

Semiconductor Physics

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1 Resistivity and Conductivity

For any material, if we connect two terminals across it, we can obtain its resistance. The resistance of any device is defined as

$$R = \rho \frac{L}{A}$$

where L is the length of the device, ρ is the resistivity, and A is the cross-sectional area.

Similarly, conductivity is obtained by

$$G = \frac{1}{R}$$

2 Quantum Mechanics

The **Wave Particle Duality** states that a particle can behave as a wave and at the same time, a wave can behave as a particle. This is captured by the **De Broglie Wavelength** which states that for any particle with momentum p , we can associate a wavelength λ such that

$$\lambda = \frac{h}{p}$$

where h is the Planck's constant.

Now, consider the hydrogen atom. It has a single proton, and a single electron on its orbital. The electron is moving with a momentum p and charge $-e$. If the electron is treated as a wave, the question is that "**What are the allowed trajectories?**"

From those, we can calculate the allowed energies, and some other parameters.

Since the ψ function represents probability of presence, if I want to have a significant presence along the orbital of the atom, then we want the circumference to be an integer multiple of the wavelength. This means that the integer multiples of the wavelength are the possible trajectories.

$$2\pi r = n\lambda = n \frac{h}{p}$$

Is the condition for having a sustainable trajectory.

We also know that the electron does not move away from the proton because it is bound by a force. That force is

$$F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{mv^2}{r} = \frac{p^2}{mr}$$

Simplifying, we get

$$2\pi r = n \frac{h}{p}$$

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = \frac{p^2}{m}$$

Obtaining p,

$$r = \frac{nh}{2\pi p}$$

$$\frac{1}{4\pi\epsilon_0} \frac{e^2(2\pi p)}{nh} = \frac{p^2}{m}$$

$$\frac{1}{2\epsilon_0} \frac{e^2}{nh} = \frac{p}{m}$$

Finally,

$$p = \frac{me^2}{2\epsilon_0 h} \frac{1}{n}$$

Plugging it back to the r equation gives

$$r = \frac{nh}{2\pi} \frac{2\epsilon_0}{me^2} \frac{nh}{n}$$

$$r = \frac{nh}{\pi} \frac{\epsilon_0}{me^2} \frac{nh}{n}$$

Finally,

$$r = \frac{\epsilon_0}{\pi me^2} n^2 h^2$$

where $n \in \mathbb{N}$ and m is the mass of the electron.

This means that both radius and momentum are quantized. They can only take certain values given that they have a bound state. It requires that the sum of the kinetic and potential energies of the particle be negative.

$$E_{total} = E_{kinetic} + E_{potential}$$

$$E_{total} = \frac{p^2}{2m} - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

$$E_{total} = \frac{p^2}{2m} - \frac{p^2}{m} = -\frac{p^2}{2m} \rightarrow E_{total} < 0$$

Substituting p to the total energy equation gives

$$E_{total} = -\frac{me^4}{8\epsilon_0^2 h^2} \cdot \frac{1}{n^2}$$

$$E_{total} = -\frac{13.6 \text{ eV}}{n^2}$$

If we have another atom of the same kind that is close to it, the energy levels of either of those atoms would change because of the **Pauli Exclusion Principle**. The principle states that you cannot have the same quantum numbers when two particles come together.

Since the two atoms occupy the same spatial area, their energy levels cannot be the same. Each energy level will split (each energy level can take two electrons : up-spin or down-spin). This is what we call degeneracy.

The next question is that: **When does the Pauli Exclusion Principle become important?** Apparently, the principle is always there. No two atoms in the universe have the same quantum numbers. It just becomes less noticeable when they are far apart and more evident when they get close.

When we create a crystal lattice, many electrons come together and we see more degeneracy. These degenerate energy levels are what we call energy bands. **Hybridization** happens within some of these bands specifically those of tetrahedral structure with a **109.5°** angle. At **0 K**, the bands are filled from the lowest to the highest energy band. Thus for a carbon atom which has 6 electrons, the **1s²** band is filled as well as the **2s¹ 2p³** hybrid band.

Consider two bands, an valence band and a conduction band. If the valence band is full, then the material is a bad conductor since it allows no electron movement. A good conductor is one that allows full electron movement from the valence band to the conduction band. A **semiconductor** is somewhat in the middle. The bands where there are no electrons can be called holes and carry a positive charge. Electrons move faster than holes. **Recombination** is the process of combining holes and electrons within a band. At the steady-state, the rate of recombination and hole generation are equal.

3 Solid-State Physics

The concept of **Doping** allows us to introduce a different element to control the ratio of electrons and holes. For example, if we use an element from the column **IV** of the periodic table, then we can choose elements from columns **III** and **V** as **Dopants**. As we increase the temperature, the number of electrons reach the number of dopants since they all become ionized. We call this number **N_D**. Similarly, we use **N_A** for the holes.

Electrons are fermions. They follow the **Fermi-Dirac Distribution** which tells us that we can obtain the probability of finding an electron with energy **E** by

$$f(E) = \frac{1}{1 + e^{\frac{E-E_f}{kT}}}$$

where **k** is the Boltzmann's constant and **T** is the absolute temperature. **E_f** defined as the energy where the probability of finding an electron is $\frac{1}{2}$. As **E - E_f** gets **> 3kT**, the more it resembles the **Boltzmann Distribution**.

$$f(E) \propto e^{\frac{-E}{kT}}$$

We can then obtain the relation

$$n = N_c e^{\frac{E_f - E_c}{kT}}$$

and

$$p = N_v e^{\frac{E_v - E_f}{kT}}$$

For an intrinsic semiconductor,

$$\boxed{n_i^2 = np}$$

3.1 Drift and Diffusion

The **Equipartition Theorem** states that in thermal equilibrium, each degree of freedom takes an energy

$$E = \frac{1}{2}kT$$

on average. For an electron, the degree of freedom is equal to 3. That translates to

$$E_{total} = \frac{3}{2}kT$$

From classical mechanics,

$$E_{kinetic} = \frac{1}{2}mv^2$$

We get,

$$\begin{aligned} \frac{1}{2}mv^2 &= \frac{3}{2}kT \\ v^2 &= \frac{3kT}{m} \end{aligned}$$

Since the mass is very small for an electron, the velocity is very high.

Consider an electric field on which the electron is placed. Assume that the field has a magnitude of E .

We know that,

$$F = \frac{dP}{dt}$$

and

$$-qE = \frac{\Delta P}{\Delta T} = \frac{m\Delta v}{\tau_c}$$

$$\Delta v = \frac{qE\tau_c}{m}$$

We can assume that the velocity, on average, is equal to

$$v_{ave} = \frac{1}{2}\Delta v = \frac{1}{2} \frac{qE\tau_c}{m} = v_{drift}$$

Which is also equal to

$$v_{drift} = -\mu_n E$$

where μ_n is the mobility.

The incremental charge that passes through a slice of a slab is obtained by

$$dq = -qnAv_d \cdot dt$$

where $-q$ is the charge of an electron, n is the electron density, A is the cross-sectional area of the slab, and v_d is the drift velocity. We know that

$$I_n = \frac{dq}{dt} = -qnAv_d$$

and

$$J_n = \frac{I}{A} = -qn v_d = qn\mu_n E$$

Similarly for holes,

$$J_p = qp\mu_p E$$

Finally,

$$J_{total} = q \cdot (n\mu_n + p\mu_p) E = \sigma E$$

where σ is the conductivity. The resistivity, ρ , is the reciprocal of σ .