CS 325 Project 4

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Project Group #4

Problem 1:

<u>The Linear Program written (mathematically) as an objective and set of constraints:</u>

Variables:

HF = hams fresh, HR = hams smoked on regular time, HO = hams smoked overtime
BF = bellies fresh, BR = bellies smoked on regular time, BO = bellies smoked overtime
PF = picnics fresh, PR = picnics smoked on regular time, PO = picnics smoked overtime

Objective:

Max profit = $8\ X_{HF} + 14\ X_{HR} + 11\ X_{HO} + 4\ X_{BF} + 12\ X_{BR} + 7\ X_{BO} + 4\ X_{PF} + 13\ X_{PR} + 9\ X_{PO}$

Set of contraints:

s.t.
$X_{HF} + X_{HR} + X_{HO} = 480$
$X_{BF} + X_{BR} + X_{BO} = 400$
$X_{PF} + X_{PR} + X_{PO} = 230$
$X_{HR} + X_{BR} + X_{PR} <= 420$
$X_{HO} + X_{BO} + X_{PO} <= 250$
$X_{HF}, X_{HR}, X_{HO}, X_{BF}, X_{BR}, X_{BO}, X_{PF}, X_{PR}, X_{PO} >= 0$

<u>A description of the environment/language/solved used to solve the LP:</u>

We used Matlab's liner programming solver linprog to solve this problem. The help page for linprog tells us:

linprog Linear programming.
X = linprog(f, A, b) attempts to solve the linear programming problem:

min f ' * x   subject to: A*x <= b

which means that we needed to get our LP into the form:

$$\min_{x} \quad f'x$$

$$s.t. \quad Ax <= b$$

where *x*, *f*, and *b* are vectors and A is a matrix. The constraints are only of the form <=, so we first had to make that so by negating our >= constraints. Also, where there was an equality in the set of constraints (like $X_{HF} + X_{HR} + X_{HO} = 480$) we know that they can produce up to that amount, so we can change those equalities to less than or equal like so:

$$\min \quad (-8\,X_{HF} - 14\,X_{HR} - 11\,X_{HO} - 4\,X_{BF} - 12\,X_{BR} - 7\,X_{BO} - 4\,X_{PF} - 13\,X_{PR} - 9\,X_{PO})$$

s.t.

$X_{HF} + X_{HR} + X_{HO} <= 480$

$X_{BF} + X_{BR} + X_{BO} <= 400$

$X_{PF} + X_{PR} + X_{PO} <= 230$

$X_{HR} + X_{BR} + X_{PR} <= 420$

$X_{HO} + X_{BO} + X_{PO} <= 250$

$-X_{HF}, -X_{HR}, -X_{HO}, -X_{BF}, -X_{BR}, -X_{BO}, -X_{PF}, X\text{-}_{PR}, -X_{PO} <= 0$

<u>And turning the linear program into matrix form:</u>

$$\min \quad [-8\ -14\ -11\ -4\ -12\ -7\ -4\ -13\ -9] \begin{bmatrix} X_{HF} \\ X_{HR} \\ X_{HO} \\ X_{BF} \\ X_{BR} \\ X_{BO} \\ X_{PF} \\ X_{PR} \\ X_{PO} \end{bmatrix}$$

$$s.t. \quad \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} X_{HF} \\ X_{HR} \\ X_{HO} \\ X_{BF} \\ X_{BR} \\ X_{BO} \\ X_{PF} \\ X_{PR} \\ X_{PO} \end{bmatrix} <= \begin{bmatrix} 480 \\ 400 \\ 230 \\ 420 \\ 250 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

<u>The code:</u>

Putting this into Matlab:

```
>> f = [-8; -14; -11; -4; -12; -7; -4; -13; -9]
>> A = [1 1 1 0 0 0 0 0 0; 0 0 0 1 1 1 0 0 0; 0 0 0 0 0 0 1 1 1; 0 1 0 0
1 0 0 1 0; 0 0 1 0 0 1 0 0 1; -1 0 0 0 0 0 0 0 0; 0 -1 0 0 0 0 0 0 0; 0 0
```

```
-1  0   0   0   0   0   0; 0   0   0  -1   0   0   0   0   0; 0   0   0   0  -1   0   0   0   0; 0   0   0   0   0  -1   0   0   0
; 0   0   0   0   0   0  -1   0   0; 0   0   0   0   0   0   0  -1   0;  0   0   0   0   0   0   0   0  -1]
>> b = [480; 400; 230; 420; 250; 0; 0; 0; 0; 0; 0; 0; 0; 0]
>> x = linprog(f, A, b)
Optimization terminated.
x =
      440.0000
        0.0000
       40.0000
        0.0000
      400.0000
        0.0000
        0.0000
       20.0000
      210.0000
>> f ' * x
ans =
      -1.0910e+04 (note: answer is negated because we negated the equation in the first place)
```

<u>The optimal solution to the linear program:</u>

Conclusion: The meat packing plant can net profit $10,900.00 a day if they sell 440 fresh ham, 0 ham smoked on regular time, 40 ham smoked on overtime, 0 fresh bellies, 400 bellies smoked on regular time, 0 bellies smoked on overtime, 0 fresh picnics, 20 picnics smoked on regular time, and 210 picnics smoked on overtime.

Problem 2:

<u>The linear program for the general problem written as an objective and set of contraints:</u>

Objective:

$$\max_{1<=i<=2} \quad |ax_i + by_i - c|$$

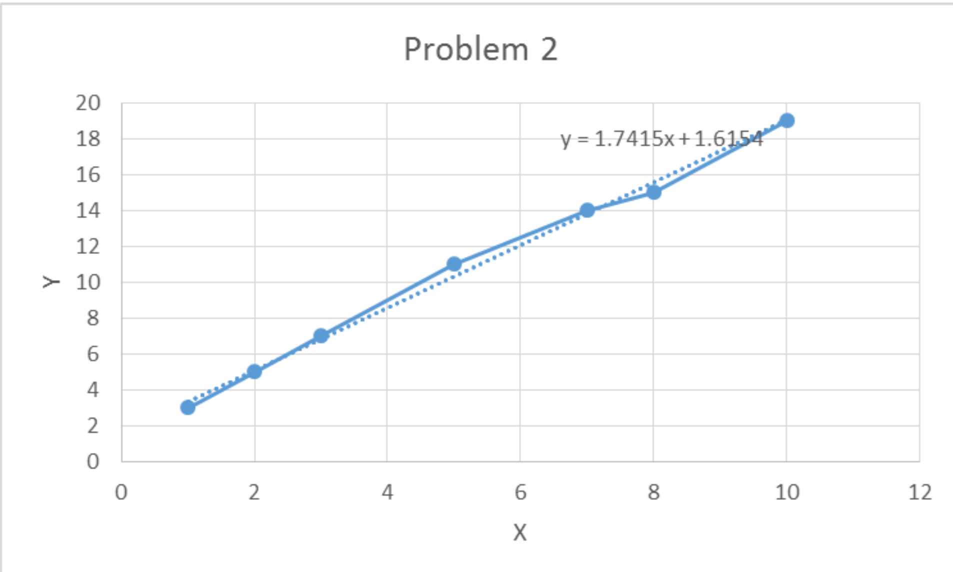
on in other words, when a set of points are linearly related, and there is not a line in the form Ax = B that hits each point in the set, find the best fit line for the points by finding $A^TAx = A^Tb$

Constraints:

Set of points: (1, 3), (2, 5), (3, 7), (5, 11), (7, 14), (8, 15), (10, 19)

<u>The best solution for the specific problem:</u>
Slope intercept form: y = 1.7415x + 1.6154
Line ax + by = c: -1.7415x + y = 1.6154

A plot of the points and your solution for *the instance*:



The code (as succinct as possible) that we used to solve the LP:

First, we set up the matrix A and b for each (x, y): $A = [x^0, x^1]$ b = [y]

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 5 \\ 1 & 7 \\ 1 & 8 \\ 1 & 10 \end{bmatrix} \qquad b = \begin{bmatrix} 3 \\ 5 \\ 7 \\ 11 \\ 14 \\ 15 \\ 19 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 5 & 7 & 8 & 10 \end{bmatrix}$$

Putting this into Matlab:
Note: AT stands for A^T

```
>> A = [ 1 1; 1 2; 1 3; 1 5; 1 7; 1 8; 1 10]
>> AT = [ 1  1  1  1  1  1   1;  1  2  3  5  7  8  10]
>> AT * A
ans =
        7     36
        36    252
```

```
>> b = [ 3; 5; 7; 11; 14; 15; 19]
>> AT * b
ans =
        74
        487
```

So the resulting matrix of $A^T * A$ and $A^T * b$ is:

[7 36 | 74]
[36 252 | 497]

Putting the matrix into matlab:

```
>> m = [7  36  74; 36  252  497]
```

We had to row reduce the equation to find the solution, so in matlab we did:

```
>> rref(m)
ans =

        1.0000         0        1.6154
           0        1.0000      1.7415
```

<u>Conclusion</u>: the best fit line is y = 1.7415x + 1.6154

Sources used:

Problem 1:
http://teaching.ust.hk/~ieem201/index_files/Tutorial%20Notes/Tutorial2.pdf

Problem 2:
http://freetext.org/Introduction_to_Linear_Algebra/Basic_Applications_Linear_Algebra/Best_Fit/

http://www.mathworks.com/help/symbolic/rref.html