Operational Planning
Charging Strategy for Electric Vehicles

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Introduction

In this project we intend to devise a strategy for charging a pool of electrical vehicles in an industrial-like microgrid. To achieve that, we model the problem as a MILP optimization with constraints. The different sections of this report give an overview of the method used to solve the optimization problem.

1.- Efficiency as a function of the charge power

The variation of the state of charge of an EV per unit time as a function of the charge power given in the problem statement is shown in Figure 1.

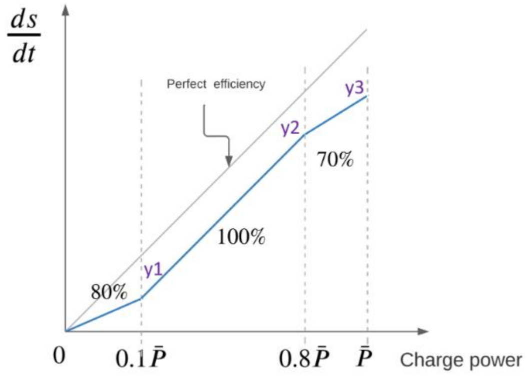

Figure 1. Variation of the state of charge of an EV as a function of the charge power

From Figure 1, the increase of state of charge is a function of efficiency and charge power, then:

$$\frac{ds}{dt} = \eta p$$

So,

$$\eta = \frac{1}{p}\frac{ds}{dt}$$

It's a piecewise function with three parts. The first part is within the range $[(0{,}0)$ to $(0.1 \cdot \bar{p},\ y_1)]$ where $y_1 = 0.08 \cdot \bar{p}$. The second part is between points $[(0.1\bar{p}, 0.08\bar{p})$ to $(0.8 \cdot \bar{p},\ y_2)]$ where $y_2 = 0.78 \cdot \bar{p}$. The last part is from point $[(0.8\bar{p}, 0.78\bar{p})$ to $(\bar{p},\ y_3)]$ where y_3 is $0.92\bar{p}$.

The expression of the piecewise function for these three parts can be written as follows:

$$\begin{cases} \frac{ds}{dt} = 0.8p; & 0 \le p < 0.1\bar{p} \\ \frac{ds}{dt} = p - 0.02\bar{p}\,; & 0.1\bar{p} \le p \le 0.8\bar{p} \\ \frac{ds}{dt} = 0.7p + 0.22\bar{p}\,; & 0.8\bar{p} \le p \le 1 \end{cases}$$

Therefore, the piecewise expression of charge efficiency as a function of the charge power is determined to be:

$$\eta(p) = \frac{1}{p}\frac{ds}{dt}\begin{cases} 0.8\,;\ 0 \le p < 0.1\bar{p} \\ 1 - \frac{0.02\bar{p}}{p}\ ;\ 0.1\bar{p} \le p \le 0.8\bar{p} \\ 0.7 + \frac{0.22\bar{p}}{p}\ ;\ 0.8\bar{p} \le p \le 1 \end{cases}$$

In the Python codes for this project, since we are dealing with a piecewise and non-linear expression of $\eta(p)$, we faced a real challenge to implement an expression that would fit the linear solver named *cbc*. Therefore, we opted for the average value of the efficiency calculated via the following integration:

$$\bar{\eta} = \frac{1}{\bar{p}} \int \frac{1}{p} \frac{ds}{dt}\, dp = \frac{1}{\bar{p}} \left[\int_{0}^{0.1\bar{p}} 0.8\, dp + \int_{0.1\bar{p}}^{0.8\bar{p}} \left(1 - \frac{0.02\bar{p}}{p}\right) dp + \int_{0.8\bar{p}}^{1} \left(0.7 + \frac{0.22\bar{p}}{p}\right) dp \right]$$

$$\boldsymbol{\bar{\eta}} = \frac{1}{\bar{p}} \left(0.08\bar{p} + 0.8\bar{p} - 0.1\bar{p} + 0.7\bar{p} - 0.56\bar{p} - 0.02\bar{p}Ln8 + 0.22\bar{p}Ln\frac{5}{4}\right) = \mathbf{0.9275}$$

Figure 2 shows the efficiency curves for three different values of max charging power [3, 6, 11]. The said values are used in our case study attempt reported a few sections below. The first part of the curves presents a linear variation whereas the second and third sections are clearly nonlinear.

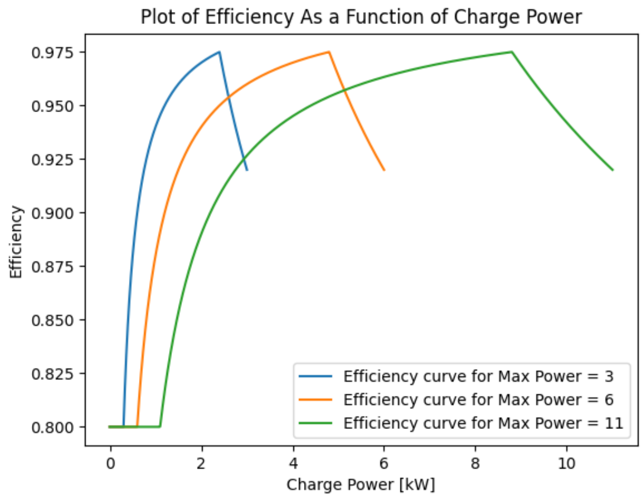

Figure 2. Charge efficiency η(p) as a function of the charge power

2. Model of the optimization problem to be solved at each time step as a MILP

2.1 Defining the Model and its Constraints

With the aim to minimize cost of electricity imports from the grid, with consideration to the penalty owed to EV owners if their desired state of charge is not met and with the willingness to maximize revenue from selling unused PV power to the main grid, we mathematically modelled our objective function as follows:

$$\forall \boldsymbol{t \ in \ control \ horizon}$$

$$\mathbf{min} \ \left[\boldsymbol{Import\ Costs_{from\_grid}} \ - \ \boldsymbol{Export\ Revenue_{to\_grid}} + \boldsymbol{\alpha} \cdot \sum \boldsymbol{Penalty}\right]$$

Where $\quad Import\ Costs_{from\_grid} \ = \ Import_{Qty} \ \cdot Import\ Price$

$$Export\ Revenue_{to\_grid} \ = \ Export_{Qty} \ \cdot Export\ Price$$

α = Trade-off Coefficient stating the system designer’s priority toward minimizing import costs, prioritizing charging the EVs or balancing the two possibilities.

$$Penalty = \left|Se_{t=T_d} - Se^{d}\right| \cdot \frac{\Delta t\left(Te^{d} - Te^{a}\right)}{\overline{C_e}} \cdot {\pi_e}^{cha}$$

And the described objective function is subjected to the following constraints:

1) **Energy Equilibrium Constraint**: Energy imports from the grid and PV generation must equal Energy used to charge the EVs plus Exports to the grid (if any)

$$Grid_{imports} + PV_{used} = Grid_{exports} + sum(ChargePower_{to\ EVs}), \forall \text{ time(t)}$$

2) **PV power constraint**: Balance between Forecast values of PV generation and the actual usage from the PV panels to charge the EVs is taken into account. Any spill overs are exported to the Grid.
$Grid_{exports} = Forecast\ PV_{qty} - Actual\ PV\ used_{qty}$, $\forall$ time(t)

3) **Avoid Energy Import & Export happening at same time**:
$Grid_{imports} <= Grid_{capacity} * (Binary_{value})$, $\forall$ time(t)
$Grid_{exports} <= Grid_{capacity} * (1 - Binary_{value})$, $\forall$ time(t)

$Binary\ Value\ set\ to$: $(0\ or\ 1)$ to constrain Energy import and export from/to grid happening at the same time.

4) **Penalty incurred to EV owners**: This constraint is added to ensure that customer's satisfaction is considered whilst trying to minimize overall cost of imports from grid.

It is mathematically represented as:

$$Penalty = abs(Soc^{desired} - Soc^{departure}) * (Priority\ Index)$$

Where $Soc^{desired}$ is the desired state of charge of the EV, $Soc^{depature}$ is the actual state of charge at departure time.

$$\text{Priority Index} = \frac{\Delta t \cdot (Depature_{time} - Arrival_{time})}{Capacity_{battery}} * (Price\ EV\ owner\ willing\ to\ pay)$$

5) **Maximum Power transfer to EVs**: At every timestep, the limit of power that can be transferred to the EVs is less or equal to its Maximum charging power.

$$Charge\ Power_{Ev} \leq Max\ Charge\ Power_{Ev}\ \forall \text{ time(t)}$$

6) **Capacity of EV constraint**: Amount energy stored in the EV at any time must be less or equal to the EVs' maximum capacity.

$$Energy\ Stored_{EV} \leq Max\ Capacity_{EV}\ \forall \text{ time(t)}$$

2.2 Charging Operation Strategy

The following charging operation strategy allows us to dictate how the state of charge of the EVs is updated with respect to our knowledge of the different possible states ("Connected", "gone" or "not yet arrived") and with respect to each time step of the simulation.

As shown in Figure 3, at the beginning of our simulation we consider three possibilities: whether the EV just arrived (connected) at start time, arrived before start time (therefore automatically connected at start time) or not connected at start time. In the first two cases, charge power is added to the EV such that its new SOC amounts to its arrival SOC plus the added charge. On the other hand, for the third case when the EV is not connected, we set its SOC for that time step to whatever the SOC value it has at its forecasted arrival time. As long as the EV is connected to the charging center and has not reached its maximum SOC, we pledge the provide it with some charge. The charging activity stops when the EV reaches its maximum SOC or at the EV's departure time from the charging center.

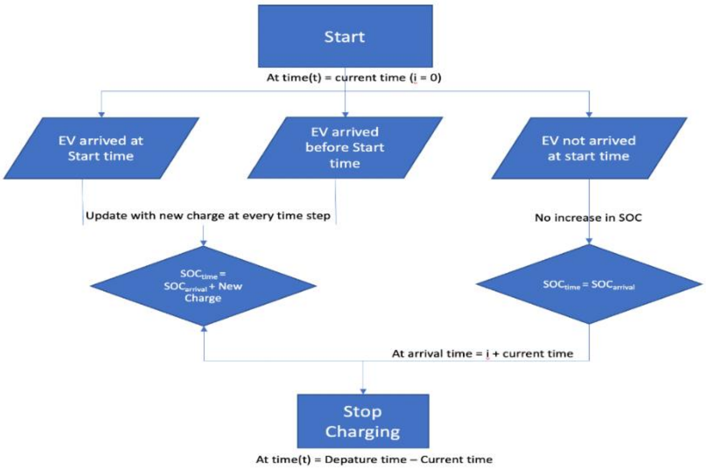

Figure 3. Flowchart of the charging operation strategy

3. Programming implementation of the problem

The Python implementation of this optimization problem with the Pyomo modelling library and cbc as an open-source MILP solver has been successfully tested and uploaded to GradeScope.

4. Case Study

We create a scenario with **3 EVs** and the following input features for each EV (Figure 4).

```
number_of_EVs = 3

SoC_init = {0: 0.0, 1: 0.0, 2: 0.83}

current_time = 7
control_horizon = 17

#control_horizon = 24 - current_time
```

```
ev1 = EV()
ev1.arrival_time = 8
ev1.departure_time = 17
ev1.arrival_soc = 0.6
ev1.departure_soc = 0.9
ev1.state = 'not arrived'
ev1.capacity = 66
ev1.max_charging_power = 6
ev1.price = 462
```

```
ev0 = EV()
ev0.arrival_time = 8
ev0.departure_time = 14
ev0.arrival_soc = 0.6
ev0.departure_soc = 0.9
ev0.state = 'not arrived'
ev0.capacity = 56
ev0.max_charging_power = 6
ev0.price = 448
```

```
ev2 = EV()
ev2.arrival_time = 6
ev2.departure_time = 13
ev2.arrival_soc = 0.6
ev2.departure_soc = 0.9
ev2.state = 'connected'
ev2.capacity = 63
ev2.max_charging_power = 3
ev2.price = 567
```

```
exp_price = [0.04, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04,
             0.04, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04, 0.04]

imp_price = [0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25,
             0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25]

forecast_PV = [0.2985, 0.5502333333333334, 1.0197, 2.0184333333333333, 2.9785, 2.0933333333333333, 3.3649,
               3.9546333333333332, 4.3033, 1.0353, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]

grid_capacity = 10
```

Figure 4. Arbitrary inputs used in the case study under consideration

We simulate with a timestep of **1hour** and observe the following, with respect to changing the Trade-off coefficient ($\boldsymbol{\alpha}$)**;**

- **<u>At Trade-off coefficient ($\boldsymbol{\alpha}$) between (0 and 0.04 = export price)</u>**: No energy is imported from the grid, and the available energy from PV is exported exclusively to the grid; with **no charge** being distributed to the EVs (Figure 6).
  <u>Implication</u>: With the low coefficient of α, there is no penalties for not charging the EVs; therefore, the model prioritizes revenue from exports to the grid in order to minimize the objective function.

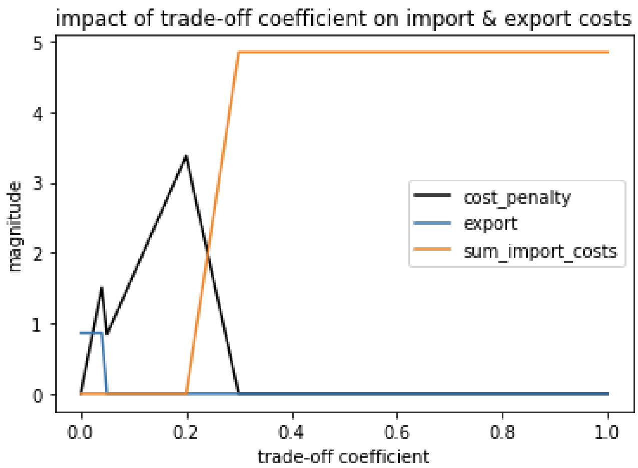

Figure 5. Variation of Import Costs, Export Revenue and Cost due to Penalty w.r.t. $\boldsymbol{\alpha}$

	Hour	Import	Export	PV Used	Forecast PV	Charge Power in Cars
0	7	0.0	0.298500	0.298500	0.298500	0.0
1	8	0.0	0.550233	0.550233	0.550233	0.0
2	9	0.0	1.019700	1.019700	1.019700	0.0
3	10	0.0	2.018433	2.018433	2.018433	0.0
4	11	0.0	2.978500	2.978500	2.978500	0.0
5	12	0.0	2.093333	2.093333	2.093333	0.0
6	13	0.0	3.364900	3.364900	3.364900	0.0
7	14	0.0	3.954633	3.954633	3.954633	0.0
8	15	0.0	4.303300	4.303300	4.303300	0.0
9	16	0.0	1.035300	1.035300	1.035300	0.0
10	17	0.0	0.000000	0.000000	0.000000	0.0
11	18	0.0	0.000000	0.000000	0.000000	0.0
12	19	0.0	0.000000	0.000000	0.000000	0.0
13	20	0.0	0.000000	0.000000	0.000000	0.0
14	21	0.0	0.000000	0.000000	0.000000	0.0
15	22	0.0	0.000000	0.000000	0.000000	0.0
16	23	0.0	0.000000	0.000000	0.000000	0.0

Figure 6. Power Flow Results in the charging center at each time step for $\boldsymbol{\alpha = 0.01}$

- **<u>At Trade-off coefficient ($\boldsymbol{\alpha}$) between (0.05 and 0.3)</u>**: At this point value, we observe the first occurrence of some charge power to the EVs but not to their desired state of charge. Still, there is no energy imports from the grid and majority of the PV energy is exported to the grid.

  <u>Implication</u>: There is no incentive to import power from the grid to satisfy the customer's EVs to their desired state of charge, as the model can still minimize overall cost, just from PV exports to the grid with little penalty costs to be paid.

- **<u>At Trade-off coefficient ($\boldsymbol{\alpha}$)beyond (0.3)</u>:** Here, we observe a spike in the energy imports from the grid, the penalty costs reduce to zero since all EVs receive full charge up to their desired state, and there is no export to the grid because all the PV forecast available has been used to help charge the EVs (Figure 7).

```
My penalty for each car is:  [0.0, 0.0, 0.0]
the sum of the penalties is: 0.0
My cost penalty: 0.0
```

Implication: At this value of trade-coefficient, the model prioritizes reducing the penalty costs because satisfying the customers by matching their desired SOC with the Departure SOC will lead to overall cost reduction, at least when compared to exports at that value of α.

	Hour	Import	Export	PV Used	Forecast PV	EV_0 Pow	EV_1 Pow	EV_2 Pow
0	7	0.000000	0.0	0.298500	0.298500	0.0000	0.000000	0.298500
1	8	10.000000	0.0	0.550233	0.550233	6.0000	4.506767	0.043467
2	9	0.000000	0.0	1.019700	1.019700	0.0000	0.000000	1.019700
3	10	0.000000	0.0	2.018433	2.018433	1.4351	0.000000	0.583333
4	11	3.393167	0.0	2.978500	2.978500	6.0000	0.000000	0.371667
5	12	0.000000	0.0	2.093333	2.093333	0.0000	0.000000	2.093333
6	13	0.000000	0.0	3.364900	3.364900	3.3649	0.000000	0.000000
7	14	0.000000	0.0	3.954633	3.954633	0.0000	3.954633	0.000000
8	15	0.000000	0.0	4.303300	4.303300	0.0000	4.303300	0.000000
9	16	0.000000	0.0	1.035300	1.035300	0.0000	1.035300	0.000000
10	17	6.000000	0.0	0.000000	0.000000	0.0000	6.000000	0.000000
11	18	0.000000	0.0	0.000000	0.000000	0.0000	0.000000	0.000000
12	19	0.000000	0.0	0.000000	0.000000	0.0000	0.000000	0.000000
13	20	0.000000	0.0	0.000000	0.000000	0.0000	0.000000	0.000000
14	21	0.000000	0.0	0.000000	0.000000	0.0000	0.000000	0.000000
15	22	0.000000	0.0	0.000000	0.000000	0.0000	0.000000	0.000000
16	23	0.000000	0.0	0.000000	0.000000	0.0000	0.000000	0.000000

Figure 7. Power Flow Results in the charging center at each time step for $\alpha = 0.7$

5. Simulation on GradeScope

By implementing the optimization problem as detailed throughout this report, we have continuously adjusted the value of the trade-off coefficient α from 0 up to $\alpha = 500$, value at which we have obtained our best accuracy of 98.48% when called upon by GradeScope's Simulator. Figure 8 displays the performance of our controller in terms of allocating values of EV SOC close enough to the desired SOC. One can also see a comparison between our realized charge power distribution and the EV real charge power. Additionally, we notice a trend between import and export energy consisting in having large imports of energy (since $\alpha = 500$, the priority is set to fully charge the EVs and reduce the penalty incurred) following a period of exportation of energy. We also observe a stepwise increase in the cumulative total cost which matches the results of our case study (see Figure 5) where the total import costs from the grid increased substantially at large values of α (meaning large quantity of power imported as well). It can also be readily noticed that the total cost spikes at exactly the time step at which the largest amount of power has been imported (Figure 8).

Figure 8. Output curves from GradeScope assessing the performance of our controller