UNIVERSITY OF THE PHILIPPINES DILIMAN
ELECTRICAL AND ELECTRONICS ENGINEERING INSTITUTE
ECE 161: Digital Signal Processing
Worksheet: Programming Exercise 02

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Lecture Section: WFX

<u>This exercise is worth 100 points.</u>

A. THE BILATERAL Z-TRANSFORM

<u>Z-Transform of Sequences</u>

1. (10pts) Determine the z-transform of the following sequences, using the definition for $Z[x(n)]$. Indicate the region of convergence for each sequence. Express $X(z)$ as a rational function in z^{-1}. Solutions may be handwritten or typewritten.

(a) $x(n) = \left(\frac{4}{3}\right)^n u(1-n)$ (b) $x(n) = 2^{-|n|} + \left(\frac{1}{3}\right)^{|n|}$

a.) $x(n) = \left(\frac{4}{3}\right)^n u[1-n]$

$$X(z) = \sum_{k=-\infty}^{\infty} \left(\frac{4}{3}\right)^k u[1-k] z^{-k}$$

$$= \sum_{k=-\infty}^{1} \left(\frac{4}{3}\right)^k z^{-k}$$

$$= \frac{4}{3} z^{-1} + 1 + \sum_{-\infty}^{-1} \left(\frac{4}{3}\right)^k z^{-k}$$

shift the signs

$$= \frac{4}{3} z^{-1} + 1 + \sum_{1}^{\infty} \left(\frac{4}{3}\right)^{-k} z^{k}$$

$$= \frac{4}{3} z^{-1} + 1 + \frac{\left(\frac{4}{3}\right)^{-1} z}{1 - \left(\frac{4}{3}\right)^{-1} z} \cdot \frac{\frac{4}{3} z^{-1}}{\frac{4}{3} z^{-1}}$$

$$\boxed{= \frac{4}{3} z^{-1} + 1 + \frac{1}{\frac{4}{3} z^{-1} - 1}}$$

ROC: $|z| < \frac{4}{3}$

b.) $x(n) = 2^{-|n|} + \left(\frac{1}{3}\right)^{|n|}$

$= \left(\frac{1}{2}\right)^{|n|} + \left(\frac{1}{3}\right)^{|n|}$

$= \left(\frac{1}{2}\right)^{-n} + \left(\frac{1}{2}\right)^{n} + \left(\frac{1}{3}\right)^{n} + \left(\frac{1}{3}\right)^{-n}$

$= \left(\frac{1}{2}\right)^{-n} + 2^{-n} + 3^{-n} + \left(\frac{1}{3}\right)^{-n}$

$= \sum_{-\infty}^{\infty} \left(\frac{1}{2}\right)^{-k} z^{-k} + \sum_{-\infty}^{\infty} 2^{-k} z^{-k} + \sum_{-\infty}^{\infty} 3^{-k} z^{-k} + \sum_{-\infty}^{\infty} \left(\frac{1}{3}\right)^{-k} z^{-k}$

since it is an absolute value function, we can consider a single interval only. we take $(0, +\infty)$

By infinite geometric series, $\left(\text{sum} \begin{cases} \frac{a_1}{1-r} & r<1 \\ \frac{a_1}{r-1} & r>1 \end{cases}\right)$

$= \frac{1}{2z^{-1}-1} + \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-\frac{1}{3}z^{-1}} + \frac{1}{3z^{-1}-1}$

with ROC:

$\left(|z| > \frac{1}{2}\right) \cap \left(|z| < 2\right) \cap \left(|z| > \frac{1}{3}\right) \cap \left(|z| < 3\right)$

$= \frac{1}{2} \; |z| < 3$

2. (10pts) Verify the z-transform expression by using Octave/Matlab. Use **deconv()** to generate the coefficients of the power-series expansion of $X(z)$. Note that computation of power-series expansion depends on causality of the signal. List the first 8 coefficients (for each letter).

   a. {}
   b. {}

3. (10pts) Determine the z-transform of the sequence using the z-transform properties and table of common transform pairs.

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$$x(n) = \left(\frac{1}{3}\right)^n u(n-2) + (0.9)^{n-3} u(n)$$

Express X(z) as a rational function in z^{-1}. Indicate the region of convergence. Use **impz()** (or **filter()** with input x(n) = δ(n)) to verify your results. Plot the first 20 samples.

3.) $x(n) = \left(\frac{1}{3}\right)^n u(n-2) + 0.9^{n-3} u(n)$

Consider the first term,

$$\left(\frac{1}{3}\right)^n u(n-2) = \left(\frac{1}{3}\right)^2 \left(\frac{1}{3}\right)^{n-2} u(n-2)$$

$$= \frac{1}{9} \left(\frac{1}{3}\right)^{n-2} u(n-2)$$

$$X(z) = \frac{1}{9} \cdot \frac{z^{-2}}{1 - \frac{1}{3}z^{-1}}$$

For the second term,

$$0.9^{n-3} u(n) = (0.9)^{-3} (0.9)^n u(n)$$

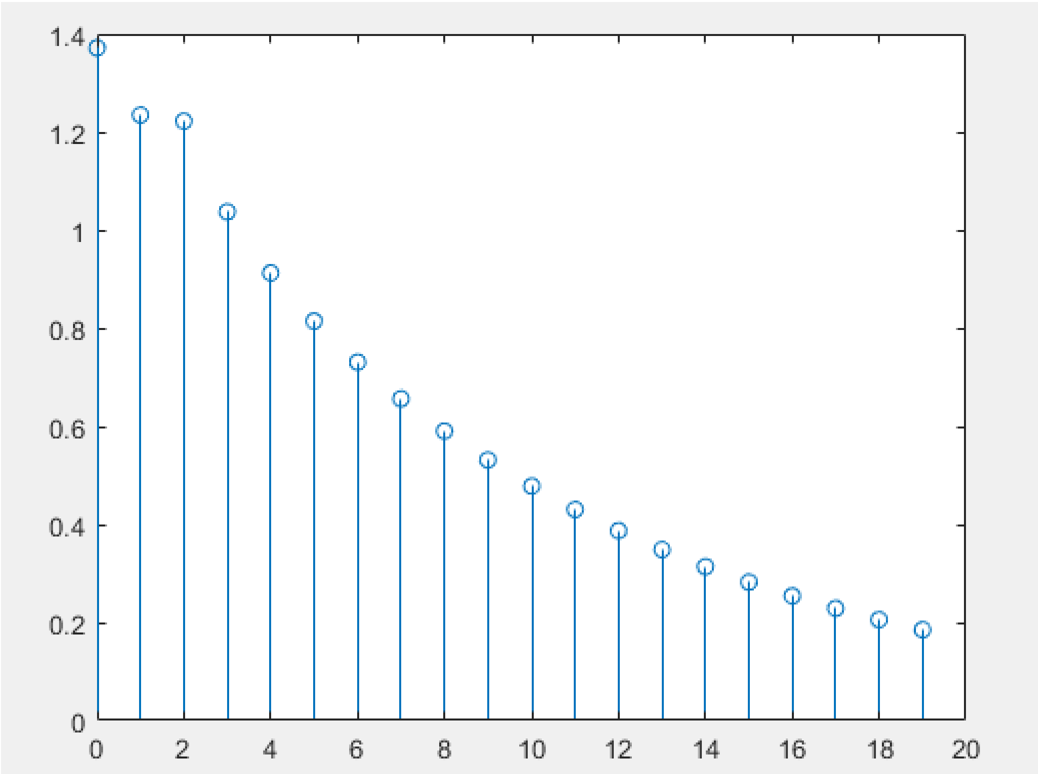
$$= \frac{1}{0.9^3} 0.9^n u(n)$$

$$X(z) = \frac{1}{0.9^3} \cdot \frac{1}{1 - 0.9z^{-1}}$$

Adding them gives

$$\boxed{X(z) = \frac{1}{9} \cdot \frac{z^{-2}}{1 - \frac{1}{3}z^{-1}} + \frac{1}{0.9^3} \cdot \frac{1}{1 - 0.9z^{-1}}}$$

ROC: $|z| > 0.9$



B. THE INVERSE Z-TRANSFORM

Inverse z-Transform

4. (10pts) Determine the inverse z-Transform of

$$X(z) = \frac{1 - z^{-1} - 4z^{-2} + 4z^{-3}}{1 - \frac{11}{4}z^{-1} + \frac{13}{8}z^{-2} - \frac{1}{4}z^{-3}}$$

Using partial-fraction expansion method. The sequence is absolutely summable. In Matlab®, the function **residuez()** can be used to solve partial fraction expansion of X(z).

```
>> [r,p,k] = residuez(num,denum);
>> r

r =

   -0.0000
  -10.0000
   27.0000

>> p

p =

    2.0000
    0.5000
    0.2500

>> k

k =

   -16
```

$$X(z) = \frac{-0}{1-2z^{-1}} - \frac{10}{1-0.5z^{-1}} + \frac{27}{1-0.25z^{-1}} - 16$$

Thus,

$$x(n) = -10(0.5)^n u(n) + 27(0.25)^n u(n) - 16\delta(n)$$

C. FOURIER TRANSFORM OF DT SIGNALS

Analysis of Signal frequency components

5. (10pts) Generate the periodic even symmetric square pulse signal x(n) from [0,1]. The period of the pulse is 1 second and a pulse width of 250 milliseconds with a sampling frequency of 8 KHz. Plot one period of the x(n) and verify if you have the correct waveform.

   a) How many samples in one period? **8000 samples**

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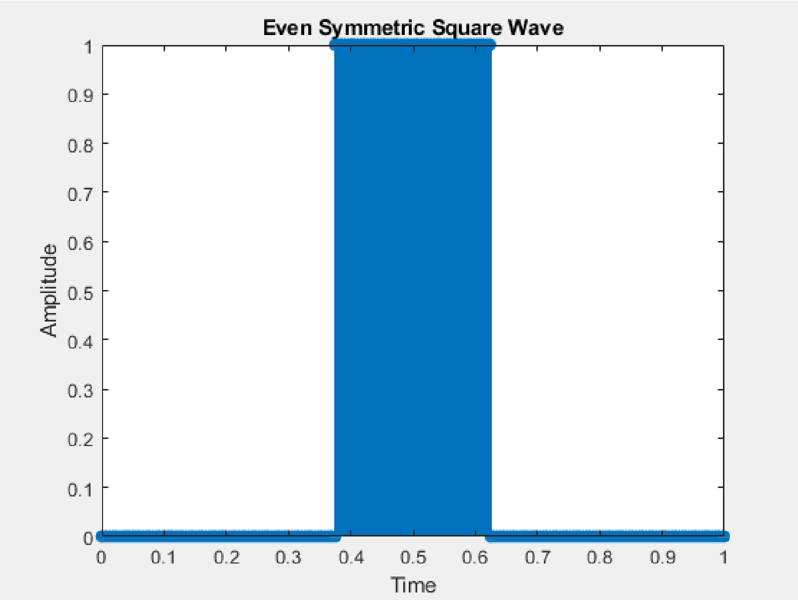

b) How many samples with a value of 1? **2000 samples**
c) How many zeros? **6000 samples**

6. (25 pts) Using the analysis equation of the Fourier series, write a program that will compute the Fourier series coefficients (complex) of the periodic square pulse signal. Plot the magnitude and phase of the first 10 Fourier series coefficients (c_k).

   a) What is the fundamental frequency of the square pulse?

$$\frac{2\pi}{N} = \frac{2\pi}{8000} = \frac{\pi}{4000}\ rad/sample$$

   b) Enumerate the magnitude and phase of the first 10 coefficients of the Fourier series (c_0, c_1, …, c_{10})

   Magnitude: {0.2500, 0.2251, 0.1592, 0.0750, 3.295e-17, 0.0450, 0.0531, 0.0322, 7.0964e-17, 0.0250}

   Phase: {0, -3.1412, 7.8540e-4, -3.1404, 2.2503, 0.0020, -3.1392, 0.0027, 1.6601, -3.1381}

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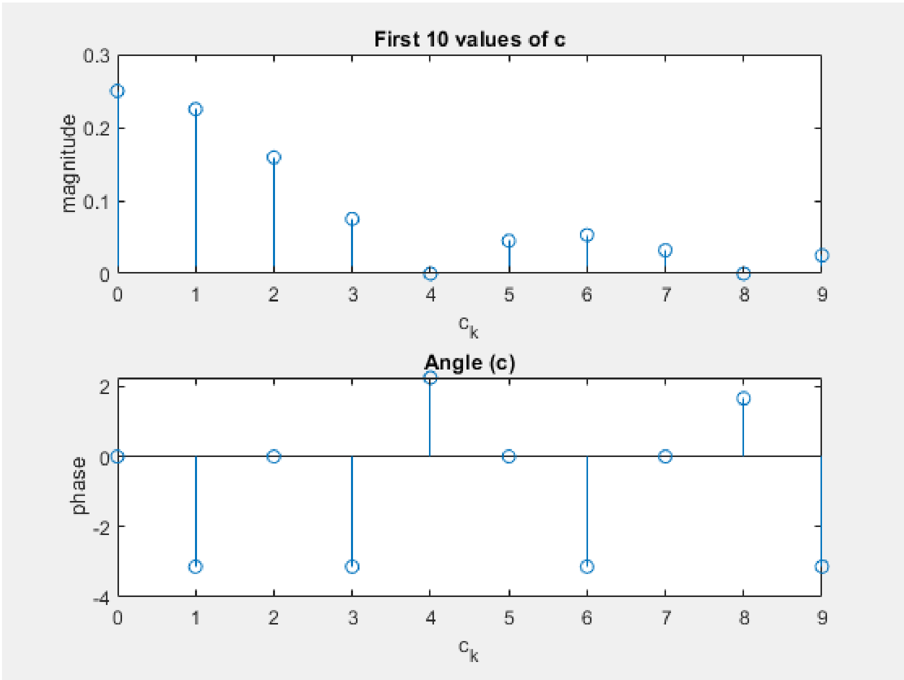
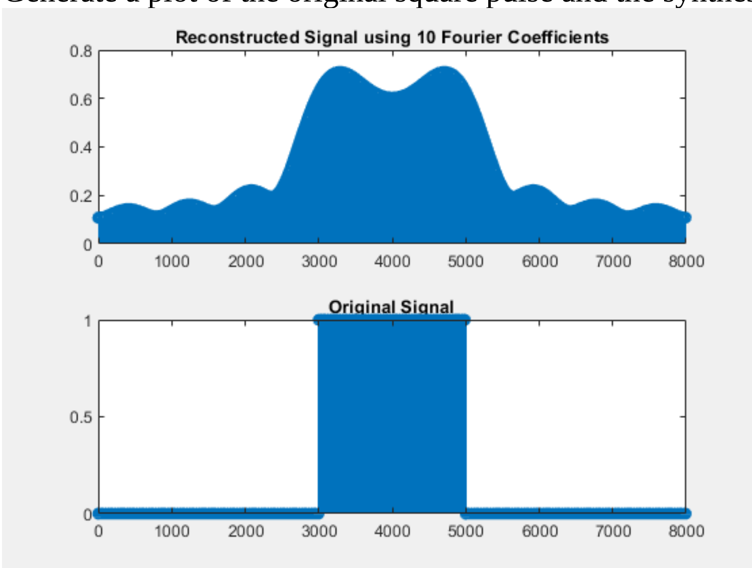


7. (25pts) Using the synthesis equation for the Fourier series, synthesize the original square pulse using the first 10 Fourier coefficients.
   a) Generate a plot of the original square pulse and the synthesized square pulse.



   b) What is the average mean square error (MSE) of the original square pulse and the synthesized pulse?

Error = 0.0701

   c) If you use 20 Fourier coefficients, what will be the average MSE?

Error = 0.0672

d) What is the effect on the fundamental frequency if I increase the pulse width to 300 ms? Explain.

- **Nothing changes on the fundamental frequency since it only depends on the number of samples. Even if the pulse width increases, the number of samples stays the same.**

e) What is the effect on the Fourier series coefficients if I change the pulse width?

- **Fourier series coefficients will change since some of the values of the square wave will change.**

f) What is the effect on the Fourier series coefficients if I change the period?

- **The Fourier series coefficients will change because of its dependence on the fundamental frequency. Depending on the period, the spacing between the samples may differ. However, since they are periodic, if the pulse width is also varied proportionally, then there will be no change in the coefficients.**

References: https://www.mathworks.com/

DECLARATION.

I accept responsibility for my role in ensuring the integrity of the work I submitted. I have not plagiarized someone else's work and turned it in as my own.

Name and Signature: Emmanuel Estallo *Date:* 4 Jun 2022