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| **Proofs and logic** |  |
| Direct proof- | Proof by contrapositive - |
| Division into cases | Transitivity - |
| Elimination - | Specialisation - |
| Inverse error - | Converse error - |
| **Uniqueness** |  |
| **Number theory** |  |
| Direct proof/Contrapositive | Pigeonhole principle |
| Constructive | Example/Counterexample – for existential statements |
| Contradiction (assume and get a contradiction) | Division into cases (**modulo**, **even/odd**, **+/-/0**) |
| **Mathematical Induction** |  |
| Strong PMI – use of every base case | PMI – 1 base case and 1 inductive step |
| Multiple base cases | PMI –inductive steps in both ways |

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| **Logical axioms** |  |
| Commutative: | Associative: |
| Distributive: | Identity: |
| Negation: | Idempotent: |
| De Morgan: | Absorption: |
| Universal bound: | Cases |
| Conditional: | Biconditional: |
| **Number system -** |  |
| Identity: | Inverse: |
| Commutative: | Associative: |
| Distributive: |  |
| **Closure properties** |  |
| **Integers:** closed under addition and multiplication | **Rational numbers:** addition, multiplication, division |
| **Even integers:** closed under addition and multiplication | **Odd integers:** closed under multiplication |

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| **Number Theory** |  |
| Tut3, q1: is even if and only if is even  *extension*: is even/odd if and only if is even/odd | Tut3, q8: If is even, and , then and |
|  | 4.1.1: If then is even |
| 4.1.2: If , then *(proven by mod cases)* | 4.1.4: Pigeonhole principle – if m pigeons go into r pigeonholes, at least one hole has more than one |
| Tut4, q5: There are no integers and with and | 4.3.6: **Standard factored form** of is where are primes, are positive integers, and |
| 5.2.1: Every positive integer can be written as the sum of distinct powers of any integer | Bernoulli inequality: |
| **Rational numbers** |  |
| 3.3.5: | 3.3.6: A rational number in its lowest term |
| **Congruence/Modulo** |  |
| Symmetric: | Transitive: |
| and for exactly one integer r such that | 𝑎≡𝑏 mod 𝑛 and 𝑐≡𝑑 mod 𝑛 ⇒𝑎+𝑐≡𝑏+𝑑 mod 𝑛 |
| and | 𝑎≡𝑏 mod 𝑛 ⇒≡ mod 𝑛 for all |
| **Absolute value** |  |
| Triangle inequality: | Tut4, q1a: |
| **Primes**: No factors except 1 and itself | **Composites**: Not a prime |
| Tut4, q4: The set of primes is infinite |  |
| 4.2.3-derived: Let be a sequence of primes. For any prime | Assn1, q5b: |
| **Irrational Numbers()** Definition: Not rational |  |
| Sum of a rational and irrational number is irrational | Product of a rational and irrational number is irrational |
| Tut3, q4a: if is irrational then is irrational  *extension*: any root of x is irrational | is an irrational number |
| is irrational | Assn1, q4b is irrational |
| **Sequences** |  |
| **AP:** | **GP:** Given a series , where |
| Sum of squares: | Product: |
| **2nd-Order Linear Homogeneous Recurrence Relation:** |  |
| Expression: | Characteristic eqn: |
| 2 roots and : | 1 root : |
| **Sets** |  |
| **Notations**: Listing {1,2,…} or Set builder {} | **Operators**: complement |
| **Laws**: Idempotent, Commutative, Associative, Distributive, De Morgan’s | Distributive law on Cartesian products: |
| **Functions** |  |
| = image of and = preimage of under , | **Injective:**  **Surjective:**  **Bijective**: Injective and Surjective |
| **Relations** |  |
| implies | **Reflexive**:  **Symmetric**:  **Transitive**: |
| **Equivalence relation** when R is reflexive, symmetric and transitive | **Equivalence classes**: |
| **Counting** |  |
| repetition allowed, order matters: (multiplication rule) | repetition allowed, order does not matter: (r-combination with repetition) |
| repetition not allowed, order matters: (r-permutation) | repetition not allowed, order does not matter: (r-combination) |
| permutations of objects with indistinguishable elements: | generalized inclusion/exclusion rule: |
| **Graphs** |  |
| Graph: A graph Consists of a *nonempty* set of vertices and set of edges | Simple graph: No loops or parallel edges |
| Bipartite graph: graph with distinct vertices and such that there are no edges between any ’s or ’s | Handshake theorem: In any graph, the total degree of a graph is twice the number of edges, and is always even |
| 10.1.9: In any graph, there are an even number of vertices with an odd degree | Trail: No repeat edges  Path: Trail with no repeat vertices |
| Closed walk: Starts and ends at same vertex  Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices | Euler circuit: Visits every edge of . must be connected and all vertices with positive even degrees |
| Isomorphism: and are isomorphic iff bijective functions and  Graph isomorphism is an equivalence relation | Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness |