Mine Crafting Lab Report 4/10/2025
Emmalynn Malixi PHYS265

I. Introduction

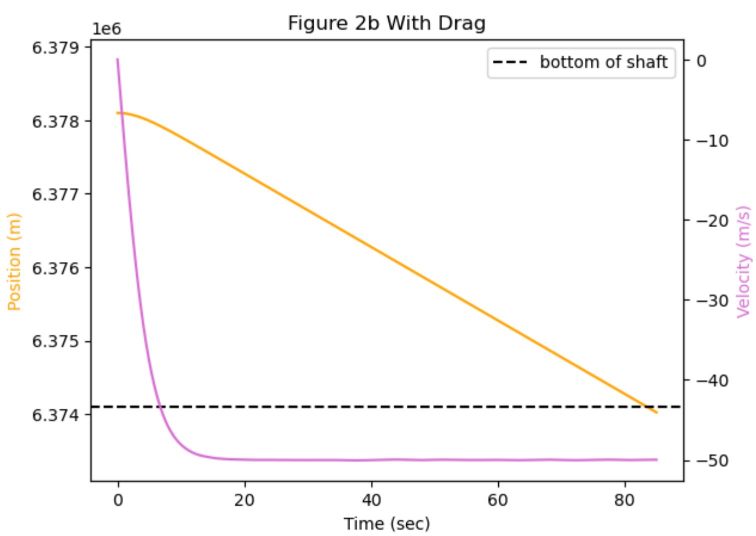
This lab report represents the work that my team and I have completed when considering a vertical mine at Earth's equator. This is potentially the deepest mine on Earth, spanning four kilometers to the bottom of the shaft. We tested this depth by dropping a test mass of one kilogram and measuring the time to hit the bottom. We progressively considered the test mass under a series of increasingly complex assumptions, such as the Coriolis Force, an infinitely deep mine, a non-uniform Earth, and a lunar mine shaft. Throughout the process we utilized values of known constants, such as the mass of the Earth, and the universal gravitational constant.

II. Calculation of Fall Time

First, we considered the simplest case of the mass drop, in which there was no drag, and calculated the time to reach the bottom of the shaft using the kinematics equation x=1/2at^2+v0t+x0. Utilizing this equation, Python found the time to hit the bottom to be 28.6 seconds. This was a very straightforward calculation as it neglected to consider additional variables, which will be discussed later in the report. Next, our team derived an equation to the first order by setting velocity equal to the change in y position with respect to t time, and applying it to the equation on the right.

$$\frac{d^2y}{dt^2} = -g + \alpha\left|\frac{dy}{dt}\right|^{\Gamma}$$

This produced an equation for the first derivative of velocity in terms of g, α, v, and Γ. We considered g to be the constant of Earth's gravity, and assuming α=0. My team and I then applied the derived equation to the Python function solve_ivp. This analytical method of finding the time resulted in the same time as the kinematic equation, with an answer of 28.6 seconds. Next, my team and I incorporated a variable g dependent on the object's distance from the center of the Earth. Again using the Python function solve_ivp, the inclusion of the variable concluded the same fall time of 28.6 seconds. This indicated that, because of the ratio of the 4km drop vs the much larger radius of the Earth, considering the variable g as a function of distance from the center did not make a significant difference. However, we did find that drag has a significant impact on fall time. To incorporate drag, my team calibrated α from an assumed terminal velocity of 50 m/s, and the force of gravity. We found that when we included variable g and the drag force, the object did not hit the bottom until after 83.5 seconds, which is a significant jump from 28.6. Therefore, although the ratio of the



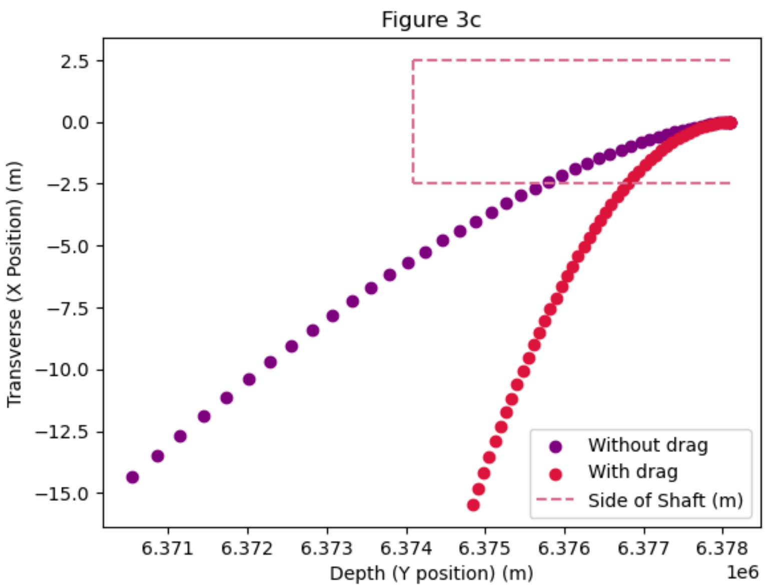
shaft depth vs the radius of Earth made the change in g essentially negligible, drag plays a large role in the time that the object will fall for.

III. Feasibility of depth measurement approach

Next, my team considered how possible the depth measurement approach was when considering the Coriolis Force, which stems from the rotation of the Earth. This risked complications in the drop process, as the object was at risk of hitting the side of the shaft before reaching the bottom. In this scenario, we considered a mine shaft on Earth's equator pointed towards its center. We first established an equation, stemming from the equation of variable g and drag, that now considers the component from the Coriolis Force, which is the equation on the right.

$$\vec{F_c} = -2m\left(\vec{\Omega} \times \vec{v}\right)$$

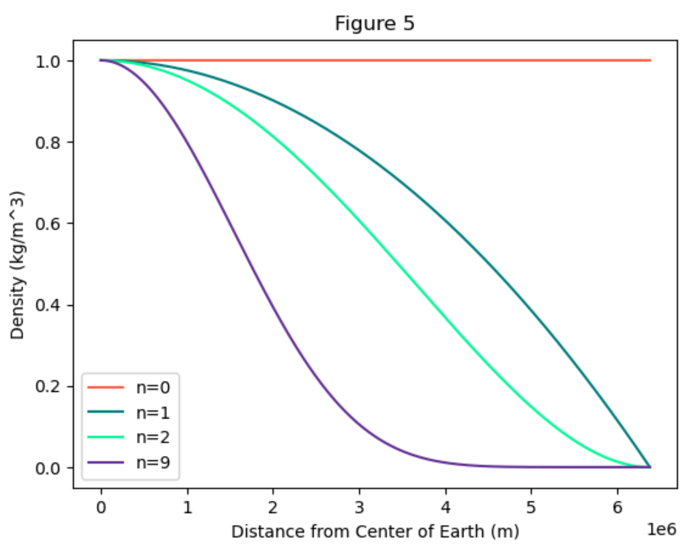
Broken into three dimensions x, y, and z, the F_c can be applied to the original equation in the vertical (y) dimension, and also now incorporates a horizontal (x) component to consider. The z dimension can be neglected, as the force will not be moving the object in this direction. My team and I then plotted the transverse position of the object as function of depth. We did this twice- one with no drag, once with. Again utilizing Python's solve_ivp, we created a scatterplot of the object's path down the shaft, and considered both it would reach the bottom, which would of couse be the depth of the shaft, and where it would impact the side on its way down. We then incorporated the drag component into the equation, and found that drag caused the object to impact the shaft earlier and higher up than the object without drag.



This is clear in Figure 3c, which represents the difference that drag makes on an object while on its path down the shaft. The purple line, representing the position of the object without drag, impacts the side of the shaft after 29.6 seconds at a depth of 1302.6 meters. The red line, representing the position of the object with drag, will hit the side of the shaft after 21.9 seconds at a depth of 2353.9 meters. Therefore the drag on the object will cause it to hit the side of the shaft earlier and higher up than if there was no drag. It is advisable to not proceed with the depth measurement technique, as the Coriolis force will cause the object to hit the side of the shaft before reaching the bottom. If the Coriolis force was considered, it would be possible to make the mineshaft wider to ensure that the object was able to fall all the way down without hitting the sides. As another technique, the shaft could theoretically be placed on a pole of the Earth, such that it is unimpacted by the Coriolis forces.

IV. Calculation of crossing times for homogenous and non-homogenous Earth

Next, my team and I calculated the crossing times for both a homogeneous and a non-homogeneous Earth scenario. We first considered the simpler scenario of a homogeneous Earth, with the theory of an infinitely deep mine. We found how long it would take the object to reach the center and then the other side of the Earth, then calculated the respective orbital speed and period, as well as the crossing time. We found the orbital period to be 5069.4 seconds, and the crossing time to be 1266.5 seconds. Therefore we found that the crossing time to the center of Earth is almost exactly a quarter of the orbital period. Next, we considered a non-homogeneous Earth. We compared four different distributions of density (ρ) based on a set of varying values of n. We utilized n=0, 1, 2, 9, with each producing a different distribution of ρ, respectively. The two extremes of this spectrum were n=0 and n=9. n=0 indicates constant density, whereas n=9 indicates the largest difference in density from the surface and the center. The forces created were significantly different, with the n=9 force spiking to above 40N when close to the center, as there was a large amount of mass at the center. Oppositely, the n=0 created a linear graph of Force vs Distance, as there is even distribution of density.



$$\rho(r) = \rho_n \left(1 - \frac{r^2}{R_\oplus^2}\right)^n$$

Next, my team and I considered the density of the moon. We calculated the density of the moon and the Earth by dividing the mass of each by its volume, and found that the moon is less dense than the Earth. We also calculated the crossing time of an object if there was a pole to pole infinite shaft on the Moon, and found that the object would reach the center of the Moon after 1625.0 seconds. This is larger than the crossing time we had found for the infinite shaft on the Earth of 1266.5 seconds. This indicates that density is inversely proportional to the cross time. In fact, fall time is inversely proportional to the square root of density.

V. Discussion and Future Work

Throughout the process of this lab, my team and I made many approximations. The constants that we used in our calculations are rounded, and may have slightly impacted our numerical results, but not in a way significant enough to affect our qualitative conclusions. Additionally, we did not consider that some parts of Earth may be denser than others based on it being perfect sphere. We considered different densities as progressing to the center of the Earth, but we assumed a uniform spherical density. In reality, the Earth is not a perfect sphere, which may affect the object side to side as it progresses through the mine shaft.