 Name (Last, First): ____________________

I will not violate the University of Toledo Code of Ethics or assist others in doing so, especially by presenting others' work as mine, or allow them to present my work as theirs. **I am better than that** and I take pride in, and responsibility for, my work. I understand that violations of the Code may result in loss of credit for the exam, the course, or even jeopardize my academic standing.

Signed:

Problem	Max	Scored
Exam grade		

- Start the exam only at the proctor's signal.
- Closed books and notes, no brought-in summary sheets, formula sheets, or any such accessories.
- No external paper allowed; if you need extra paper, please ask the proctor for it.
- Only basic sci. calculators allowed; no graphing, matrix, or CAS calculators.
- If needed, use both sides of each sheet for your answers. Clearly indicate where the answer is written, if it is not in the space provided for it.

Question 1

Compute the solution of the ODE using the Laplace transform:

$$3\frac{d^2x(t)}{dt^2} + 12\frac{dx(t)}{dt} + 15x(t) = e^{-t}, \quad x(0) = 0, \quad \frac{dx(0)}{dt} = 1.$$

Laplace integral:
$\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$
Properties, given that
$\mathscr{L}\{f(t)\} = F(s), \mathscr{L}\{g(t)\} = G(s)$:

- Linearity:
  $\mathscr{L}\{c_1 f(t) + c_2 g(t)\}$
  $= c_1 F(s) + c_2 G(s)$
- s-shift:
  $\mathscr{L}\{e^{at} f(t)\} = F(s-a)$
- t-shift:
  $\mathscr{L}\{\mathscr{U}(t-a) f(t-a)\} = e^{-sa} F(s)$
- Derivative in t: If $\mathscr{L}\{y(t)\} = Y(s)$, then

$$\mathscr{L}\{y'(t)\} = sY(s) - y(0),$$
$$\mathscr{L}\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$$

$f(t)$	$F(s) = \mathscr{L}\{f(t)\}$
$\delta(t)$	1
1	$\frac{1}{s}$
$\mathscr{U}(t)$	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{at}\sin(\omega t)$	$\frac{\omega}{(s-a)^2+\omega^2}$
$e^{at}\cos(\omega t)$	$\frac{s-a}{(s-a)^2+\omega^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$

Question 2

(a) Compute: $F(s) = \mathscr{L}\left\{3t^2\, \mathscr{U}(t-2)\right\}$

(b) Compute and **sketch**: $g(t) = \mathscr{L}^{-1}\left\{-\frac{1}{s}e^{-s} - \frac{2}{s^2}e^{-3s}\right\}$

Laplace integral:
$\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$
Properties, given that
$\mathscr{L}\{f(t)\} = F(s), \mathscr{L}\{g(t)\} = G(s)$:

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  $\mathscr{L}\{c_1 f(t) + c_2 g(t)\}$
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$f(t)$	$F(s) = \mathscr{L}\{f(t)\}$
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Question 3

(a) Determine the matrix $\boldsymbol{M}$ and vector function $\underline{f}(t)$ so that the following ODE system is equivalent to $\dot{\underline{v}}(t) = \boldsymbol{M}\underline{v}(t) + \underline{f}(t)$ with $\underline{v}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$.

$\dot{x} = 6x + y + 6t$

$\dot{y} = 4x + 3y - 10t + 4$

(b) Determine what value **R** makes the following **candidate** a solution of the matrix ODEs from (a) by plugging the candidate into the matrix ODE. If no values **R** do the trick, state so. Support your answer by showing work.

$$\underline{v}(t) = \begin{bmatrix} 1 \\ -4 \end{bmatrix} e^{2t} + \begin{bmatrix} \mathbf{R} \\ 6 \end{bmatrix} t + \begin{bmatrix} -4/7 \\ 10/7 \end{bmatrix}$$

Question 4

(a) Use row reduction (Gauss elimination) to solve the following system of linear equations. If needed, use $C_1, C_2, \ldots$ as free parameters in the solution. Label row operations in your solution procedure.

$$\begin{aligned} 4X - 4Y - 4Z &= 0 \\ 4X - 3Y - 2Z &= 3 \\ -2X + Y &= -3 \end{aligned}$$

(b) Compute eigenvalues of $\begin{bmatrix} -3 & 1 \\ -4 & -3 \end{bmatrix}$