Optimal Neural Network Feature Selection for Forecasting of Spatial-Temporal Series

Eurico Covas, Emmanouil Benetos

Abstract—We show empirical evidence on how to construct the optimal feature selection or architecture for the input layer of a feedfoward neural network for the propose of forecasting spatial-temporal signals. The approach is based on results from dynamical systems theory, namely the non-linear embedding theorems. We demonstrate it for a variety of two-dimensional signals with one spatial dimension and one time dimension, and show that the optimal input layer seems to consist of a two dimensional grid, with spatial/temporal lags determined by the minimum of the mutual information of the spatial/temporal signals and the number of points taken in space/time decided by the embedding dimension of the signal. We present evidence of this conjecture by running a Monte Carlo simulation of several combinations of input layer architectures and show that the one predicted by the non-linear embedding theorems seems to be optimal or close of optimal. In total we show evidence in four unrelated systems: a series of coupled Hénon maps; a series of couple Ordinary Differential Equations (Lorenz-96) phenomenologically modelling atmospheric dynamics; the Kuramoto-Sivashinsky equation and finally real data from sunspot areas in the Sun (in latitude and time) from 1870 to today.

Index Terms—IEEE, IEEEtran, journal, LATEX, paper.

I. Introduction

GIVEN a physical data set one of the most important questions one can pose is:"Can we predict the future?" This question can be put forward irrespectively of the fact that we may already have some insight or even be certain on what the exact model behind some or all the observed variables is. For example, for chaotic dynamical systems [1], [2], we may even have the underlying dynamics but still find it hard to predict the future, given that chaotic systems have exponential sensitivity to initial conditions. The more chaotic a system is (as measured by the positiveness of their largest Lyapunov exponents [3], [4]) the harder it gets to predict the future, even within very short time horizons. In the limit case of a random system, it is not possible to predict the future at all, although one can opine on certain future statistics [5]. For the case of weakly chaotic systems, there is an extensive literature on forecasting methods ranging from linear approximations [6]; truncated functional expansion series [7], [8]; non-linear embeddings [9]; auto-regression methods [10]; hidden markov models [11] to state-of-the-art neural networks and deep learning methodologies [12].

Most literature on forecasting chaotic signals is dedicated to a single time series, or treat a collection of related time series as a non-extended set, i.e. a multivariate set of discrete as opposed to continous time series. For forecasting spatial-temporal chaos we refer the reader to [13]–[19] and references therein. Even rarer are attempts to forecast spatial-temporal chaos using neural networks and deep learning methodologies [20]–[27], although this field of research is clearly growing at the moment. Nonetheless, this area of research is of importance, as most physical systems are spatially extended, e.g. the atmospheric system driving the weather [28]; the solar dynamo driving the Sun's sunspots [29]; and the influence of sunspots on the Earth's magnetic field via the solar wind, coronal mass ejections and solar flares – the so called space weather [30]–[44].

The reasons why spatial-temporal chaos is so difficult to forecast are: first the Hausdorff dimension of the attractor [45] – usually quite large; and second how to choose the variables to use for forecasting, i.e., is there enough information on the same point back in time to derive the future of that particular point, or do spatial correlations and spatial propagation affect it in a way that one must take into account some spatial and temporal neighbours set to forecast the future, and if so, can that set be defined and how can it be constructed? It is this last question that we investigate in this article, in the particular context of spatial-temporal forecasting using neural networks. Feature extraction and the design of the architecture of the input layer for a neural network is considered to be an art form, relying mostly on trial and error and domain knowledge. For forecasting of time series a simple approach consists of designing the input layer as a vector of previous data using a time delay, the time delay neural network method [46]–[51]. For spatial-temporal series, one can generalize it to include temporal and spatial delays [19], [20]. This is where the connection to dynamical systems can be useful. In 1981, Takens established the theoretical background [52] in the Takens embedding theorem for a mathemmatical method to reconstruct the dynamics of the underlying attractor of a chaotic dynamical system from a time ordered sequence of data observations. Notice the reconstruction conserves the properties of the original dynamical system up to a diffeomorphism. Further developments established a series of theorems by [53], [52], [54] and [55]. These theorems serve the basis for a non-linear embedding and forecasting on the original variables. The theorems and related articles propose to use a time delay based on the first minima of the *mutual information* - see [56]–[58]) and to use the number of points using the method of *method of false nearest neighbours* detection suggested by [59] and reported in detail in [57], [60]–[62]. Using this theoretical framework, we propose that this non-linear embedding method can be used to indicate what the

E. Covas is with the CITEUC, Geophysical and Astronomical Observatory, University of Coimbra, 3040-004, Coimbra, Portugal, and Queen Mary University London, 10 Godward Square, Mile End Rd, London E1 4FZ, e-mail: (eurico.covas@mail.com)

E. Benetos is with Queen Mary University London, 10 Godward Square, Mile End Rd, London E1 4FZ, e-mail: (emmanouil.benetos@qmul.ac.uk)

best way is to contruct the input layer for a neural network in both space and time.

In this paper we show empirical evidence for an optimal architecture of the input layer of a neural network which tries to forecast or predict a spatial-temporal signal. Here we show the empiral evidence for fours particular cases of two-dimensional data series s_m^n, where byy two-dimensional we mean a scalar field which can be defined by a $N \times M$ matrix with components $s_m^n \in \mathbb{R}$.

A. Neural networks for time-series forecasting

Describe the use of feedforward artificial neural networks, then recurrent networks and other methods (put all references). Put all references to pure time only forecasts [63]. DO WE REALLY NEED THIS SECTION?

B. Neural networks for spatial-temporal forecasting

There has been several attemps to forecast spatial-temporal data series in the literature (ADD MORE REFERENCES [21], [22], [64]). The references above confine themselves to attemptS to forecast the actual full scalar field s_m^n, as opposed to the ongoing research on pattern recognition in moving images (2D and 3D), which attempt to pick particular features in images (e.g. car, pedestrian, bicycle, person) and to forecast where those features will be in subsequent images within the particular moving sequence (ADD REFERENCES) or research on word sequences (ADD REFERENCES).

C. Input layer architecture for neural networks for spatial-temporal forecasting

All of the references above on neural network forecasting of spatial-temporal data series either use a simple delay based architecture for the input layer, or use a time delay based on the first minima of the *mutual information* as proposed in [56]–[58]) and/or use the number of points dictated by the embedding theorems by [53], [52], [54] and [55] using the method of *method of false nearest neighbours* detection suggested by [59] and reported in detail in [57], [60]–[62]. There are other researchers who use more complex neural networks without being explicity about time or spatial delays [65]. ADD REFERENCES.

However, as far as we are aware, all the references to using the embedding theorems and the related mutual information method and the false nearest neighbours method seems to be not justified, i.e., the approach is explained but not proven either theoretically or empirically. Here we use two examples of spatial-temporal signals, one a real world data series, another a synthetic signal, to demonstrate that there seems to be empirical evidence for an underlying theorem for what the optimal neural network feature selection for forecasting of spatial-temporal series should be.

Takens established the theoretical background [52] in the Takens embedding theorem. Further developments established a series of theorems by [53], [52], [54] and [55].

Some authors discuss the use of either mutual information and/or embedding dimension as a constraint on the input layer, i.e. the feature selection. [48], [50], [51], [75], [78], [81], [106], [108], [114], [117]–[129] used the mutual information and/or the false nearest neighbours' methods to estimate the suitable embedding parameters.

[72], [73], [78], [108], [114], [117], [119] attempted to forecast the solar sunspot number using neural networks and they used the embedding dimension of the sunspot time series as way to define the architecture of the input layer.

[130] generalize the mutual information approach to higher dimensions but do not connect it to the problem of the neural network input layer architecture optimization.

There are also papers [131] that try to use neural networks to determine the optimal embedding and time delay for the purposes of local reconstruction of states with a view to forecast.

There are also papers [18] that use Support Vector Machines (SVMs) to forecast in space and time and use time delays and embedding approachs to define the states vectors.

In fact Parlitz and Merkwirth [16] in the ESANN 2000 meeting proceddings paper mentioned that local reconstruction of states "…may also serve as a starting point for deriving local mathematical models in terms of polynomials, radial basis functions or neural networks.". Here we attempt to show empirical evidence that this is not just a starting point, but the optimal neural network input architecture.

Eurico Covas
March 12, 2018

II. Neural Network architecture

The neural network architecture we choose to demonstrate our possible conjecture is a form of the basic feedforward neural network, sometimes called the time-delayed neural network [46], trained using the so-called back-propagation algorithm [132]–[134]. We focus on spatial-temporal series, so we have extended the usual time-delayed neural network to be a time and space delayed network. The overall architecture of the network is depicted in detail in Fig. 1.

Under this architecture, we use the ideas proposed in [16] to construct a grid of input values which are then fed to the neural network to produce a single output, the future state. Formally, let $n = 1, ..., N$ and $m = 1, ..., M$. Consider a spatial-temporal data series $\mathbf{s}$ which can be defined by a $N \times M$ matrix with components $s_m^n \in \mathbb{R}$. To these components, we will call *states* of the spatial-temporal series. Consider a number $2I \in \mathbb{N}$ of neighbours in space of a given s_m^n and a number $J \in \mathbb{N}$ of temporal past neighbours relative to s_m^n (see Fig. 2 for details). For each s_m^n, we define the input (feature) vector $\mathbf{x}(s_m^n)$ with components given by s_m^n, its $2I$ spatial neighbours and its J past temporal neighbours, and with K and L being the spatial and temporal lags:

$$\mathbf{x}(s_m^n) = \{s_{m-IK}^n, ..., s_m^n, ..., s_{m+IK}^n, s_{m-IK}^{n-L}, ..., s_m^{n-L}, ..., s_{m+IK}^{n-L}, ...$$
$$\cap \, s_{m-IK}^{n-JL}, ..., s_m^{n-JL}, ..., s_{m+IK}^{n-JL}\} \quad (1)$$

So, the input is a $(2I + 1)(J + 1)$ vector $\mathbf{x}(s_m^n)$ and the target (output) to train the network is the value s_m^{n+1}. In Fig. 2 we show the details of this method. We train the network using stochastic gradient back-propagation (using momentum

input grid for s(n,m) used to forecast s(n+1,m)

target

s_m^{n+1}

hidden

input

w(i,j)

w'(i,j)

output

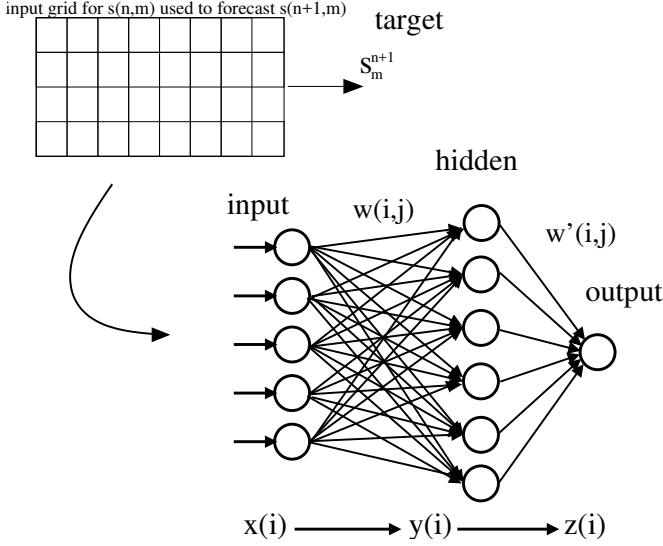$x(i) \longrightarrow y(i) \longrightarrow z(i)$

Fig. 1. Forecasting method illustration. The neural network is made of an input layer, one or more hidden layer(s) and one output layer. In this article, we use only one hidden layer and the output layer is made of a single neuron. Each input pattern $x(i)$ is sent to the input layer, then each of the hidden neurons values is calculated from the sum of the product of the weights by the inputs $\sum w(i,j)x(i)$ and passed via the activation function. Then the output is made by the product of the second set of weights times the hidden node values $\sum w'(i,j)y(i)$ again passed to another (or the same) activation function. Each input pattern $x(i)$ is actually a matrix constructed using an embedding space of spatial and temporal delays, calculated from the actual physical spatial-temporal data values $s(n,m)$. After many randomly chosen input patterns are passed via the neural network, the weights hopefully converge to an optimal training value.

and weigh decay for regularization) by running a stochastic batch where we randomly sample pairs of inputs and outputs from the training set: $\mathbf{x}(s_m^n)$ and s_m^{n+1}, respectively. Then at test time we choose inputs $\mathbf{x}(s_m^n)$, such that $n = N_{train}$, N_{train} being the number of temporal slices on the training set.

As for the back-propagation hyperparameters, we included an adaptive learning rate $\eta_n = \eta/(1 + n/10000)$, where the parameter η is the initial learning rate and η_n is the learning rate used at time step n, we included a momentum α and a weight decay ρ. In addition we use one hidden layer with N_h nodes. A further hyperparameter is the choice of the activation function, we use either ReLu or a sigmoid function depending on the test case we are working with. We also normalize the data before passing it throught the neural network, in most cases we scale it in linear fashion $x \rightarrow \alpha_{nor} + x/\beta_{nor}$, and in the case of real physical data as we will see later, we scale it in logarithmic fashion it by $x \rightarrow \alpha_{nor} + \frac{\ln(1+x)}{\beta_{nor}}$, where x is the inital data, and α_{nor} and β_{nor} are the arbitrary shift and scaling constants, respectively. For the weight (and bias) initialization we choose random numbers with a constant distribution between $[0, 1]$ and shifted by α_{rng} and scaled by β_{rng}. The final hyperparameter is the number of steps taken on the stochastic gradient descent (equivalent to a mini-batch approach of $N_{batch} = 1$) which we denote by N_{steps}. All of these hyperparameters are calibrated and fixed before we do any simulations with respect to the parameters I, J, K, L, which are autocalibrated by the below mentioned methods derived from dynamical systems theory. In this sense these are not

hyperparameters of the neural network.

We then compare the goodness of fit by first visual inspection and second by numerically calculating the so-called structural similarity $SSIM(x, y)$ which has been proposed by [135] and used already in the context of spatial-temporal forecasting in [19], [20]. It has also been used in the context of deep learning used for enhancing resolution on two dimensional images [136] and restoring missing data in images [137]. For details on the SSIM measure see [135], [138], [139]. The SSIM index is an metric quantity used to calculate the perceived quality of digital images and videos. It allows two images to be compared and provides a value of their similarity - a value of SSIM = 1 corresponds to the case of two perfectly identical images. We use it by calculating the $SSIM(x, y)$ between the entire test set and the predicted set.

Although most papers using neural networks for forecasting in time, or in space and time (ADD REFERENCES) may use implicitly (convolutional ones?) or explicitly (TDNNs) (ADD REFERENCES), as far as the authors are aware there has been no firm evidence, either theoretical or empirical on how to choose the inputs for a neural network (see however [140] who show how the forecasting error for a pure time series prediction changes with the delay and the number of time delay points used as an input).

Here we propose that the optimal time delay/spatial delays (L and K, respectively) must be the ones based on the first minima of the *mutual information* [56]–[58] and that the optimal number of temporal/spatial points to use (J and I, respectively) must be the ones based on the the method of *method of false nearest neighbours* detection [57], [59]–[62]. We conjecture that as any set of architectures "approach" this optimal architecture, then the SSIM $\rightarrow$ 1. In the case of finite training sets and/or noisy training sets $SSIM \rightarrow x < 1$, where x is the best forecast possible given the data set. Visually we believe that the SSIM versus some reasonable metric constructed to represent the distance between any architecture and the optimal architecture will show a skewed bell shape as depicted in Fig. 3. In this conjecture we use the most obvious candidate to represent the distance between any architecture and the optimal architecture, the euclidian distance between the architecture four parameters $d_e = \sqrt{(I - I^*)^2 + (J - J^*)^2 + (K - K^*)^2 + (L - L^*)^2}$.

III. MONTE CARLO RESULTS

In order to empirically substantiate our conjecture we take four examples of spatial-temporal series and attempt to forecast using our feedforward neural network. First we calculate the optimal time delay/spatial delays (L^* and K^*, respectively) using the first minima of the mutual information optimal number of temporal/spatial points to use (J^* and I^*, respectively) using the method of method of false nearest neighbours. We then use a Monte Carlo simulation on each one of our four examples, sampling randomly values of I, J, K, L and calculating the values $d_e(I, J, K, L)$ and $SSIM(I, J, K, L)$.

We first take a physical system example, a real data example, and then we progress from "simpler" systems (maps) capable of generating spatial-temporal chaos to more "complex"
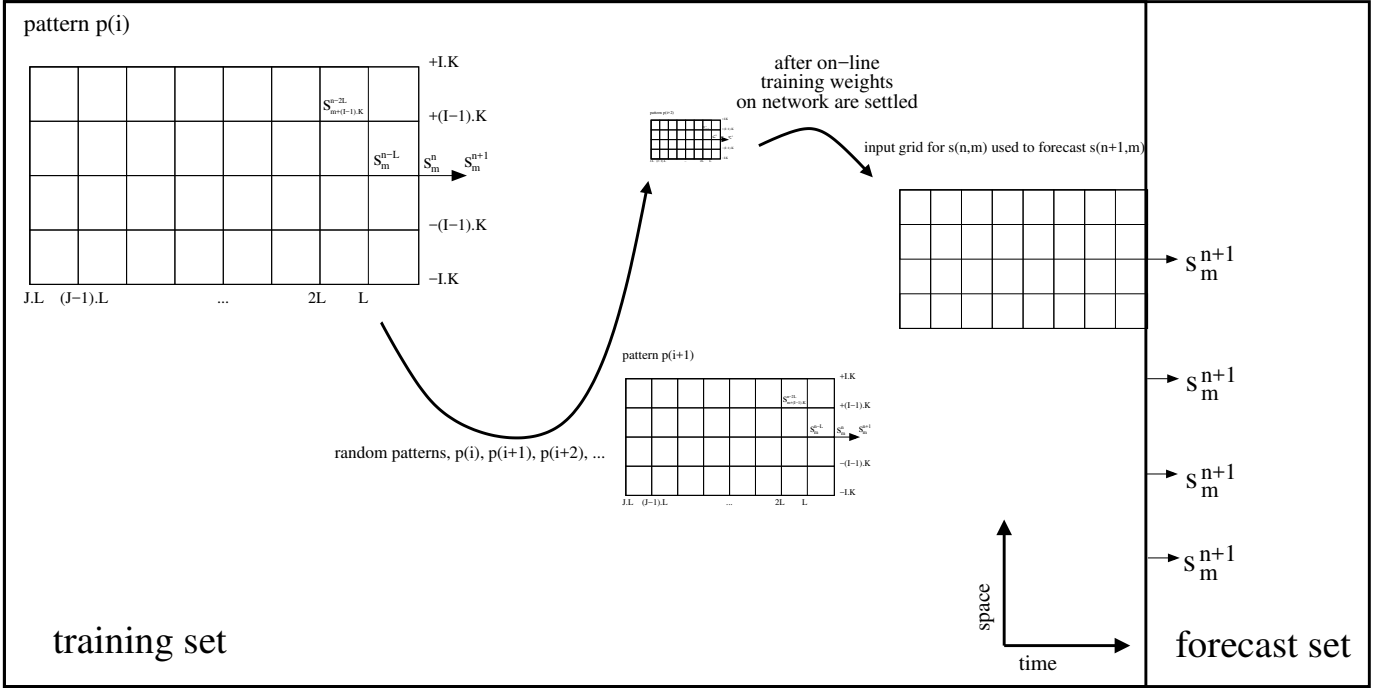
Fig. 2. Forecasting method illustration. One constructs an embedding space using space and time delays, then assemble randomly positioned grid input patterns within the training set to pass to the neural network (in this figure we show 3 randomly selected input patterns). The input is a $(2I + 1)(J + 1)$ vector $\mathbf{x}(s_m^n)$ and the target (output) to train the network is the value s_m^{n+1}. After training with a sequence of patterns $p(i), p(i + 1), p(i + 2), \ldots$ then the patterns adjacent to the forecast set are used to calculate the outputs to compare against the forecast. To forecast the $n + 2$ slice we concatenate the previously predicted $n + 1$ and progress accordingly.
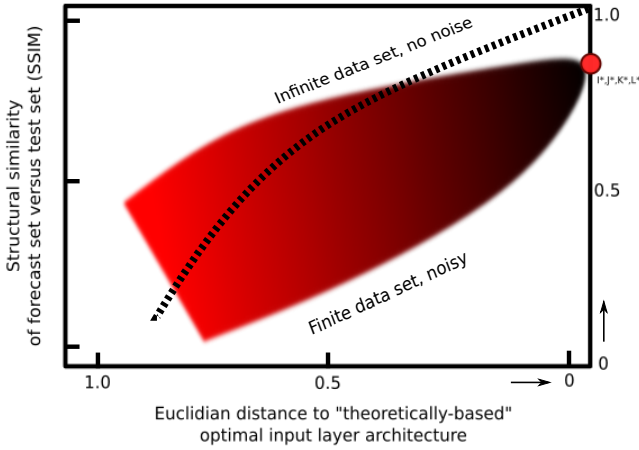

Fig. 3. Our main conjecture. For a infinite noiseless training set, the SSIM approaches SSIM $\rightarrow 1$. For real data sets, there is a dispersion of the SSIM versus some reasonable metric constructed to represent the distance between any architecture (e.g. $d_e \sqrt{(I - I^*)^2 + (J - J^*)^2 + (K - K^*)^2 + (L - L^*)^2}$).

systems (Ordinary Differential Equations - ODEs) to really "complex" systems (Partial Differential Equations - PDEs). This is partially motivated by results in the literature that show that general universalities are present in different levels of simplification of physical models, from the original PDEs to truncated ODE expansions (e.g. Galerkin expansions [141]) to the most extreme simplications such as maps which capture

the essence of the problem. In all cases we take examples with one spatial and one temporal dimension. However we believe that our conjecture will extend to multiple spatial and one temporal dimensional systems.

A. Sunspot data - a physical system example

The first example we take is a physical real data example based on a previous paper of one of us [20], where a neural network using this type of architecture above (Fig. 1) was used to forecast sunspot areas $A(t, \theta)$ in our Sun in both space (θ latitude) and time (Carrington Rotation index).

Several authors [66]–[117] have already attempted to use neural networks to forecast aspects of the sunspot cycle, although none in both space and time, having restricted themselves to using these neural networks to forecast mostly either the sunspot number or the sunspot areas as a function of time. There are a few examples (add references) of actual spatial-temporal forecasts using neural networks (see mine and the one on MDI/HDI data).

We take as a "training set" the data from the year 1874 to approximately 1997 (i.e. the first 1646 Carrington Rotations). We then attempt to reproduce or forecast the sunspot area butterfly diagram from Carrington Rotation 1921 to 2162 (the last one corresponding approximately to the year 2015); that is, we use 1646 time slices (~122.92 years) to reproduce the next 242 time slices (~18.07 years). The training set corresponds to around 12 solar cycles (cycle 11 to 22), while the "forecasting set" (or validation set) equates to around 1.5 cycles (cycle 23 and half of cycle 24). The entire dataset, including the training

and forecasting sets, is a grid $x_j^i = x(i,j)$, with $i = 1888$ and $j = 50$. The training set is a grid $x(1646, 50)$. For this case the optimal values were $I^* = 2$, $J^* = 6$, $K^* = 9$ and $L^* = 70$. The hyperparameters of the neural network were: $N_h = 70$, $\eta = 0.3$, $\alpha = 0.01$, $\rho = 0$, a logarithmic normalization of the inputs scaled with $\alpha_{nor} = 10$ and $\beta_{nor} = 0$, weight initialization with $\alpha_{rng} = 10^{-2}$ and $\beta_{rng} = -0.5$ and $N_{\text{steps}} = 1,000,000$ We used the logistic sigmoid function as the activation on both the hidden and output layers. The results are depicted in Fig. 4. It shows a dispersion as conjectured and a convergence to the highest SSIM value we could obtain for this particular slicing of the training and test sets SSIM = 0.836876152.
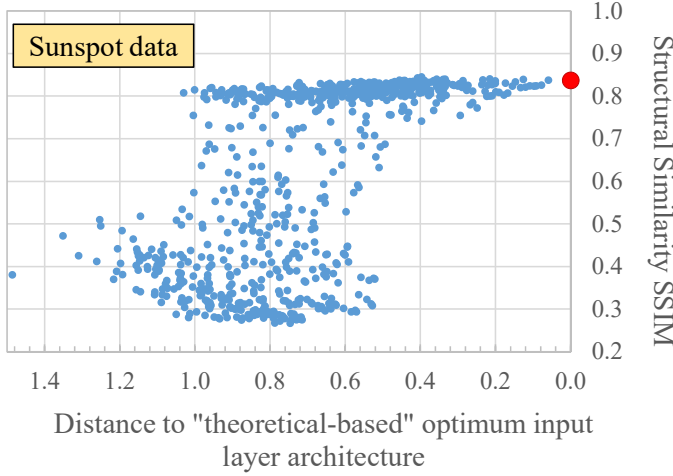

Fig. 4. Monte Carlo simulation of different architectures of the input layer for the neural network forecast for the sunspot data. It shows the structural similarity (SSIM) against how far (in an Euclidean space metric) the particular parameters of a particular run were from the supposely optimal architecture parameters (red dot).

B. Coupled Hénon maps - a discrete-time dynamical system

Motivated by having a real case from a physical system, we then tried to investigate if this same conjecture holds in a very simplifed example of a spatial-temporal model. Coupled maps are widely used as models of spatial-temporal chaos and pattern/structure formation [142]–[144].

Following [16], [17] we then take a lattice of $M = 100$ coupled Hénon maps:

$$u_m^{n+1} = 1 - 1.45 \left[\frac{1}{2} u_m^n + \frac{u_{m-1}^n + u_{m+1}^n}{4} \right]^2 + 0.3 v_m^n, \quad (2)$$

$$v_m^{n+1} = u_m^n.$$

with fixed boundary conditions $u_1^n = u_M^n = \frac{1}{2}$ and $v_1^n = v_M^n = 0$.

We run the simulation for $N = 531$ time steps, and divided the set into $N_{\text{train}} = 500$ time steps for the training set and $N_{\text{test}} = 31$ time steps for the test set. The other parameters of the neural network were: $N_h = 10$, $\eta = 0.1$, $\alpha = 0$, $\rho = 0$, a linear input normalization scaling with $\alpha_{nor} = 2.947992$, $\beta_{nor} = 0.515$, $\alpha_{rng} = 10^{-3}$, $\beta_{rng} = -0.5$ and $N_{\text{steps}} = 1,000,000$. We used the ReLu function as the activation on both the hidden and output layers.

For this case the optimal values were $I^* = 1$, $J^* = 3$, $K^* = 2$ and $L^* = 3$. The results are depicted in Fig. 4. It shows a dispersion as conjectured and a convergence to the highest SSIM value we could obtain for this particular slicing of the training and test sets SSIM = 0.71139101.
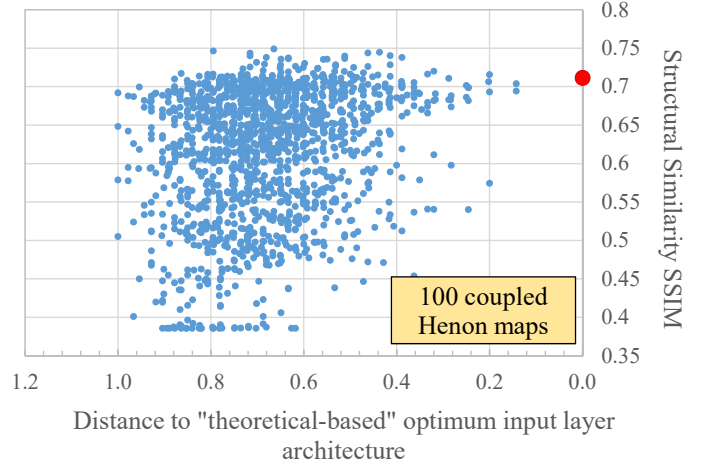

Fig. 5. Monte Carlo simulation of different architectures of the input layer for the neural network forecast for a series of 100 coupled Hénon maps. It shows the structural similarity (SSIM) against how far (in an Euclidean space metric) the particular parameters of a particular run were from the supposely optimal architecture parameters (red dot). The green line (trendline) seems to show that as the parameters of a randomly choosen architecture get close to the supposely optimal architecture ones, the SSIM converges to what seems to be the best possible forecast value given the limited dataset.

Results seems to suggest the same structure as depicted in our conjecture diagram and in the previous results for sunspots.

C. Coupled Ordinary Differential Equations - Lorenz-96 model

For the spatially extended ODEs model we used a well known 40 couple ODE dynamical system proposed by Edward Lorenz in 1996 [145]

$$lCr \frac{d x_j}{d t} = (x_{j+1} - x_{j-2}) x_{j-1} - x_j + F, j = 1, \ldots, N = 40, \quad (3)$$

where $x_{-1} = x_{N-1}$, $x_0 = x_N$ and $x_{N+1} = x_1$. We use the forcing $F = 5$ to get some interesting behaviour in space and time. We used a time step $\Delta t = 0.05$ and we have integrated this equation using Matlab (ADD REFERENCE).

We run the simulation for $N = 531$ time steps, and divided the set into $N_{\text{train}} = 500$ time steps for the training set and $N_{\text{test}} = 31$ time steps for the test set. The other parameters of the neural network were: $N_h = 10$, $\eta = 0.05$, $\alpha = 0.001$, $\rho = 0$, a linear normalization input scaling with $\alpha_{nor} = 10$ and $\beta_{nor} = 0.430$, weight initialization with $\alpha_{rng} = 10^{-3}$ and $\beta_{rng} = -0.5$ and $N_{\text{steps}} = 100,000$. We used the ReLU function as the activation on both the hidden and output layers.

For this case the optimal values obtained before the Monte Carlo simulation were $I^* = 2$, $J^* = 2$, $K^* = 1$ and $L^* = 9$. The results are depicted in Fig. 6. It shows a dispersion as conjectured and a convergence to the highest SSIM value we could obtain for this particular slicing of the training and test sets SSIM = 0.861844038. Results seems to suggest the same
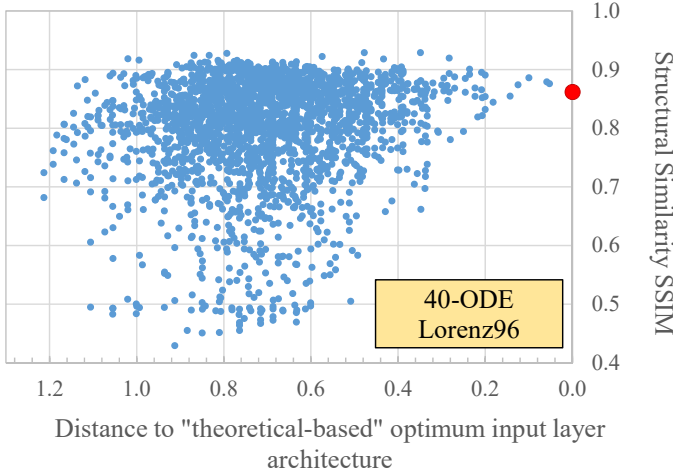
Fig. 6. Monte Carlo simulation of different architectures of the input layer for the neural network forecast for the 40-ODE Lorenz 96 system. It shows the structural similarity (SSIM) against how far (in an Euclidean space metric) the particular parameters of a particular run were from the supposely optimal architecture parameters (red dot). The green line (trendline) seems to show that as the parameters of a randomly choosen architecture get close to the supposely optimal architecture ones, the SSIM converges to what seems to be the best possible forecast value given the limited (and noisy) dataset.
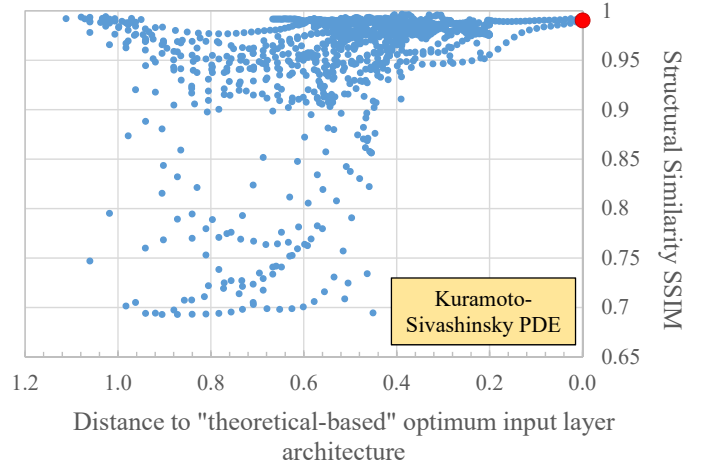

Fig. 7. Monte Carlo simulation of different architectures of the input layer for the neural network forecast for Kuramoto-Sivashinsky with $L = 22$ system. It shows the structural similarity (SSIM) against how far (in an Euclidean space metric) the particular parameters of a particular run were from the supposely optimal architecture parameters (red dot). The green line (trendline) seems to show that as the parameters of a randomly choosen architecture get close to the supposely optimal architecture ones, the SSIM converges to what seems to be the best possible forecast value given the limited (and noisy) dataset.

structure as depicted in our conjecture diagram and in the previous results for sunspots and the coupled Hénon maps.

D. Partial Differential Equations - Kuramoto-Sivashinsky model

Finally we take a full PDE system, the Kuramoto-Sivashinsky model [146], [147], a very well known system capable of spatiatemporal chaos and complex spatial-temporal dynamics. It is a fourth-order nonlinear PDE introduced in the 1970s by Yoshiki Kuramoto and Gregory Sivashinsky to model the diffusive instabilities in a laminar flame front.

The model is described by the following equation:

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^4 u(x,t)}{\partial x^4} + \frac{\partial^2 u(x,t)}{\partial x^2} + u(x,t)\frac{\partial u(x,t)}{\partial x}, \quad (4)$$

where $x \in [-\frac{L}{2}, +\frac{L}{2}]$. We have integrated this equation using Matlab (ADD REFERENCE) using a time step of $\Delta t = 0.5$ and $L = 22$ fourier modes.

We run the simulation for $N = 531$ time steps, and divided the set into $N_{\text{train}} = 500$ time steps for the training set and $N_{\text{test}} = 31$ time steps for the test set. The other parameters of the neural network were: $N_h = 50$, $\eta = 0.1$, $\alpha = 0$, $\rho = 0$, a linear normalization input scaling with $\alpha_{nor} = 5.8472$ and $\beta_{nor} = 0.5$, weight initialization with $\alpha_{rng} = 10^{-3}$ and $\beta_{rng} = -0.5$ and $N_{\text{steps}} = 1,000,000$. We used the ReLU function as the activation on both the hidden and output layers.

The results of the Monte Carlo simulation can be see below in Fig. 7. For this case the optimal values obtained before the Monte Carlo simulation were $I^* = 1$, $J^* = 2$, $K^* = 2$ and $L^* = 39$. It shows a dispersion as conjectured and a convergence to the highest SSIM value we could obtain for this particular slicing of the training and test sets a surprising high value of SSIM = 0.990264382. Results seems to suggest the same

structure as depicted in our conjecture diagram and in the previous results for sunspots, the coupled Hénon maps.

IV. CONCLUSION

We have shown empirical evidence for the existence of an optimal architecture of the input layer of feedforward neural networks used for forecasting spatial-temporal series. We believe that the selection of the features of the input layer can be uniquely determined by the data itself, using two techniques from dynamical systems embedding theory: the mutual information and the false neighbours methods. The former procedure determines the temporal and spatial delays to take when selecting features, while the latter procedure determines the number of data points in space and time to take as inputs. We conjecture that this optimal architecture gives the best forecast, as measured by a standard image similarity index and imply that the shape of the dispersion of points on a Monte Carlo simulation across all possible architectures in the forecast similarity versus distance to optimal architecture was a skewed bell shape with the highest value being the optimal architecture/maximum similarity index.

In order to substantiate our conjecture, we choose four independently related systems, in order of complexity: a set of spatially extended coupled maps; a set of spatially extended ODEs; a one-dimensional spatial PDE and a real spatial-temporal data set from sunspots areas in our Sun. In all four cases, we were able to first use the mutual information and the false neighbours methods to determine the four parameters defining the input layer architecture[1]. After calibration of the hyperparameters we then were able to forecast reasonably the

---

[1]We have four parameters for the input layer architecture in these cases with one temporal and one spatial dimension. For higher dimensional systems, there will be more parameters, the exact number being double the number of dimensions of the system.

test test. We then show that for a Monte Carlo simulation across all possible architectures, the neural network did not, as expected, forecast as well as for the specific set of four parameters defining the input layer architecture. The Monte Carlo simulations show that the shape of the distribution of points in the forecast similarity versus distance to optimal architecture was a skewed bell shape with the highest value being the optimal architecture/maximum similarity index (subject to noise), as conjectured.

Given how important spatial-temporal systems are and how we want to forecast the future as accurately as possible, it is quite important to attempt to reduce the number of hyperparameters in neural network prediction, and try to constraint the architecture from the data properties only. If indeed our conjecture turns out to be true, it would remove the input layer architecture as another free variable in the already complex process of choosing the details of the neural network to use for forecasting.

In this paper we have focused first and foremost in establishing empirical evidence for our conjecture, within a simple framework of feedforward neural networks with one hidden layer for the purpose of prediction in one spatial and one temporal dimensions. Naturally, there are many clear extensions to our research. First to include more complex architectures, with deeper networks with more hidden layers, to possible tackle systems which are hyperchaotic. Second, to attempt to extend the conjecture with empirical evidence in high dimensions, e.g. 3+1 dimensional weather systems. Third, to extend the conjecture to more complex and fashionable neural networks models, such as recurrent neural networks, particularly echo state networks and long short-term memory networks. Fourth and last but not least, to prove the conjecture (HOW? Markov Chains?).

REFERENCES

[1] R. L. Devaney, *An Introduction to Chaotic Dynamical Systems, 2nd Edition*. CRC Press, 2003. [Online]. Available: https://www.amazon.com/Introduction-Chaotic-Dynamical-Systems-2nd/dp/0813340853?SubscriptionId=0JYN1NVW651KCA56C102&tag=techkie-20&linkCode=xm2&camp=2025&creative=165953&creativeASIN=0813340853

[2] P. Manneville, *Instabilities, Chaos and Turbulence: AN Introduction to Nonlinear Dynamics and Complex Systems*. World Scientific Press, Oct. 2004.

[3] A. Wolf, J. B. Swift, H. L. Swinney, and J. A. Vastano, "Determining Lyapunov exponents from a time series," *Physica D Nonlinear Phenomena*, vol. 16, pp. 285–317, Jul. 1985.

[4] H. Kantz, "A robust method to estimate the maximal Lyapunov exponent of a time series," *Physics Letters A*, vol. 185, pp. 77–87, Jan. 1994.

[5] I. I. Gikhman and A. V. Skorokhod, *Introduction to the Theory of Random Processes (Dover Books on Mathematics)*. Dover Publications, 1996. [Online]. Available: https://www.amazon.com/Introduction-Theory-Random-Processes-Mathematics/dp/0486693872?SubscriptionId=0JYN1NVW651KCA56C102&tag=techkie-20&linkCode=xm2&camp=2025&creative=165953&creativeASIN=0486693872

[6] J. Makhoul, "Linear prediction: A tutorial review," *Proceedings of the IEEE*, vol. 63, no. 4, pp. 561–580, April 1975.

[7] M. J. D. Powell, "Algorithms for approximation," J. C. Mason and M. G. Cox, Eds. New York, NY, USA: Clarendon Press, 1987, ch. Radial Basis Functions for Multivariable Interpolation: A Review, pp. 143–167. [Online]. Available: http://dl.acm.org/citation.cfm?id=48424.48433

[8] D. S. Broomhead and D. Lowe, "Multivariable functional interpolation and adaptive networks," *Complex Systems*, vol. 2, 1988.

[9] J. D. Farmer and J. J. Sidorowich, "Predicting chaotic time series," *Phys. Rev. Lett.*, vol. 59, pp. 845–848, Aug 1987. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.59.845

[10] G. Box, G. M. Jenkins, and G. Reinsel, *Time Series Analysis: Forecasting & Control (3rd Edition)*. Prentice Hall, 1994. [Online]. Available: https://www.amazon.com/Time-Analysis-Forecasting-Control-3rd/dp/0130607746?SubscriptionId=0JYN1NVW651KCA56C102&tag=techkie-20&linkCode=xm2&camp=2025&creative=165953&creativeASIN=0130607746

[11] L. Rabiner and B. Juang, "An introduction to hidden markov models," *IEEE ASSP Magazine*, vol. 3, no. 1, pp. 4–16, Jan 1986.

[12] M. Lǎngkvist, L. Karlsson, and A. Loutfi, "A review of unsupervised feature learning and deep learning for time-series modeling," *Pattern Recognition Letters*, vol. 42, pp. 11 – 24, 2014. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0167865514000221

[13] D. M. Rubin, "Use of forecasting signatures to help distinguish periodicity, randomness, and chaos in ripples and other spatial patterns," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 2, no. 4, pp. 525–535, 1992. [Online]. Available: https://doi.org/10.1063/1.165894

[14] U. Parlitz and G. Mayer-Kress, "Predicting low-dimensional spatiotemporal dynamics using discrete wavelet transforms," *Phys. Rev. E*, vol. 51, pp. R2709–R2711, Apr 1995. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevE.51.R2709

[15] C. López, A. Álvarez, and E. Hernández-García, "Forecasting confined spatiotemporal chaos with genetic algorithms," *Phys. Rev. Lett.*, vol. 85, pp. 2300–2303, Sep 2000. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.85.2300

[16] U. Parlitz and C. Merkwirth, "Nonlinear prediction of spatio-temporal time series," in *ESANN*, 2000.

[17] U. Parlitz and C. Merkwirth, "Prediction of Spatiotemporal Time Series Based on Reconstructed Local States," *Physical Review Letters*, vol. 84, pp. 1890–1893, Feb. 2000.

[18] Y. Xia, H. Leung, and H. Chan, "A prediction fusion method for reconstructing spatial temporal dynamics using support vector machines," *IEEE Trans. on Circuits and Systems*, vol. 53-II, pp. 62–66, 2006.

[19] E. Covas, "Spatial-temporal forecasting the sunspot diagram," *A&A*, vol. 605, p. A44, Sep. 2017.

[20] E. Covas, N. Peixinho, and J. Fernandes, "Neural network forecast of the sunspot diagram," *ArXiv e-prints*, Jan. 2018.

[21] P. L. McDermott and C. K. Wikle, "An Ensemble Quadratic Echo State Network for Nonlinear Spatio-Temporal Forecasting," *ArXiv e-prints*, Aug. 2017.

[22] ——, "Bayesian Recurrent Neural Network Models for Forecasting and Quantifying Uncertainty in Spatial-Temporal Data," *ArXiv e-prints*, Nov. 2017.

[23] M. Raissi, P. Perdikaris, and G. E. Karniadakis, "Physics Informed Deep Learning (Part II): Data-driven Discovery of Nonlinear Partial Differential Equations," *ArXiv e-prints*, Nov. 2017.

[24] ——, "Physics Informed Deep Learning (Part I): Data-driven Solutions of Nonlinear Partial Differential Equations," *ArXiv e-prints*, Nov. 2017.

[25] Z. Long, Y. Lu, X. Ma, and B. Dong, "PDE-Net: Learning PDEs from Data," *ArXiv e-prints*, Oct. 2017.

[26] M. Raissi and G. E. Karniadakis, "Hidden physics models: Machine learning of nonlinear partial differential equations," *Journal of Computational Physics*, vol. 357, pp. 125–141, Mar. 2018.

[27] M. Raissi, "Deep Hidden Physics Models: Deep Learning of Nonlinear Partial Differential Equations," *ArXiv e-prints*, Jan. 2018.

[28] P. Lynch, *The Emergence of Numerical Weather Prediction: Richardson's Dream*. Cambridge University Press, 2006. [Online]. Available: https://www.amazon.com/Emergence-Numerical-Weather-Prediction-Richardsons/dp/0521857295?SubscriptionId=0JYN1NVW651KCA56C102&tag=techkie-20&linkCode=xm2&camp=2025&creative=165953&creativeASIN=0521857295

[29] E. N. Parker, *Cosmical Magnetic Fields: Their Origin and their Activity (The International Series of Monographs on Physics)*. Oxford University Press, 1979.

[30] E. Sabine, "On Periodical Laws Discoverable in the Mean Effects of the Larger Magnetic Disturbances," *Philosophical Transactions of the Royal Society of London Series I*, vol. 141, pp. 123–139, 1851.

[31] ——, "On Periodical Laws Discoverable in the Mean Effects of the Larger Magnetic Disturbances. No. II," *Philosophical Transactions of the Royal Society of London Series I*, vol. 142, pp. 103–124, 1852.

[32] S. T. Suess, "The solar wind during the Maunder minimum," *Planet. Space Sci.*, vol. 27, pp. 1001–1013, Jul. 1979.

[33] J. A. Eddy, "The Maunder Minimum - A reappraisal," *Sol. Phys.*, vol. 89, pp. 195–207, Nov. 1983.

[34] E. N. Parker, "The passage of energetic charged particles through interplanetary space," *Planet. Space Sci.*, vol. 13, pp. 9–49, Jan. 1965.

[35] D. C. Wilkinson, M. A. Shea, and D. F. Smart, "A Case History of Solar and Galactic Space Weather Effects on the Geosynchronous Communications Satellite TDRS-1," *Advances in Space Research*, vol. 26, pp. 27–30, 2000.

[36] E. S. Babayev, "Some results of investigations on the space weather influence on functioning of several engineering-technical and communication systems and human health," *Astronomical and Astrophysical Transactions*, vol. 22, pp. 861–867, Jun. 2003.

[37] K. Schatten, "Fair space weather for solar cycle 24," *Geophys. Res. Lett.*, vol. 32, p. L21106, Nov. 2005.

[38] J. G. Kappenman, "An overview of the impulsive geomagnetic field disturbances and power grid impacts associated with the violent Sun-Earth connection events of 29-31 October 2003 and a comparative evaluation with other contemporary storms," *Space Weather*, vol. 3, p. S08C01, Aug. 2005.

[39] R. E. Turner, "Space Weather Challenges Intrinsic to the Human Exploration of Space," *Washington DC American Geophysical Union Geophysical Monograph Series*, vol. 165, p. 367, 2006.

[40] D. H. Hathaway and R. M. Wilson, "Geomagnetic activity indicates large amplitude for sunspot cycle 24," *Geophys. Res. Lett.*, vol. 33, p. L18101, Sep. 2006.

[41] G. Cornélissen, R. Tarquini, F. Perfetto, K. Otsuka, M. Gigolashvili, and F. Halberg, "Investigation of Solar about 5-Month Cycle in Human Circulating Melatonin: Signature of Weather in Extraterrestrial Space?" *Sun and Geosphere*, vol. 4, pp. 55–59, Dec. 2009.

[42] H.-S. Choi, J. Lee, K.-S. Cho, Y.-S. Kwak, I.-H. Cho, Y.-D. Park, Y.-H. Kim, D. N. Baker, G. D. Reeves, and D.-K. Lee, "Analysis of GEO spacecraft anomalies: Space weather relationships," *Space Weather*, vol. 9, p. 06001, Jun. 2011.

[43] M. West, D. Seaton, M. Dominique, D. Berghmans, B. Nicula, E. Pylyser, K. Stegen, and J. De Keyser, "Space Weather and Particle Effects on the Orbital Environment of PROBA2," in *EGU General Assembly Conference Abstracts*, ser. EGU General Assembly Conference Abstracts, vol. 15, Apr. 2013, pp. EGU2013–10 865.

[44] C. J. Schrijver, "Socio-Economic Hazards and Impacts of Space Weather: The Important Range Between Mild and Extreme," *Space Weather*, vol. 13, pp. 524–528, Sep. 2015.

[45] P. Grassberger, "Generalizations of the Hausdorff dimension of fractal measures," *Physics Letters A*, vol. 107, pp. 101–105, Jan. 1985.

[46] A. Waibel, T. Hanazawa, G. Hinton, K. Shikano, and K. J. Lang, "Readings in speech recognition," A. Waibel and K.-F. Lee, Eds. San Francisco, CA, USA: Morgan Kaufmann Publishers Inc., 1990, ch. Phoneme Recognition Using Time-delay Neural Networks, pp. 393–404.

[47] K. Luk, J. E. Ball, and A. Sharma, "A study of optimal model lag and spatial inputs to artificial neural network for rainfall forecasting," *Journal of Hydrology*, vol. 227, no. 1, pp. 56–65, 2000.

[48] R. J. Frank, N. Davey, and S. P. Hunt, "Time series prediction and neural networks," *Journal of Intelligent and Robotic Systems*, vol. 31, no. 1, pp. 91–103, 2001.

[49] K. J. Oh and K. jae Kim, "Analyzing stock market tick data using piecewise nonlinear model," *Expert Systems with Applications*, vol. 22, no. 3, pp. 249 – 255, 2002.

[50] Z. Sheng, L. Hong-Xing, G. Dun-Tang, and D. Si-Dan, "Determining the input dimension of a neural network for nonlinear time series prediction," *Chinese Physics*, vol. 12, no. 6, p. 594, 2003.

[51] A. H. Ghaderi, B. Bharani, and H. Jalalkamali, "Embedding dimension as input dimension of artificial neural network: A study on stock prices time series." *International Journal of Modern Physics and Applications*, vol. 1, no. 3, pp. 64–72, 2015.

[52] F. Takens, "Detecting strange attractors in turbulence," *Lecture Notes in Mathematics, Berlin Springer Verlag*, vol. 898, p. 366, 1981.

[53] H. Whitney, "Differentiable manifolds," *Ann. Math. (2)*, vol. 37, no. 3, pp. 645–680, July 1936, mR:1503303. Zbl:0015.32001. JFM:62.1454.01.

[54] R. Mañé, "On the dimension of the compact invariant sets of certain non-linear maps," *Lecture Notes in Mathematics, Berlin Springer Verlag*, vol. 898, p. 230, 1981.

[55] T. Sauer, J. A. Yorke, and M. Casdagli, "Embedology," *Journal of Statistical Physics*, vol. 65, pp. 579–616, Nov. 1991.

[56] A. M. Fraser and H. L. Swinney, "Independent coordinates for strange attractors from mutual information," *Phys. Rev. A*, vol. 33, pp. 1134–1140, Feb. 1986.

[57] H. Abarbanel, *Analysis of Observed Chaotic Data*, ser. Institute for Nonlinear Science. Springer New York, 1997.

[58] H. Kantz and T. Schreiber, *Nonlinear time series analysis*, ser. Cambridge nonlinear science series. Cambridge, New York: Cambridge University Press, 1997, originally published: 1997.

[59] M. B. Kennel, R. Brown, and H. D. I. Abarbanel, "Determining embedding dimension for phase-space reconstruction using a geometrical construction," *Phys. Rev. A*, vol. 45, pp. 3403–3411, Mar. 1992.

[60] J. M. Martinerie, A. M. Albano, A. I. Mees, and P. E. Rapp, "Mutual information, strange attractors, and the optimal estimation of dimension," *Phys. Rev. A*, vol. 45, pp. 7058–7064, May 1992.

[61] H. D. I. Abarbanel, R. Brown, J. J. Sidorowich, and L. S. Tsimring, "The analysis of observed chaotic data in physical systems," *Reviews of Modern Physics*, vol. 65, pp. 1331–1392, Oct. 1993.

[62] H. D. I. Abarbanel and J. P. Gollub, "Analysis of Observed Chaotic Data," *Physics Today*, vol. 49, p. 86, Nov. 1996.

[63] W. S. McCulloch and W. Pitts, "A logical calculus of the ideas immanent in nervous activity," *The bulletin of mathematical biophysics*, vol. 5, no. 4, pp. 115–133, Dec 1943. [Online]. Available: https://doi.org/10.1007/BF02478259

[64] A. Ghaderi, B. M. Sanandaji, and F. Ghaderi, "Deep forecast: Deep learning-based spatio-temporal forecasting," in *The 34th International Conference on Machine Learning (ICML), Time series Workshop*, 2017.

[65] J. Cao, D. J. Farnham, and U. Lall, "Spatial-temporal wind field prediction by Artificial Neural Networks," *ArXiv e-prints*, Dec. 2017.

[66] H. C. Koons and D. J. Gorney, "A sunspot maximum prediction using a neural network," *EOS Transactions*, vol. 71, p. 677, May 1990.

[67] A. S. Weigend, "Connectionist Architectures for Time Series Prediction of Dynamical Systems," Ph.D. dissertation, STANFORD UNIVERSITY., 1991.

[68] A. S. Weigend, B. A. Huberman, and D. E. Rumelhart, "Predicting Sunspots and Exchange Rates with Connectionist Networks," in *Nonlinear modeling and forecasting*, M. Casdagli and S. Eubank, Eds. Addison-Wesley, 1992, pp. 395–432.

[69] K. MacPherson, "Neural network computation techniques applied to solar activity prediction," *Advances in Space Research*, vol. 13, pp. 447–450, Sep. 1993.

[70] K. P. Macpherson, A. J. Conway, and J. C. Brown, "Prediction of solar and geomagnetic activity data using neural networks," *J. Geophys. Res.*, vol. 100, pp. 21 735–21 744, Nov. 1995.

[71] A. Conway, "Echoed time series predictions, neural networks and genetic algorithms," *Vistas in Astronomy*, vol. 38, pp. 351–356, 1994.

[72] H. D. Navone and H. A. Ceccatto, "Forecasting chaos from small data sets: a comparison of different nonlinear algorithms," *Journal of Physics A: Mathematical and General*, vol. 28, no. 12, p. 3381, 1995.

[73] R. A. Calvo, H. A. Ceccato, and R. D. Piacentini, "Neural network prediction of solar activity," *ApJ*, vol. 444, pp. 916–921, May 1995.

[74] T. Koskela, M. Lehtokangas, J. Saarinen, and K. Kaski, "Time series prediction with multilayer perceptron, fir and elman neural networks," in *In Proceedings of the World Congress on Neural Networks*. Press, 1996, pp. 491–496.

[75] F. Fessant, C. Pierret, and P. Lantos, "Comparison of Neural Network and McNish and Lincoln Methods for the Prediction of the Smoothed Sunspot Index," *Sol. Phys.*, vol. 168, pp. 423–433, Oct. 1996.

[76] F. Fessant, S. Bengio, and D. Collobert, "On the prediction of solar activity using different neural network models," *Annales Geophysicae*, vol. 14, pp. 20–26, Jan. 1996.

[77] Y. R. Park, T. J. Murray, and C. Chen, "Predicting sunspots using a layered perceptron neural network." *IEEE Transactions on Neural Networks*, vol. 7, pp. 501–505, Mar. 1996.

[78] D. R. Kulkarni, A. S. Pandya, and J. C. Parikh, "Modeling and predicting sunspot activity-state space reconstruction + artificial neural network methods," *Geophys. Res. Lett.*, vol. 25, pp. 457–460, 1998.

[79] A. J. Conway, K. P. Macpherson, G. Blacklaw, and J. C. Brown, "A neural network prediction of solar cycle 23," *J. Geophys. Res.*, vol. 103, pp. 29 733–29 742, Dec. 1998.

[80] A. J. Conway, "Time series, neural networks and the future of the Sun," *New A Rev.*, vol. 42, pp. 343–394, Oct. 1998.

[81] P. Verdes, M. Parodi, P. Granitto, H. Navone, R. Piacentini, and H. Ceccatto, "Predictions of the maximum amplitude for solar cycle 23 and its subsequent behavior using nonlinear methods," *Solar Physics*, vol. 191, no. 2, pp. 419–425, 2000.

[82] P. F. Verdes, P. M. Granitto, and H. A. Ceccatto, "Secular Behavior of Solar Magnetic Activity: Nonstationary Time-Series Analysis of the Sunspot Record," *Sol. Phys.*, vol. 221, pp. 167–177, May 2004.

[83] H. Lundstedt, "Solar activity predicted with artificial intelligence," *Washington DC American Geophysical Union Geophysical Monograph Series*, vol. 125, pp. 201–204, 2001.

[84] M. Small and C. K. Tse, "Minimum description length neural networks for time series prediction," *Phys. Rev. E*, vol. 66, no. 6, p. 066701, Dec. 2002.

[85] A. V. Mordvinov, N. G. Makarenko, M. G. Ogurtsov, and H. Jungner, "Reconstruction of Magnetic Activity of the Sun and Changes in Its Irradiance on a Millennium Timescale Using Neurocomputing," *Sol. Phys.*, vol. 224, pp. 247–253, Oct. 2004.

[86] A. Gholipour, C. Lucas, B. N. Araabi, and M. Shafiee, "Solar activity forecast: Spectral analysis and neurofuzzy prediction," *Journal of Atmospheric and Solar-Terrestrial Physics*, vol. 67, pp. 595–603, Apr. 2005.

[87] A.-F. A. Attia, R. H. Abdel-Hamid, and M. Quassim, "A genetic-based neuro-fuzzy approach for prediction of solar activity," in *Modeling and Systems Engineering for Astronomy*, ser. Proc. SPIE, S. C. Craig and M. J. Cullum, Eds., vol. 5497, Sep. 2004, pp. 542–552.

[88] M. S. Quassim and A. F. Attia, "Forecasting the global temperature trend according to the predicted solar activity during the next decades," *Mem. Soc. Astron. Italiana*, vol. 76, p. 1030, 2005.

[89] A.-F. Attia, R. Abdel-Hamid, and M. Quassim, "Prediction of Solar Activity Based on Neuro-Fuzzy Modeling," *Sol. Phys.*, vol. 227, pp. 177–191, Mar. 2005.

[90] M. Mirmomeni, M. Shafiee, C. Lucas, and B. N. Araabi, "Introducing a new learning method for fuzzy descriptor systems with the aid of spectral analysis to forecast solar activity," *Journal of Atmospheric and Solar-Terrestrial Physics*, vol. 68, pp. 2061–2074, Dec. 2006.

[91] M. S. Quassim, A.-F. Attia, and H. K. Elminir, "Forecasting the Peak Amplitude of the 24th and 25th Sunspot Cycles and Accompanying Geomagnetic Activity," *Sol. Phys.*, vol. 243, pp. 253–258, Jul. 2007.

[92] A.-F. Attia, H. A. Ismail, and H. M. Basurah, "A Neuro-Fuzzy modeling for prediction of solar cycles 24 and 25," *Ap&SS*, vol. 344, pp. 5–11, Mar. 2013.

[93] J.-X. Xie, C.-T. Cheng, K.-W. Chau, and Y.-Z. Pei, "A hybrid adaptive time-delay neural network model for multi-step-ahead prediction of sunspot activity," *International Journal of Environment and Pollution*, vol. 28, no. 3-4, pp. 364–381, 2006.

[94] G. Maris and A. Oncica, "Solar Cycle 24 Forecasts," *Sun and Geosphere*, vol. 1, no. 1, pp. 8–11, Mar. 2006.

[95] F. Emmert-Streib and M. Dehmer, "Nonlinear Time Series Prediction Based on a Power-Law Noise Model," *International Journal of Modern Physics C*, vol. 18, pp. 1839–1852, 2007.

[96] A. S. Pandya, D. R. Kulkarni, and J. C. Parikh, "Study of time series prediction under noisy environment," in *Applications and Science of Artificial Neural Networks III*, ser. Proc. SPIE, S. K. Rogers, Ed., vol. 3077, Apr. 1997, pp. 116–126.

[97] H. Lundstedt, M. Wik, and P. Wintoft, "Synoptic Solar Magnetic Fields: Explored and Predicted," *AGU Fall Meeting Abstracts*, Dec. 2006.

[98] M. Wik, "Multiresolution Analysis and Prediction of Solar Magnetic Flux," in *37th COSPAR Scientific Assembly*, ser. COSPAR Meeting, vol. 37, 2008, p. 3467.

[99] D. Gang and Z. Shi-Sheng, "Sunspot number prediction based on process neural network with time-varying threshold functions [j]," *Acta Physica Sinica*, vol. 2, p. 099, 2007.

[100] J. Uwamahoro, L.-A. McKinnell, and P. J. Cilliers, "Forecasting solar cycle 24 using neural networks," *Journal of Atmospheric and Solar-Terrestrial Physics*, vol. 71, pp. 569–574, Apr. 2009.

[101] C. Francile and M. L. Luoni, "Hacia la predicción del Número R de Wolf de manchas solares utilizando Redes Neuronales con retardos temporales," *Boletin de la Asociacion Argentina de Astronomia La Plata Argentina*, vol. 53, pp. 241–244, 2010.

[102] M. A. Parodi, H. A. Ceccatto, R. D. Piacentini, and P. J. García, "Actividad solar del ciclo 23. Predicción del máximo y fase decreciente utilizando redes neuronales," *Boletin de la Asociacion Argentina de Astronomia La Plata Argentina*, vol. 43, pp. 23–24, 1999.

[103] A. Ajabshirizadeh, N. Masoumzadeh Jouzdani, and S. Abbassi, "Neural network prediction of solar cycle 24," *Research in Astronomy and Astrophysics*, vol. 11, pp. 491–496, Apr. 2011.

[104] A. Ajabshirizadeh and M. Juzdani Nafiseh, "Sunspot Number and Solar Radio Flux Prediction by Artificial Neural Network Method." in *38th COSPAR Scientific Assembly*, ser. COSPAR Meeting, vol. 38, 2010, p. 2.

[105] S. Chattopadhyay, D. Jhajharia, and G. Chattopadhyay, "Trend estimation and univariate forecast of the sunspot numbers: Development and comparison of ARMA, ARIMA and Autoregressive Neural Network models," *Comptes Rendus Geoscience*, vol. 343, pp. 433–442, Jul. 2011.

[106] C. Jiang and F. Song, "Sunspot forecasting by using chaotic time-series analysis and narx network." *JCP*, vol. 6, no. 7, pp. 1424–1429, 2011.

[107] Y. Maslennikova and V. Bochkarev, "Solar activity prediction using artificial neural network and singular spectrum analysis," in *39th COSPAR Scientific Assembly*, ser. COSPAR Meeting, vol. 39, Jul. 2012, p. 1194.

[108] R. Chandra and M. Zhang, "Cooperative coevolution of elman recurrent neural networks for chaotic time series prediction," *Neurocomput.*, vol. 86, pp. 116–123, Jun. 2012.

[109] D.-C. Park and D.-M. Woo, "Prediction of sunspot series using bilinear recurrent neural network," in *Information Management and Engineering, 2009. ICIME'09. International Conference on*. IEEE, 2009, pp. 94–98.

[110] T.-H. Kim, D.-C. Park, D.-M. Woo, W. Huh, C.-H. Yoon, H.-U. Kim, and Y. Lee, "Sunspot series prediction using a multiscale recurrent neural network," in *Signal Processing and Information Technology (ISSPIT), 2010 IEEE International Symposium on*. IEEE, 2010, pp. 399–403.

[111] J. D. Moghaddam, A. Mosallanezhad, and M. Teshnehlab, "Sunspot prediction by a time delay line recurrent fuzzy neural network using emotional learning," in *Fuzzy Systems (IFSC), 2013 13th Iranian Conference on*. IEEE, 2013, pp. 1–5.

[112] G. Chattopadhyay and S. Chattopadhyay, "Monthly sunspot number time series analysis and its modeling through autoregressive artificial neural network," *European Physical Journal Plus*, vol. 127, p. 43, Apr. 2012.

[113] Z.-G. Liu and J. Du, "Sunspot time sequences prediction based on process neural network and quantum particle swarm," in *Multimedia Information Networking and Security (MINES), 2012 Fourth International Conference on*. IEEE, 2012, pp. 233–236.

[114] A. Gkana and L. Zachilas, "Sunspot numbers: Data analysis, predictions and economic impacts," *Journal of Engineering Science and Technology Review*, vol. 8, no. 1, pp. 79–85, 2015, cited By 1.

[115] M. Parsapoor, U. Bilstrup, and B. Svensson, "Prediction of solar cycle 24," in *2015 International Joint Conference on Neural Networks, IJCNN 2015, Killarney, Ireland, July 12-17, 2015*. IEEE, 2015, pp. 1–8.

[116] M. Parsapoor, J. Brooke, and B. Svensson, "A new computational intelligence model for long-term prediction of solar and geomagnetic activity," in *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence, January 25-30, 2015, Austin, Texas, USA.*, B. Bonet and S. Koenig, Eds. AAAI Press, 2015, pp. 4192–4193.

[117] R. Archana, A. Unnikrishnan, and R. Gopikakumari, "Computation of state space evolution of chaotic systems from time series of output, based on neural network models," *International Journal of Engineering Research and Development*, vol. 2, pp. 49–56, Jul. 2012.

[118] M. Annunziato, S. Pizzuti, and L. S. Tsimring, "Analysis and prediction of spatio-temporal flame dynamics," in *Proceedings of the IEEE Workshop on Nonlinear Dynamics of Electronic Systems: Catania, Italy, 18-20 May 2000*, G. Setti, R. Rovatti, and G. Mazzini, Eds. World Scientific, 2000, pp. 117–121.

[119] L. Zachilas and A. Gkana, "On the verge of a grand solar minimum: A second maunder minimum?" *Solar Physics*, vol. 290, no. 5, pp. 1457–1477, 2015.

[120] Y. Sun, V. Babovic, and E. S. Chan, "Multi-step-ahead model error prediction using time-delay neural networks combined with chaos

theory," *Journal of Hydrology*, vol. 395, no. 1–2, pp. 109 – 116, 2010.

[121] S.-C. Huang, P.-J. Chuang, C.-F. Wu, and H.-J. Lai, "Chaos-based support vector regressions for exchange rate forecasting," *Expert Systems with Applications*, vol. 37, no. 12, pp. 8590 – 8598, 2010.

[122] R. Battiti, "Using mutual information for selecting features in supervised neural net learning," *IEEE Transactions on Neural Networks*, vol. 5, no. 4, pp. 537–550, Jul 1994.

[123] S. BuHamra, N. Smaoui, and M. Gabr, "The box–jenkins analysis and neural networks: prediction and time series modelling," *Applied Mathematical Modelling*, vol. 27, no. 10, pp. 805 – 815, 2003.

[124] R. Chandra and M. Zhang, "Cooperative coevolution of elman recurrent neural networks for chaotic time series prediction," *Neurocomputing*, vol. 86, pp. 116–123, 2012.

[125] Y. S. Maslennikova and V. V. Bochkarev, "Training algorithm for neuro-fuzzy network based on singular spectrum analysis," *CoRR*, vol. abs/1410.1151, 2014.

[126] T. Sauter, B. Weitzenkamp, and C. Schneider, "Spatio-temporal prediction of snow cover in the black forest mountain range using remote sensing and a recurrent neural network," *International Journal of Climatology*, vol. 30, no. 15, pp. 2330–2341, 2010.

[127] D. R. Kulkarni, J. C. Parikh, and A. S. Pandya, "Dynamic Predictions from Time Series Data - An Artificial Neural Network Approach," *International Journal of Modern Physics C*, vol. 8, pp. 1345–1360, 1997.

[128] P. S. Lucio, F. C. Conde, I. F. A. Cavalcanti, A. I. Serrano, A. M. Ramos, and A. O. Cardoso, "Spatiotemporal monthly rainfall reconstruction via artificial neural network - case study: south of Brazil," *Advances in Geosciences*, vol. 10, pp. 67–76, Apr. 2007.

[129] P. Maass, T. Koehler, J. Kalden, R. Costa, U. Parlitz, C. Merkwirth, and J. Wichard, *Mathematical Methods for Forecasting Bank Transaction Data*, ser. DFG-Schwerpunktprogramm 1114, Mathematical methods for time series analysis and digital image processing. Zentrum für Technomathematik, 2003.

[130] G. Simon and M. Verleysen, "High-dimensional delay selection for regression models with mutual information and distance-to-diagonal criteria," *Neurocomput.*, vol. 70, no. 7-9, pp. 1265–1275, Mar. 2007.

[131] M. Ragulskis and K. Lukoseviciute, "Non-uniform attractor embedding for time series forecasting by fuzzy inference systems," vol. 72, pp. 2618–2626, 12 2009.

[132] P. J. Werbos, "Applications of advances in nonlinear sensitivity analysis," in *System Modeling and Optimization*, R. F. Drenick and F. Kozin, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 1982, pp. 762–770.

[133] D. E. Rumelhart, G. E. Hinton, and R. J. Williams, "Learning representations by back-propagating errors," *Nature*, vol. 323, pp. 533–536, Oct. 1986.

[134] P. J. Werbos, "Backpropagation through time: what it does and how to do it," *Proceedings of the IEEE*, vol. 78, no. 10, pp. 1550–1560, Oct 1990.

[135] Z. Wang, A. C. Bovik, H. R. Sheikh, and E. P. Simoncelli, "Image quality assessment: From error visibility to structural similarity," *IEEE TRANSACTIONS ON IMAGE PROCESSING*, vol. 13, no. 4, pp. 600–612, 2004.

[136] C. Dong, C. Change Loy, K. He, and X. Tang, "Image Super-Resolution Using Deep Convolutional Networks," *ArXiv e-prints*, Dec. 2015.

[137] Q. Zhang, Q. Yuan, C. Zeng, X. Li, and Y. Wei, "Missing Data Reconstruction in Remote Sensing image with a Unified Spatial-Temporal-Spectral Deep Convolutional Neural Network," *ArXiv e-prints*, Feb. 2018.

[138] Z. Wang and A. C. Bovik, "Mean squared error: Lot it or leave it? A new look at Signal Fidelity Measures," *IEEE Signal Processing Magazine*, vol. 26, pp. 98–117, Jan. 2009.

[139] D. Brunet, E. R. Vrscay, and Z. Wang, "On the Mathematical Properties of the Structural Similarity Index," *IEEE Transactions on Image Processing*, vol. 21, pp. 1488–1499, Apr. 2012.

[140] F. Liu, C. Quek, and G. S. Ng, "Neural network model for time series prediction by reinforcement learning," in *Proceedings. 2005 IEEE International Joint Conference on Neural Networks, 2005.*, vol. 2, July 2005, pp. 809–814 vol. 2.

[141] E. Covas, R. Tavakol, P. Ashwin, A. Tworkowski, and J. M. Brooke, "In-out intermittency in partial differential equation and ordinary differential equation models," *Chaos*, vol. 11, pp. 404–409, Jun. 2001.

[142] K. Kaneko, "Towards Thermodynamics of Spatiotemporal Chaos," *Progress of Theoretical Physics Supplement*, vol. 99, pp. 263–287, 1989.

[143] G. Mayer-Kress and K. Kaneko, "Spatiotemporal chaos and noise," *Journal of Statistical Physics*, vol. 54, pp. 1489–1508, Mar. 1989.

[144] K. Kaneko, *Theory and Applications of Coupled Map Lattices (Nonlinear Science: Theory and Applications)*. Wiley, 1993. [Online]. Available: https://www.amazon.com/Applications-Coupled-Lattices-Nonlinear-Science/dp/047193741X?SubscriptionId=0JYN1NVW651KCA56C102&tag=techkie-20&linkCode=xm2&camp=2025&creative=165953&creativeASIN=047193741X

[145] L. EN, "Predictability: A problem partly solved," vol. 1, 01 1995.

[146] Y. Kuramoto and T. Tsuzuki, "Persistent Propagation of Concentration Waves in Dissipative Media Far from Thermal Equilibrium," *Progress of Theoretical Physics*, vol. 55, pp. 356–369, Feb. 1976.

[147] G. I. Sivashinsky, "Nonlinear analysis of hydrodynamic instability in laminar flames I. Derivation of basic equations," *Acta Astronautica*, vol. 4, pp. 1177–1206, 1977.

Eurico Covas Biography text here.

John Doe Biography text here.

Jane Doe Biography text here.