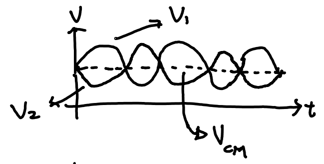
Bipolar Differential Pairs

- Differential Signals
  - Vary by equal and opposite amounts
  - Same average DC value



- V_{CM}: common-mode level

$V_1(t) = V_0 \sin\omega t + V_{CM}$

$V_2(t) = -V_0 \sin\omega t + V_{CM}$

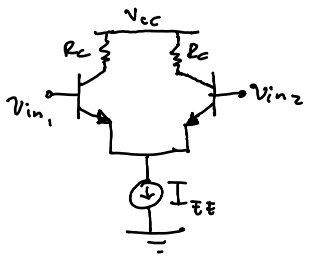
Notice that:

$V_1 - V_2 = 2V_0 \sin\omega t$

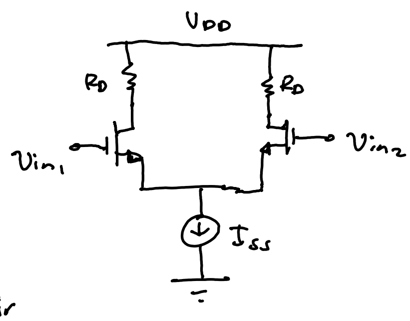
$V_{sig} = \frac{V_1 - V_2}{2} = V_0 \sin\omega t$

The immediate question is how do we generate differential signals???

- the Differential Pair

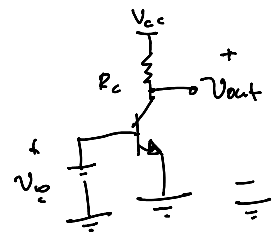


BJT Differential Pair



MOS Differential Pair

- Review of Common Emitter Configuration

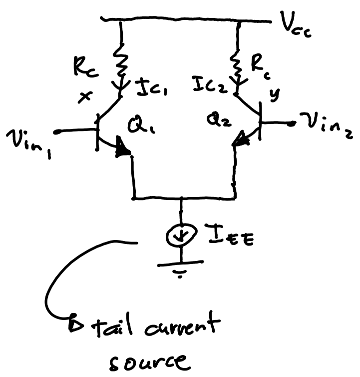


What is V_{out}???

$I_C = I_S \exp\left(\frac{V_{in}}{V_T}\right)$

$\underline{\underline{V_{out} = V_{DD} - I_C R_C}}$

Consider the circuit



Case 1: The bases of Q_1 and Q_2 are shorted and connected to a battery.

$V_{in_1} = V_{in_2} = V_{CM,in}$

KCL: $I_{C_1} + I_{C_2} = I_{EE}$, if $I_{C_1} = I_{C_2}$, then $I_{C_1} = I_{C_2} = \frac{1}{2} I_{EE}$

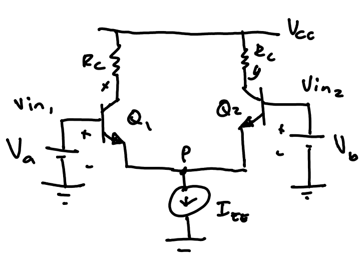
Voltage Drop across $R_C = \frac{1}{2} I_{EE} R_C$

$V_x = V_{CC} - \frac{1}{2} I_{EE} R_C = V_y$

$\therefore$ If $\underbrace{V_{in_1} - V_{in_2}}_{\text{Differential Input Voltage}} = 0$, then $\underbrace{V_x - V_y}_{\text{Differential Output Voltage}} = 0$

$\therefore$ If there is perturbation in V_{in_1} and V_{in_2}, the circuit is oblivious to that change. the differential pair rejects changes in input CM level.

Case 2: V_{in_1} and V_{in_2} are not the same $(V_{in_1} - V_{in_2} \neq 0)$



Guess: Q_1 is on and has some current

Question: Is Q_2 ON???

Assume $V_{BE,1} = 800$ mV, $V_a = 2.5$V, $V_b = 1.5$V

thus, $V_P = V_a - V_{BE,1} = 1.7$V.

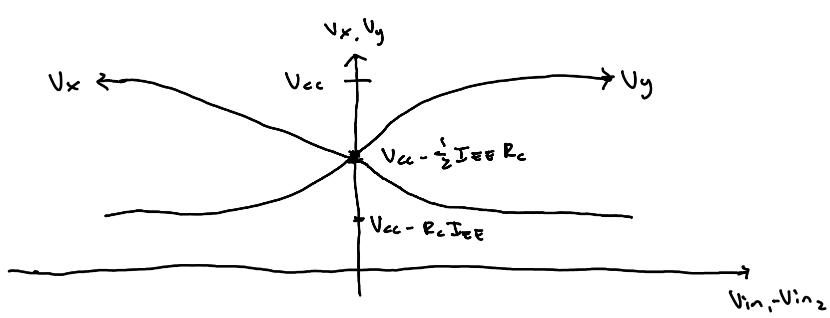
Q_2 is off since $V_{E,2} > V_{B,2}$.

$\therefore$ All of I_{EE} flows through Q_1

In response to an imbalance at the input, there came an imbalance to the output.

$V_x = V_{CC} - R_C I_{EE}$, $V_y = V_{CC}$ (since $I_{C,2} = 0$)

First Input-Output Characteristic

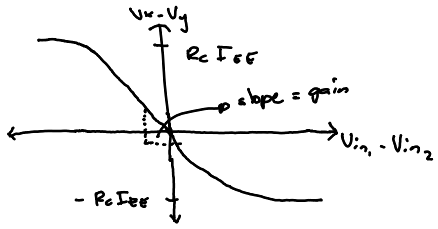


- Characteristic Plot is similar to the plot of differential signals.

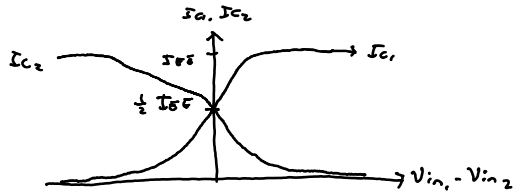
- This is just an "intuitive" approach to the differential pair.

Large and Small Signal Analysis of Bipolar Differential Pair

• Large-Signal Analysis

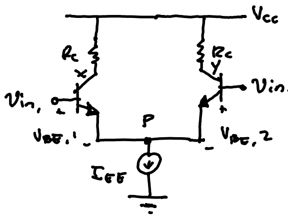


• The slope is the gain of an amplifier

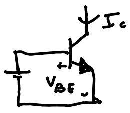


• We have obtained graphs for $V_x - V_y$ and I_{C_1}, I_{C_2} as a function of $V_{in_1} - V_{in_2}$

Objective: Derive equations for I_{C_1}, I_{C_2}, V_x, V_y, and $V_x - V_y$ as a function of $V_{in_1} - V_{in_2}$.



Remember that:



$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right)$

① If $V_{in_1} = V_{in_2}$, the current splits equally.

② If $V_{in_1} - V_{in_2}$ is positive enough, I_{C_1} and I_{C_2} differs.

Notice that:

$V_{in_1} - V_{BE,1} = V_{in_2} - V_{BE,2}$

$V_{in_1} - V_{in_2} = V_{BE,1} - V_{BE,2}$

$= V_T \ln\left(\frac{I_{C_1}}{I_S}\right) - V_T \ln\left(\frac{I_{C_2}}{I_S}\right)$

$= V_T \ln\left(\frac{I_{C_1}}{I_{C_2}}\right)$

* These transistors are identical ($I_{S_1} = I_{S_2}$)

$I_{C_1} + I_{C_2} = I_{EE}$

$I_{C_1} = I_{C_2} \exp\left(\frac{V_{in_1} - V_{in_2}}{V_T}\right)$

$I_{C_2} + I_{C_2} \exp\left(\frac{V_{in_1} - V_{in_2}}{V_T}\right) = I_{EE}$

$$I_{C_2} = \frac{I_{EE}}{1 + \exp\left(\frac{V_{in_1} - V_{in_2}}{V_T}\right)}$$

similarly,

$$I_{C_1} = \frac{I_{EE}}{1 + \exp\left(\frac{V_{in_2} - V_{in_1}}{V_T}\right)}$$

where $V_T = 26mV$ @ $300^\circ K$

$V_x = V_{CC} - R_C I_{C_1}$

$V_y = V_{CC} - R_C I_{C_2}$

$\Rightarrow V_x - V_y = R_C I_{C_2} - R_C I_{C_1}$

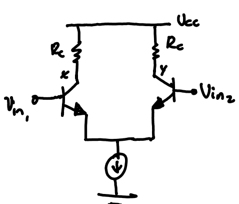
$$R_C I_{EE} \left[\frac{1}{1 + \exp\left(\frac{V_{in_1} - V_{in_2}}{V_T}\right)} - \frac{1}{1 + \exp\left(\frac{V_{in_2} - V_{in_1}}{V_T}\right)}\right]$$

or $V_x - V_y = -R_C I_{EE} \tanh\left(\frac{V_{in_1} - V_{in_2}}{2V_T}\right)$

• The plot is similar to the inverted hyperbolic tangent.

If $\frac{V_{in_1} - V_{in_2}}{V_T} \ll 1$, then by the Taylor expansion, $\tanh\left(\frac{V_{in_1} - V_{in_2}}{2V_T}\right) \approx \frac{V_{in_1} - V_{in_2}}{2V_T}$ $\Rightarrow V_x - V_y \approx -R_C I_{EE}\left(\frac{V_{in_1} - V_{in_2}}{2V_T}\right)$, slope $= -\frac{R_C I_{EE}}{2V_T}$

• Small-signal Analysis



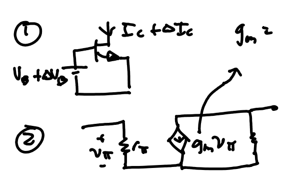
If V_{in_1} increases by ΔV, V_2 decreases by ΔV, then,

$V_x - V_y = -R_C I_{EE} \tanh\left(\frac{2\Delta V}{2V_T}\right)$

$\approx \frac{-R_C I_{EE}}{2V_T}(2\Delta V)$

If $\frac{R_C I_{EE}}{2V_T} \gg 1$, amplification happens

Basic Concepts:



$g_m = \frac{\Delta I_C}{\Delta V_B} = \frac{I_C}{V_T}$

lemma: If V_{in_1} and V_{in_2} change by equal and opposite amounts, then the tail node voltage does not change. (If the change is small)

$V_x = V_{CC} - R_C I_{C_1}$

$V_y = V_{CC} - R_C I_{C_2}$

$V_x - V_y = -R_C[I_{C_1} - I_{C_2}] = -R_C I_{EE}\left(\frac{\Delta V}{V_T}\right)$

$I_{C_1} - I_{C_2} = I_{EE}\frac{\Delta V}{V_T}$, and $I_{C_1} + I_{C_2} = I_{EE}$

thus,

$$\begin{array}{rl} & I_{C_1} - I_{C_2} = I_{EE}\frac{\Delta V}{V_T} \\ + & I_{C_1} + I_{C_2} = I_{EE} \\ \hline & 2I_{C_1} = I_{EE}\left(1 + \frac{\Delta V}{V_T}\right) \end{array}$$

$I_{C_1} = \frac{1}{2} I_{EE}\left(1 + \frac{\Delta V}{V_T}\right)$

$\Delta I_{C_1} = \frac{I_{EE}}{2V_T}\Delta V$

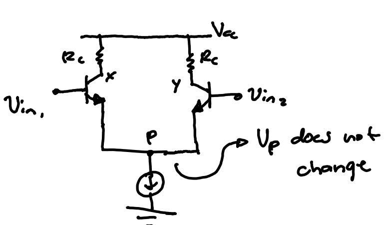
$g_m = \frac{\partial I_C}{\partial V_{BE}}$

$\partial I_C = g_m \partial V_{BE}$

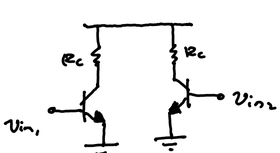
$$\Delta V_{BE} = \frac{\left(\frac{I_{EE}}{2V_T}\Delta V\right)}{\frac{I_{EE}}{2V_T}} = \Delta V$$

∴ If the voltage in V_{B_1} increases by ΔV, then $V_{BE,1}$ changes by the same amount. thus, V_P remains the same.

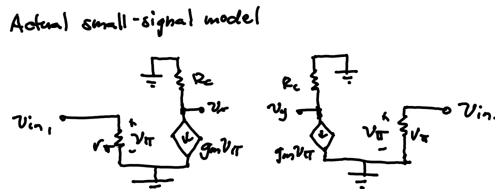
• Small-signal Behavior



• V_P is considered virtual ground since it does not change with time.



Actual small-signal model



$v_x = -g_m R_C v_{in_1}$

$v_y = -g_m R_C v_{in_2}$

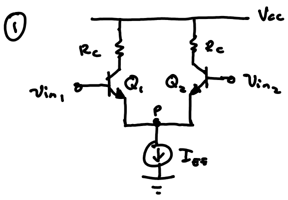
$v_x - v_y = -g_m R_C (v_{in_1} - v_{in_2})$

$A_v = \frac{v_x - v_y}{v_{in_1} - v_{in_2}} = -g_m R_C$

More on Bipolar Differential Pairs

Additional Examples:

①



Let $I_{EE} = 1\,mA$
$R_C = 1k\Omega$
$I_S = 2\times10^{-18}\,A$

Questions:
- Determine the bias conditions
- Construct the I/O characteristic
- Obtain the voltage gain.

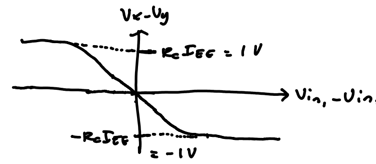
Solution:
- Bias Conditions,

$I_{C_1} = I_{C_2} = \frac{1}{2} I_{EE}$

since $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$,

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right)$

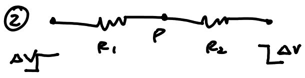
$V_{BE,1} = V_{BE,2} = 862\,mV$



- small-signal gain $= -g_m R_C$

$= \frac{-R_C I_{EE}}{2V_T} = -19.2$

②



What happens to V_P ???

Answer

Let $R_1 = R_2 = R$

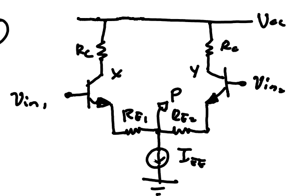
$I = \frac{2\Delta V}{2R} = \frac{\Delta V}{R}$

Voltage Drop across R_1

$\Rightarrow \frac{\Delta V}{R} R = \Delta V$

If we increase V_{in_1} by ΔV,
the voltage drop across R_1
increases by ΔV. $\Rightarrow V_P$ is constant

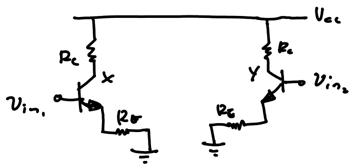
③



* Bipolar Differential Pair with Emitter Degeneration

Let $R_{E_1} = R_{E_2} = R_E$

- Since V_P does not change with time, we can draw two separate circuits



Thus,

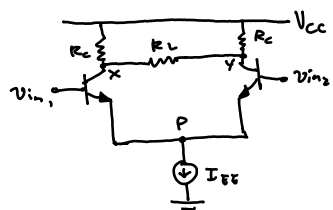
$$\frac{V_X - V_Y}{V_{in_1} - V_{in_2}} = \frac{-R_C}{\frac{1}{g_m} + R_E}$$

- $\frac{-R_C}{\frac{1}{g_m} + R_E} = \frac{-g_m R_C}{1 + g_m R_E} \Rightarrow$ less slope

gain decreases around $V_{in_1} - V_{in_2} = 0$

$\therefore$ Emitter degeneration improves linearity

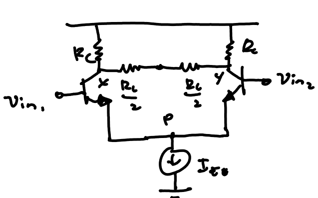
④



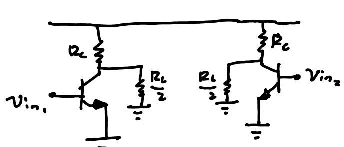
If V_{in_1} and V_{in_2} changes differentially,
V_X and V_Y also changes differentially.

* the circuit is grounded along the point of symmetry.

We can replace R_L with 2 series resistors



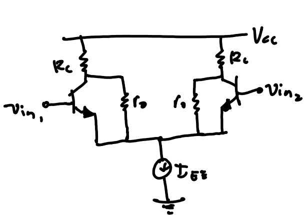
$\Rightarrow$



$\therefore A_v = -g_m \left(R_C \,\|\, \frac{R_L}{2}\right)$

we decomposed R_L to find the point of symmetry.

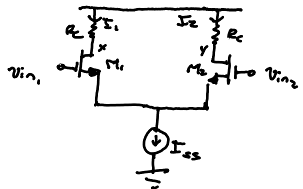
$\therefore$ If early effect is considered,



$A_v = -g_m (R_C \| r_o)$

as far as small signals are concerned.

Large Signal Analysis of MOS Differential Pair



Cases:

① V_{in1} and V_{in2} are shorted. They are connected to V_{CM}.

Thus, $I_1 = I_2 = \frac{1}{2} I_{SS}$.

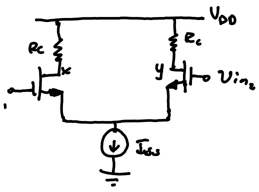
$V_X = V_Y = V_{DD} - \frac{R_D I_{SS}}{2}$

If $V_{in1} = V_{in2}$, the differential pair is in equilibrium.

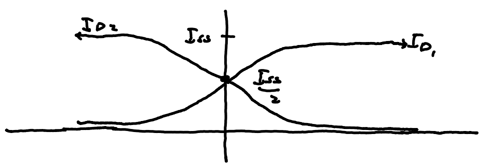
$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$

$V_{GS} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} + V_{TH}$

where $\sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$ is the equilibrium overdrive voltage

② 

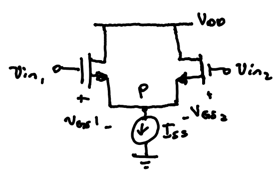
② If V_{in1} and V_{in2} are different, the characteristic plot looks like this:



* If V_{in1} is much greater than V_{in2}, All of I_{SS} flows through M_1 and M_2 is off.

• Condition: MOS should be in saturation

Question: What is the minimum value of $V_{in1} - V_{in2}$ at which one transistor turns off???



If M_2 is off, then

$V_{GS2} = V_{TH}$.

If all of I_{SS} flows through M_1,

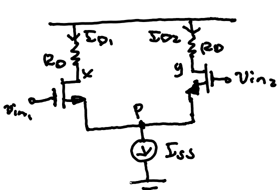
$V_{GS1} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} + V_{TH}$

Thus, $V_P = V_{in1} - V_{GS1} = V_{in2} - V_{GS2}$

$V_{in1} - V_{in2} = V_{GS1} - V_{GS2}$

$= \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$

• Large Signal Analysis



By KVL,

$V_{in1} - V_{in2} = V_{GS1} - V_{GS2}$

Assume that both of the transistors are on.

$V_{in1} - V_{in2} = \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} + V_{TH} - \left(\sqrt{\frac{2 I_{D2}}{\mu_n C_{ox} \frac{W}{L}}} + V_{TH} \right)$

similarly,

$V_X = V_{DD} - R_D I_{D1}$

$V_Y = V_{DD} - R_D I_{D2}$

$V_X - V_Y = -R_D (I_{D1} - I_{D2})$

$I_{D1} + I_{D2} = I_{SS}$

$(V_{in1} - V_{in2})^2 = \frac{2 I_{D1}}{\mu_n C_{ox} \frac{W}{L}} + \frac{2 I_{D2}}{\mu_n C_{ox} \frac{W}{L}} - \frac{2\sqrt{4 I_{D1} I_{D2}}}{\mu_n C_{ox} \frac{W}{L}}$

$(V_{in1} - V_{in2})^2 - \frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}} = \frac{-4\sqrt{I_{D1}(I_{SS} - I_{D1})}}{\mu_n C_{ox} \frac{W}{L}}$

If we solve the equation:

$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4 I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$

Observations:

① $I_{D1} - I_{D2} = 0$ if $V_{in1} = V_{in2}$

② If $V_{in1} - V_{in2}$ exceeds a certain amount, one transistor turns off. The equation is no longer valid.

* If $V_{in1} - V_{in2} > \frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}$, one transistor turns off.

Question:

• What happens if I_{SS} is doubled?

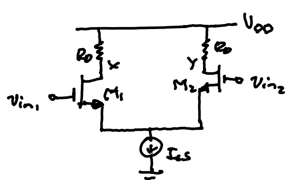
∴ slope changes to

$-\sqrt{2} R_D \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}$.

∴ Increases linearity around the origin.

• What happens if W is doubled?

∴ circuit becomes sharper but less linear.

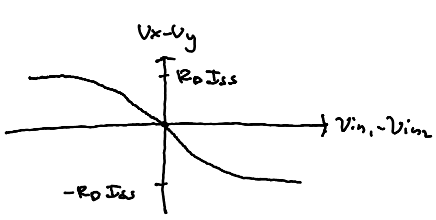


$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4 I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$

this equation is valid iff:

$|V_{in1} - V_{in2}| \leq \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$

$V_X - V_Y = -R_D (I_{D1} - I_{D2})$



If $V_{in1} - V_{in2} \ll \frac{4 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}$

$\Rightarrow V_X - V_Y = -R_D \mu_n C_{ox} \frac{W}{L} \sqrt{\frac{4 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$

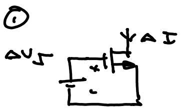
$V_X - V_Y \approx -R_D \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} (V_{in1} - V_{in2})$

slope $= -\sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} \; R_D$

Small Signal Analysis of MOS Differential Pair

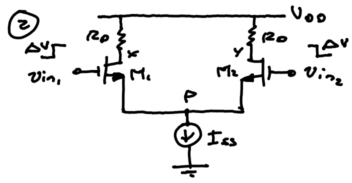
• Small-signal Behavior of MOS Differential Pair

Few points

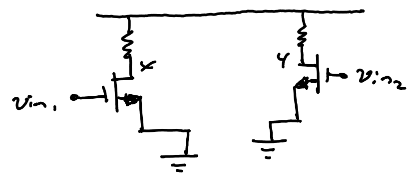
① 

$\Rightarrow \frac{\Delta I}{\Delta V} = g_m$

$\Delta I = g_m \Delta V$

② 

• Small-signal equivalent



V_{GS_1} changes by

$$\frac{\sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}\ \Delta V}{\sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}} = \Delta V$$

thus, V_P is constant and is equivalent to an AC ground.

$$v_x - v_y = -R_D \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}\ (2\Delta V)$$

$$= -R_D (I_{D_1} - I_{D_2})$$

$$\therefore I_{D_1} - I_{D_2} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}\ (2\Delta V)$$

$$+ \quad I_{D_1} + I_{D_2} = I_{SS}$$

$$2 I_{D_1} = I_{SS} + \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}\ (2\Delta V)$$

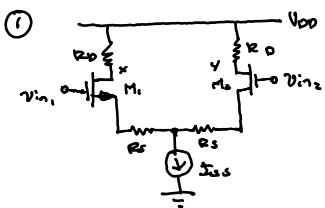
$$I_{D_1} = \frac{I_{SS}}{2} + \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}\ (\Delta V)$$

$$\frac{v_x}{v_{in_1}} = -g_m R_D, \quad \frac{v_y}{v_{in_2}} = -g_m R_D$$

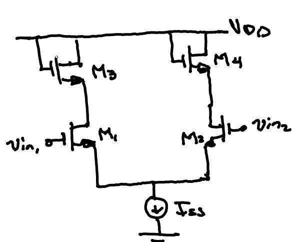
thus,

$$\frac{v_x - v_y}{v_{in_1} - v_{in_2}} = -g_m R_D$$

• Examples:

① 

$$\frac{v_x}{v_{in_1}} = \frac{-R_D}{\frac{1}{g_m} + R_S} \quad \text{or} \quad \frac{v_x - v_y}{v_{in_1} - v_{in_2}} = \frac{-R_D}{\frac{1}{g_m} + R_S}$$

② 

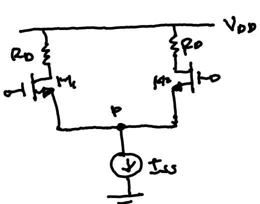
Note that

A diode connected device acts as a resistor

$R_{in} = \frac{1}{g_m}$

Note: Neglecting channel-length modulation

$$A_v = \frac{-g_{m_1}}{g_{m_3}}$$

③ 

What if $W \to 2W$??

$-g_m \Rightarrow \sqrt{2}\, g_m$

$A_v \Rightarrow \sqrt{2}\, A_v$

④ What happens if $I_{SS} \Rightarrow 2 I_{SS}$ and $W \Rightarrow 2W$???

$g_m \Rightarrow 2 g_m$

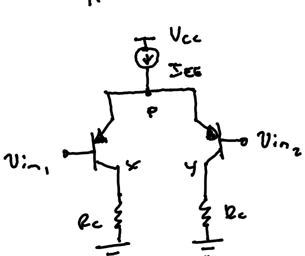
$A_v \Rightarrow 2 A_v$

⑤ What happens if temperature rises???

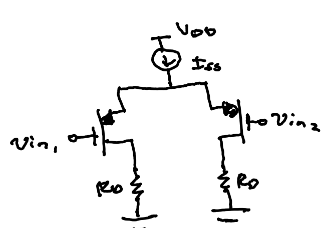
$g_m \downarrow$ since $\mu_n \downarrow$

$\Rightarrow |A_v| \downarrow$

• P-type Differential Pairs



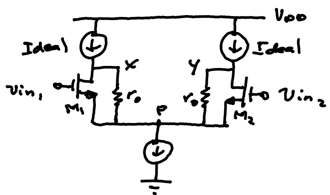
• the calculations are similar to the NPN BJT



• All calculations are similar to NMOS

High - Gain Differential Pairs

- We need a load that doesn't satisfy Ohm's law.

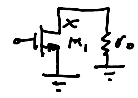


Approach I: V_P is constant

$\Rightarrow$ P is AC GND.

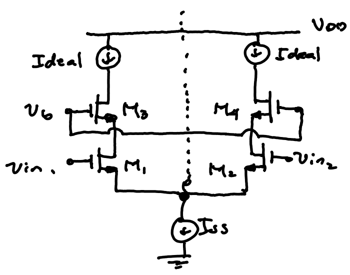
$\therefore A_v = -g_m r_o$

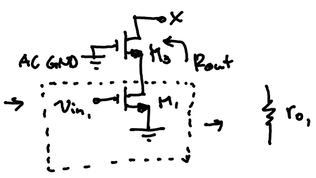
the circuit is equivalent to



where $A_v = -g_m r_o$

- Differential Pair with Cascode load



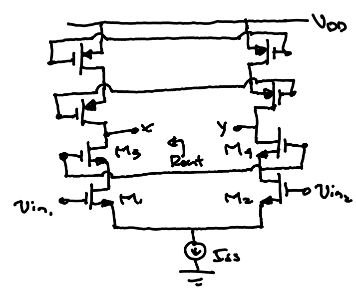


$\rightarrow R_{out} = (1+g_{m_3} r_{o3}) r_{o1} + r_{o3}$

$\approx g_{m_3} r_{o3} r_{o1}$

$\therefore A_v = -g_{m_1} r_{o1} g_{m_3} r_{o3}$

* Ideal current sources can be realized by a PMOS

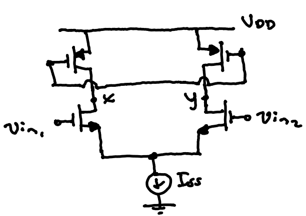


$G_m = g_{m_1}$

$R_{out} = \left[(1+g_{m_3} r_{o3}) r_{o1} + r_{o3}\right] \,\|\, \left[(1+g_{m_7} r_{o7}) r_{o5} + r_{o7}\right]$

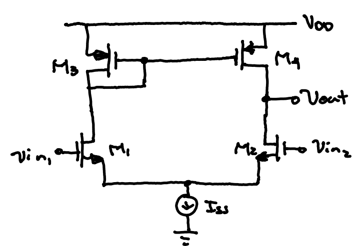
$\therefore A_v = -G_m R_{out}$

- Differential Pair with Active load



- What if I want to connect X and Y to a single-ended input stage??
  - If X is connected and Y is left out then A_v is halved.

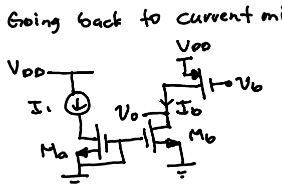
- Consider



If V_{in_1} increases by ΔV and V_{in_2} decreases by the same amount, then $I_{D1} \uparrow$ and $I_{D2} \downarrow$.

$\therefore V_{out}$ increases.

- Going back to current mirrors,



NMOS current mirror

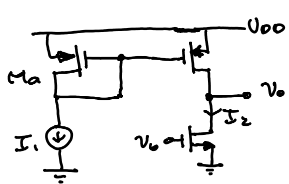
If $I_1 \rightarrow I_1 + \Delta I_1$, V_o decreases.

why?

$V_o = V_{DD} - I_D(Z)$ for some impedance Z.

$I_D \uparrow = V_o \downarrow$

Conversely, for a PMOS current mirror,



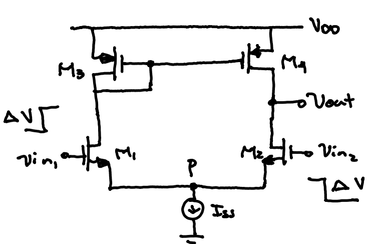
If $I_1 \rightarrow I_1 + \Delta I_1$, V_o increases.

why?

$V_o = I_D(Z)$ for some impedance Z.

thus, $I_D \uparrow = V_o \uparrow$.

Going back,

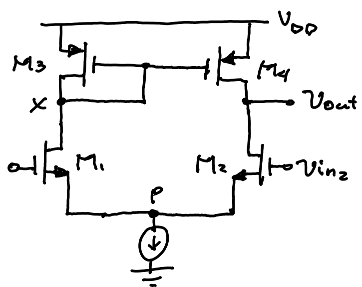


- V_{out} wants to go up if V_1 increases.
- V_{out} wants to go up if V_2 decreases

$\rightarrow V_o$ wants to go up (intuitively)

Small Signal Behavior of Differential Pair with Active Load

Consider the circuit



$\rightarrow$ Change in V_X < Change in V_{out}

- $A_v = \dfrac{V_{out}}{V_{in1} - V_{in2}}$

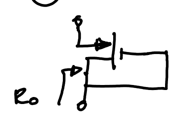
- This circuit has two signal paths
  - Path from M_1
  - Path from M_2

Questions:

1. How much is the voltage change at X?
2. How much is the voltage change at the output?

Answers:

1. the equivalent impedance of



$\Rightarrow R_o$ is $\frac{1}{g_m} \| r_o$

thus, $\Delta V_X = \Delta I \left(\frac{1}{g_m} \| r_o\right)$

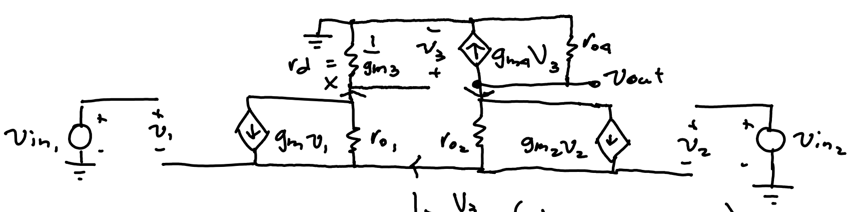
$\approx \dfrac{\Delta I}{g_m}$

2. Resistance seen by the output node is $r_{on} \| r_{op}$, thus,

$\Delta V_{out} = \Delta I (r_{on} \| r_{op})$

- Small-signal Voltage Gain

* P is not AC GND!!!



KCL at the output:

$$\frac{v_3}{r_d} + \frac{v_{out}}{r_{o4}} + g_{m4} v_3 = 0$$

$$v_3 = \frac{-v_{out}}{r_{o4}(2g_{mp})} = \frac{-v_{out}}{r_{o,p}(2g_{mp})}$$

KCL at X:

$$g_m v_1 + \frac{v_3 - v_p}{r_{o1}} + \frac{v_3}{r_d} = 0$$

KCL at output:

$$g_m v_2 + \frac{v_{out} - v_p}{r_{o2}} - \frac{v_3}{r_d} = 0$$

(−)

$$g_{m,N}(v_1 - v_2) + \frac{v_3 - v_{out}}{r_{o,N}} + \frac{2v_3}{r_d} = 0$$

$v_1 - v_2 = v_{in1} - v_{in2}$, thus $\quad g_{m,N}(v_{in1} - v_{in2}) + \dfrac{v_3 - v_{out}}{r_{o,N}} + \dfrac{2v_3}{r_d} = 0$

since $v_3 = \dfrac{-v_{out}}{r_{o,p}(2g_{mp})}$, $\quad g_{m,N}(v_{in1} - v_{in2}) + \dfrac{-v_{out}}{2g_{mp} r_{op} r_{oN}} - \dfrac{v_{out}}{r_{oN}} - \dfrac{2v_{out}}{2g_{mp} r_{op} r_d} = 0$

$$g_{m,N}(v_{in1} - v_{in2}) = v_{out}\left[\underbrace{\frac{1}{2g_{mp} r_{op} r_{oN}}}_{\text{negligible } \approx 0} + \frac{1}{r_{oN}} + \frac{1}{r_{op}}\right]$$

Thus, $\dfrac{v_{out}}{v_{in1} - v_{in2}} = -g_{m,N}(r_{o,N} \| r_{o,P})$ $\rightarrow$ same as the gain of fully differential output!!!