

ECE 161: Digital Signal Processing Programming Exercise 04

INSTRUCTIONS. This is an open-notes, open-books exercise. Put your answers on the space provided on this document and save it as a PDF. In all your figures, write your full name as the title of your plot. Submit the PDF and the M-file in the UVLe submission bin with the filename *ECE161_PE04_*<*surname*>.*pdf* and *ECE161_PE04_*<*surname*>.*m*

A. FIR Filter Design by Windowing

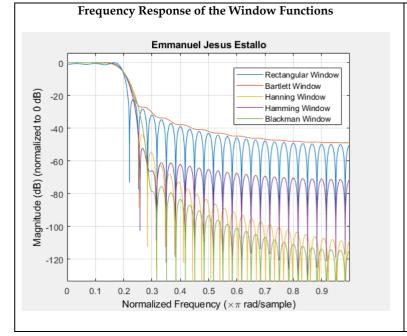
The unit impulse response h(n) obtained using the windowing method in designing FIR filters can be expressed as $h(n) = h_d(n)w(n)$, where w(n) is the window function, and

$$h_d(n) = \frac{\sin \omega_c \left(n - \frac{M-1}{2}\right)}{\pi \left(n - \frac{M-1}{2}\right)}, \qquad 0 \le n \le M-1, n \ne \frac{M-1}{2}$$

For the case where *M* is selected to be odd,

$$h_d\left(\frac{M-1}{2}\right) = \frac{\omega_c}{\pi}, \qquad n = \frac{M-1}{2}$$

- 1. With M = 61 and a cutoff frequency of $\omega_c = 0.2\pi$, generate the FIR filter coefficients h(n) with the following window functions. Normalize the filters such that $\max |H(\omega)| = 1$. Superimpose the plots of the magnitude response (in dB) for each window function used. Write your full name as the title of the plot. Describe your observations in applying different windowing functions based on the passband ripple, the stopband attenuation, and the transition region.
 - a. Rectangular
 - b. Bartlett
 - c. Hanning
 - d. Hamming
 - e. Blackman



Observations

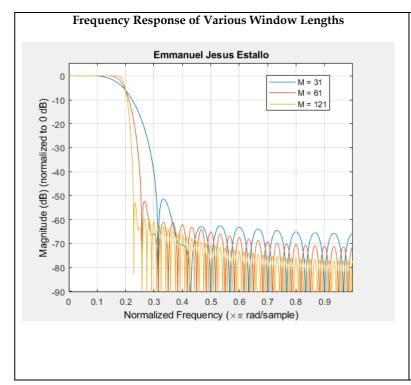
The Bartlett, Hanning, and Hamming windows have almost the same transition region. Blackman window has the largest transition region; rectangular window – smallest.

The Bartlett window has no sidelobes. In the passband, the rectangular window produces the largest ripple.



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2. With a cutoff frequency of $\omega_c = 0.2\pi$ and using a Hamming window, generate the FIR filter coefficients h(n) with M = 31, M = 61, and M = 121. Normalize your filter such that $\max(|H(\omega)|) = 1$. Superimpose the plots of the magnitude response (in dB) for each window length used. Write your full name as the title of your plot. Describe your observations in employing various window lengths based on the passband ripple, the stopband attenuation, and the transition region.



Observations

Increasing M also increases the number of sidelobes in the stopband. Also, it provides a shorter transition region. As we can see, the transition region of M = 121 is far smaller than the transition region of M = 31. The slope is steeper. In the passband, the ripples decrease.

In a sense, increasing the window length also makes the filter better.

B. FIR Filter Design by Frequency Sampling

In this part, we will try to obtain the impulse response h(n) by the frequency sampling technique. To retrieve h(n) based on the frequency samples $H\left(\frac{2\pi k}{M}\right)$, $k=0,1,...,\frac{M-1}{2}$, we use the following linear equations.

$$\sum_{n=0}^{M-1)/2} a_{kn}h(n) = H_r(\omega_k)$$

where

$$a_{kn} = \begin{cases} 2\cos\omega_k \left(\frac{M-1}{2} - n\right), & n \neq \frac{M-1}{2} \\ 1, & n = \frac{M-1}{2} \end{cases}$$

Suppose we have the following frequency samples.



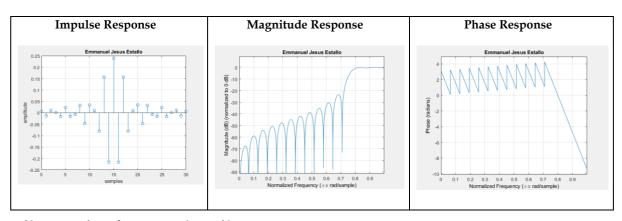
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$$H_r\left(\frac{2\pi k}{M}\right) = \begin{cases} 0, & k = 0,1,\dots,11\\ 0.7, & k = 12\\ 1, & k = 13,14,15 \end{cases}$$

1. Determine the order M and the approximate cutoff frequency w_c .

M=31	$\omega_c = 0.79\pi$

2. Determine the impulse response h(n) by solving the set of linear equations. Note: use the symmetry conditions of a linear phase FIR filter to obtain the remaining filter coefficients. Include a *stem* plot of the h(n) you have calculated with your full name as the title of your plot. Also plot the magnitude (in dB) and the phase response of H(z) with your full name as the title of your plots.



C. IIR Filter Design from Analog Filters

An analog lowpass filter is to be designed that requires the following specifications:

$$0.99 \le |H(\Omega)| \le 1$$
, $0 \le f \le 1500 \, Hz$
 $0 \le |H(\Omega)| \le 0.01$, $f \ge 2000 \, Hz$

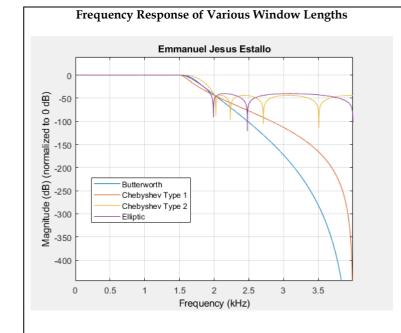
1. Determine the lowest-order filter for the following kinds of filter that satisfies the specifications. You may use the appropriate function that computes for *N*, e.g. *buttord* for a Butterworth filter.

Kind of Filter	N
Butterworth	23
Chebyshev Type I	10
Chebyshev Type II	10
Elliptic	6

2. Suppose that $f_s = 8000 \, Hz$. Design the IIR digital filters for the four kinds of filter by using their appropriate functions, e.g. *butter* for Butterworth filters. Plot their corresponding magnitude responses (in dB) in a single plot with your full name as the title of your plot. Describe your observations in applying different analog filter types based on the passband ripple, the stopband attenuation, the transition region, and the minimum filter order to satisfy the specifications.



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Observations

The minimum order that satisfies the conditions are:

Butterworth: 17 Chebyshev Type 1: 8 Chebyshev Type 2: 8

Elliptic: 5

The Butterworth filter has the least ripples. The passband is almost flat for the Butterworth and Chebyshev Type 2. All filters satisfy the transition region frequencies. There are ripples in the stopband of the Chebyshev Type 2 and the Elliptic