 Name (Last, First): _____

I will not violate the University of Toledo Code of Ethics or assist others in doing so, especially by presenting others' work as mine, or allow them to present my work as theirs. **I am better than that** and I take pride in, and responsibility for, my work. I understand that violations of the Code may result in loss of credit for the exam, the course, or even jeopardize my academic standing.

Signed:

Problem	Max	Scored
Exam grade		

- Start the exam only at the proctor's signal.
- Closed books and notes, no brought-in summary sheets, formula sheets, or any such accessories.
- No external paper allowed; if you need extra paper, please ask the proctor for it.
- Only basic sci. calculators allowed; no graphing, matrix, or CAS calculators.
- If needed, use both sides of each sheet for your answers. Clearly indicate where the answer is written, if it is not in the space provided for it.

Question 1

Given the ODE

$$2\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 8y(x) = 0$$

In all cases, use **real basis** (sines, cosines, etc.) where appropriate.

(a) Calculate the general solution.

(b) Calculate the particular solution that satisfies $y(0) = 4,\ y'(0) = 0$.

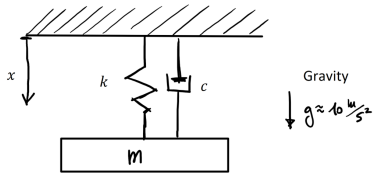
Question 2

Find the general solution of the ODE:

$$\frac{d^2x}{dt^2} + 4x(t) = -16\,t + 4 + 3\sin(4t)$$

In all cases, use **real basis** (sines, cosines, etc.) where appropriate.

Question 3



A loading platform of mass $m = 10\text{kg}$ that can move only along a vertical axis is supported by:

- a spring of undetermined spring constant k, and
- a damper whose force is proportional to the speed at which it is being stretched or compressed with constant of proportionality $c = 20\text{Ns/m}$.

Take gravitational acceleration to be $g \approx 10\text{ms}^{-2}$.

(a) Determine the ODE for this mechanical system with undetermined k. As needed, use free-body diagrams, force balance, etc.

(b) Calculate the value of k if you know that the platform sits at $x_{EQ} = 1\text{m}$ in the equilibrium (static) state.

(c) Calculate the general solution for the motion of the platform.

(d) Sketch the solution $x(t)$ that you calculated under (c) for an arbitrary non-equilibrium initial condition. Make sure your sketch demonstrates at least (i) presence/absence of oscillations, (ii) growth/decay of the amplitude, and (iii) the limit as $t \to \infty$.

In all cases, use **real basis** (sines, cosines, etc.) where appropriate.

Question 4

Explain: **What is resonance**, from physical and mathematical standpoints.

What are the possible values of k in the equation below that result in resonance (**show your work**):

$$2\frac{d^2x}{dt^2} + 4kx(t) = \sin(2t) + e^{3t}$$

Form the **candidate for the forced solution** for the resonant case (that is, when k is set to the value in which resonance occurs). Show your work. (No need to find values of the coefficients, leave them as $A, B, C, \ldots$, just forming the candidate is enough.)