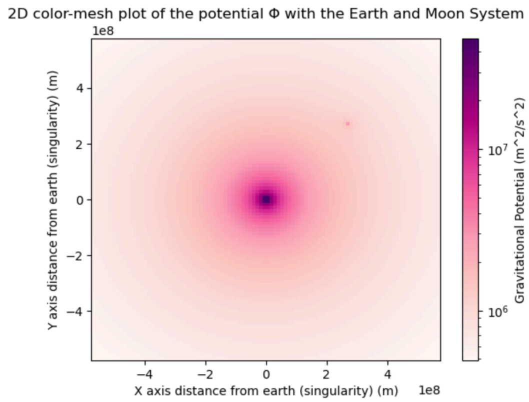
• Gravitational Constant (G) 6.67 × 10−11 m3/kg/s2
Mass of the Earth (Me) 5.9 × 1024 kg
• Mass of Apollo Command Module (Ma) 5500 kg
• Radius of the Moon (Rm) 1737 km
• Burn Rate of S-1C (m) 1.3 × 104 kg/s
• Dry Mass of S-1C (mf) 7.5 × 105 kg

• Gravitational Acceleration (g) 9.81 m/s2 •
• Mass of the Moon (Mm) 7.3 × 1022 kg
• Radius of the Earth (Re) 6378 km
• Distance from Earth to Moon (D) 3.8 × 108 m
• Wet Mass of S-1C (m0) 2.8 × 106 kg
• Exhaust Velocity of S-1C (ve) 2.4 × 103 m/s

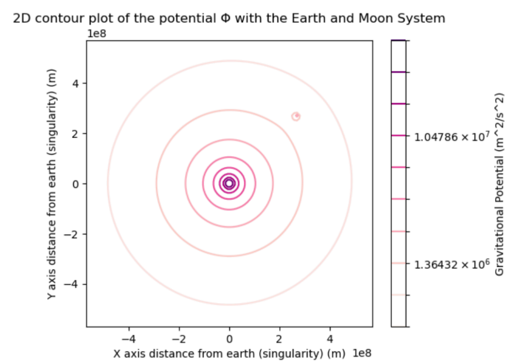
II. The Gravitational Potential of the Earth-Moon System

Gravitational potential is defined as the energy per unit mass an object has due to a force of gravity at a given distance. This can be represented in an equation to calculate the gravitational potential energy at a distance r from a mass M with Φ(r)=−GM/r. When calculating the potential between the Earth and the Moon, We first defined an equation U as my gravitational potential energy value, and set it to return the equation Φ(r)=−GM/r. We set U to be in terms of M, as well as the locations of both the Earth and Moon in two dimensions: xm, ym, and x, y. We then defined r in terms of these coordinates, using Pythagorean theorem to establish the distance between the Moon and the Earth's cores. We then graphed the absolute value of my equation U vs the distance between the Earth and the Moon. Assuming y=0, we plotted the 150% of the distance of the Earth to the Moon, or 1.5*D. We achieved this utilizing the fig, ax=plt.subplots method, and then adding more alterations to ax from there, such as the x1=np.linspace(), and ax.plot(x1,np.abs(U())). We used Me as the attractive mass, since it was previously defined as the mass of Earth. The graph above represents the relationship between the gravitational potential energy and the distance from Earth. As the graph depicts, the gravitational potential energy decreases as the distance increases. This gives the Apollo team information about the gravitational influence that Earth has on objects around it in respect to the distance between their cores.





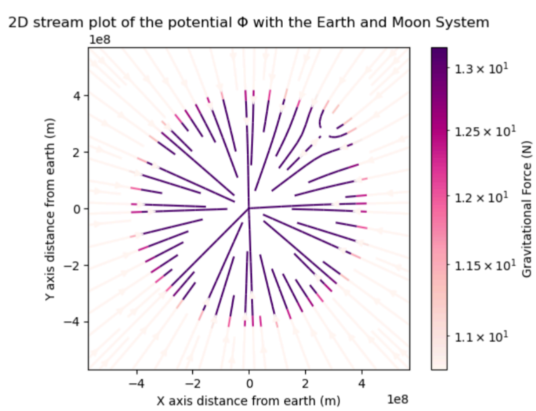
We also graphed a 2D color-mesh plot (left) of the potential energy with the Earth at the origin (0,0). We considered the absolute value of the potential energy over the range -1.5*D to 1.5*D, where D is the distance between the core of the Earth and the Moon. We applied a logarithmic normalization so that the range of values would be presented in a neater manner, that would allow us to observe the Moon's impact. The color bar provided context for the graph, and the graph was also made square to provide equal scale on the x vs y plane and to prevent distortion. In respect to this plane, Earth is at (0,0), where the colors are the darkest and where the potential is the greatest. The pink dot to the right and up from the center represents the location of the moon. This is at ($D/\sqrt{2}$, $D/\sqrt{2}$), respectively,



We also provided a visualization (right) of the gravitational potential relationship utilizing a contour plot. We again used a logarithmic scale to prevent all of the values from bunching up around the Earth and Moon. Similar to the previous plot, the Earth is at (0,0), and the Moon is represented by the dot within the circular spaces at ($D/\sqrt{2}$, $D/\sqrt{2}$) where D is the constant defined previously.

III. The Gravitational Force of the Earth-Moon System

Our team also calculated the gravitational force of the Earth and Moon system. Gravitational force is defined as an attractive force between two masses. Because of Newton's third law, we know that every action has an equal and opposite reaction, and therefore the forces between two objects are equal, and



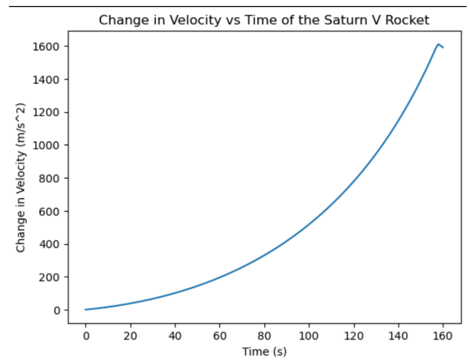
therefore the gravity they pull on each other are the same magnitude, although in different directions. This relationship depends on each object's mass, and the distance between their centers. This can be represented by the equation $F=-GMm/r^2$. To calculate this, we defined an equation F for force in terms of each objects mass, and their locations. We then represented this in a 2D streamplot to represent the gravitational force that the Earth-Moon system would exert on the Apollo 11 command module. We considered over the range -1.5*D to 1.5*D. We again utilized a logarithmic scale, both for the x,y scale and the colorbar. This was crucial to the mission, as it provided the engineering team with essential information about the impact space forces would have on a launched spacecraft.

IV. Projected Performance of the Saturn V Stage 1

The Saturn V rocket is the launch craft designated for the Apollo missions. The launch of this vehicle consists of three stages, the first of which being Stage 1 (S-IC). S-IC is the initial liftoff into and journey throughout Earth's lower atmosphere. Powered by five F-1 engines, the rocket's performance is modeled in our code, tracking the engines' burn time, the resulting velocity change, and the altitude after burnout. This signals the end of S-IC.

To calculate the burn time, we created a function that divides the total mass lost by the rate of fuel consumption, known as the mass flow rate. In the constants as defined before, this is represented by (m0-mf)/m. The rocket takes off with a mass that includes both its body and fuel, but as the engines burn, fuel is lost at a constant rate (m). This function determines how long the F-1 engines can fire before the fuel runs out. This information is essential to allow our engineers to time Stage 2.

Next, we defined a function to find the change in velocity, or in other words how fast the rocket will accelerate. According to the Rocket Equation, a rocket's velocity increases when fuel is burned and exhaust gases are expelled, propelling the rocket through space. The faster the exhaust gases are expelled, the more efficiently the rocket gains speed. We graphed this by defining an equation dv in terms of time, wet mass, dry mass, burn rate, exhaust speed, and gravitational constant g. As shown on the right, the graph depicts that as time goes on, the change in velocity increases. At 158 seconds, which is the burn time calculated previously, the change in velocity becomes zero, as represented by the sudden dip in the graph. However, gravity from nearby planets or moons works against the rocket's acceleration and therefore impacts its velocity. This is why the previous calculations of the gravitational effects of Earth and the Moon were essential- they are crucial to determine if the rocket will reach the required velocity for Stage 2.



Finally, we calculated the rocket's final altitude after Stage 1 separation. This is done by finding the velocity over time and using an integral to sum the small altitude changes at each moment. Since velocity isn't constant or changing at a constant rate during the rocket's ascent, our engineering team integrated from t=0 to t=T where T is the time of burnout, as calculated previously.

V. Discussion and Future Work

The first prototype of Saturn V burned for 160 seconds and achieved an altitude of 70km. In comparison, our functions discovered a burn time of 158 seconds and an altitude of 74.1km. Our burn time had a percent error of about 1.25%, which is fairly accurate, while the altitude had an error a little under 6%. Our calculations being slightly off was likely to do with the approximations that we made throughout our calculations. For example, our engineering team ignored air resistance, which would contribute to slowing down the capsule and decreasing its altitude, therefore contributing to our overestimation of the final position. We also assumed a constant gravitational field, without considering variation with respect to altitude. To make our calculations more accurate, we could utilize an equation that would consider air resistance, air density, the shape of the rocket, and the gravitational field as an exact altitude with respect to Earth's center. With the revisions in place, I am certain that our team will be able to continue to facilitate the advancement of our country into space, and into the future.