

Nomenclature

State variables:

V_T	-Total velocity, m/s
α	-angle of attack, rad
β	-angle of sideslip, rad
ϕ	-roll angle, rad
θ	-pitch angle, rad
ψ	-yaw angle, rad
p	-body-axis roll rate, rad/s
q	-body-axis pitch rate, rad/s
r	-body-axis yaw rate, rad/s
x_E	-x-position w.r.t. earth, m
y_E	-y-position w.r.t. earth, m
z_E	-z-position w.r.t. earth, m
h	-altitude, m

Control variables:

δ_{th}	-throttle setting
δ_e	-elevator deflection, rad
δ_a	-aileron deflection, rad
δ_r	-rudder deflection, rad

Parameters:

δ_f	-leading edge flap deflection, rad
ρ	-air density, kg/m ³
b	-reference wing span, m
\bar{c}	-mean aerodynamic chord, m
C_{lT}	-total rolling moment coefficient
C_{mT}	-total pitching moment coefficient
C_{nT}	-total yawing moment coefficient
C_{XT}	-total axial force coefficient
C_{YT}	-total lateral force coefficient
C_{ZT}	-total normal force coefficient
F_T	-total force on aircraft, N
M_T	-total moment on aircraft, N.m

F_E	-total engine thrust, N
M_E	-total engine moment, N.m
F_w	-force generated by wind, N
F_{wxb}	-force generated by wind in x_b -axis direction, N
F_{wyb}	-force generated by wind in y_b -axis direction, N
F_{wzb}	-force generated by wind in z_b -axis direction, N
F_{pxb}	-propeller effects on force in x_b -axis direction, N
F_{pyb}	-propeller effects on force in y_b -axis direction, N
F_{pzb}	-propeller effects on force in z_b -axis direction, N
M_{pxb}	-propeller effects on moment around x_b -axis, N
M_{pyb}	-propeller effects on moment around y_b -axis, N
M_{pzb}	-propeller effects on moment around z_b -axis, N
g	-gravitational constant, m/s ²
g_1	-gravity component in x_w -axis direction, m/s
g_2	-gravity component in y_w -axis direction, m/s
g_3	-gravity component in z_w -axis direction, m/s
g_{1b}	-gravity component in x_b -axis direction, m/s
g_{2b}	-gravity component in y_b -axis direction, m/s
g_{3b}	-gravity component in z_b -axis direction, m/s
h_E	-engine angular momentum. kg.m ² /s
I_x	-roll moment of inertia, kg.m ²
I_y	-pitch moment of inertia, kg.m ²
I_z	-yaw moment of inertia, kg.m ²
I_{xz}	-product moment of inertia, kg.m ²
I_{xy}	-product moment of inertia, kg.m ²
I_{yz}	-product moment of inertia, kg.m ²
\bar{L}	-rolling moment, N.m
\bar{M}	-pitching moment, N.m
\bar{N}	-yawing moment, N.m
m	-total aircraft mass, kg
M	-Mach number
p_s	-static pressure, Pa
\bar{q}	-dynamic pressure. Pa
S	-reference wing area, m ²
u	-velocity in x_b -axis direction, m/s
v	-velocity in y_b -axis direction, m/s
w	-velocity in z_b -axis direction, m/s
u_w	-velocity of wind in x_b -axis direction, m/s
v_w	-velocity of wind in y_b -axis direction, m/s
w_w	-velocity of wind in z_b -axis direction, m/s
\dot{u}_w	-acceleration of wind in x_b -axis direction, m/s
\dot{v}_w	-acceleration of wind in y_b -axis direction, m/s
\dot{w}_w	-acceleration of wind in z_b -axis direction, m/s
γ	-flight path angle, rad

T	-temperature, K
a	-sound speed, m/s
μ	-air viscosity, Pa.s
x_{cg}	-center of gravity location, m
x_{cgr}	-reference c.g. location, m
\bar{X}	-axial force component, N
\bar{Y}	-lateral force component, N
\bar{Z}	-normal force component, N
D	-drag force, N
Y	-side force, N
L	-lift force, N

1 Aircraft Dynamics

In this section the nonlinear dynamical model of the ATR aircraft is derived. Mass and geometric data are given in Table 2.

1.1 Reference Frames

Before describing the equations of motion of an aircraft, some frame of reference is needed to describe the motion in. The most commonly used reference frames are the earth-fixed reference frame F_E and the body-fixed reference frame F_B . Both reference frames are right-handed and orthogonal. In the earth-fixed reference frame the Z_E -axis points to the center of the earth, the X_E -axis points in some arbitrary direction, e.g. the north, and the Y_E -axis is perpendicular to the X_E -axis. This frame is useful for describing the position and orientation of the aircraft. In the body-fixed reference frame, the origin is at the aircraft center of gravity, while the X_B -axis points forward through the nose, the Y_B -axis through the starboard (right) wing and the Z_B -axis downwards.

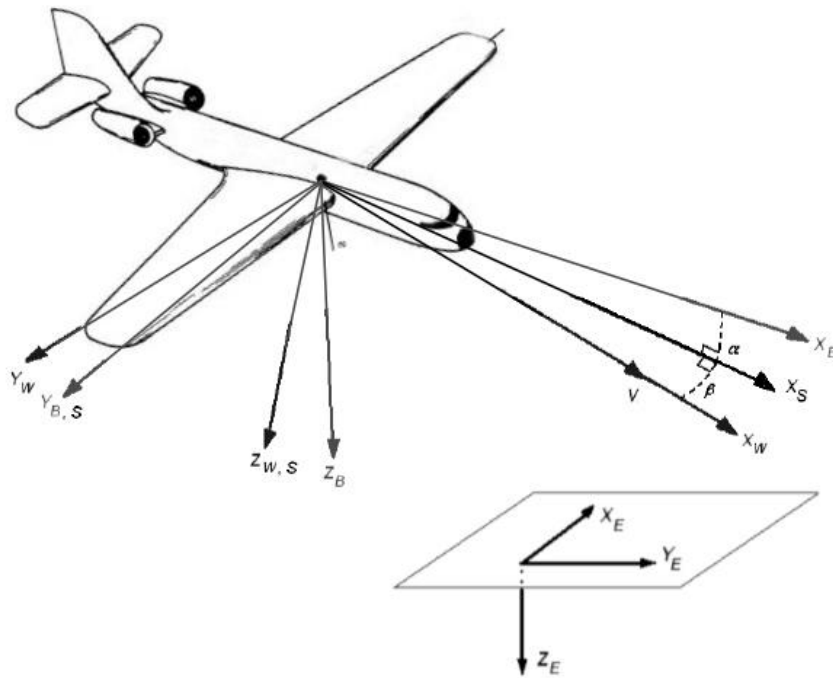


Figure 1 Aircraft reference frames: earth-fixed reference frame, body-fixed reference frame, stability-axes reference frame and wind-axes reference frame

Two other frames are indicated in Figure 1, these are the stability-axes reference frame F_S and the wind-axes reference frame F_W . The stability-axes reference frame is obtained from the body-fixed reference frame by a left-handed rotation through angle of attack α , and is used for analyzing the effect of perturbations from steady-state flight. The wind-axes reference frame

is obtained from the stability-axes reference frame by a rotation around the z-axis through sideslip angle β . The lift, drag and side forces are defined naturally in this reference frame. Using this reference frame can be convenient when describing the equations of motion.

The transformation from a vector in the body-fixed reference frame to the stability-axes reference frame can be described by a rotation over angle of attack α :

$$\mathbf{T}_s = T_{s/b} \mathbf{T}_b = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix} \mathbf{T}_b \quad (1)$$

and the transformation from a vector in the body-fixed reference frame to the wind-axes reference frame can be described by a rotation over side slip angle β , followed by a rotation over angle of attack α :

$$\mathbf{T}_w = T_{w/b} \mathbf{T}_b \quad (2)$$

where

$$\begin{aligned} T_{w/b} &= \begin{bmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} T_{s/b} \\ &= \begin{bmatrix} \cos \alpha \cos \beta & \sin \beta & \sin \alpha \cos \beta \\ -\cos \alpha \sin \beta & \cos \beta & -\sin \alpha \sin \beta \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}. \end{aligned} \quad (3)$$

1.2 Aircraft Variables

A number of assumptions has to be made, before proceeding with the derivation of the equations of motion:

1. The aircraft is a rigid-body, which means that any two points on or within the airframe remain fixed with respect to each other. This assumption is quite valid for fighter aircraft.
2. The earth is flat and non-rotating and regarded as an inertial reference. This assumption is valid when dealing with control design of aircraft, but not when analyzing inertial guidance systems.
3. The mass is constant during the time interval over which the motion is considered, the fuel consumption is neglected during this time-interval. (To calculate the fuel consumption, you have to give me the performance of engine and the variation of location of center of gravity.)
4. The mass distribution of the aircraft is symmetric relative to the $X_B O Z_B$ -plane, this implies that the products of inertia I_{yz} and I_{xy} are equal to zero. This assumption is valid for most aircraft.

Under the above assumptions the motion of the aircraft has 6 degrees of freedom (rotation and translation in 3 dimensions). The aircraft dynamics can be described by its position, orientation, velocity and angular velocity over time. $\mathbf{P}_E = (x_E, y_E, H)$ is the

position vector expressed in an earth-fixed coordinate system. V is the velocity vector given by $V = (u, v, \omega)$, where u is the longitudinal velocity, v the lateral velocity and ω the normal velocity. The orientation vector is given by $\Phi = (\phi, \theta, \psi)$, where ϕ is the roll angle, θ the pitch angle and ψ the yaw angle, and the angular velocity vector is given by $\omega = (p, q, r)$, where p, q and r are the roll, pitch and yaw angular velocities.

The relation between the attitude vector Φ and the angular velocity vector ω is given by

$$\dot{\Phi} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \frac{\sin \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{bmatrix} \omega \quad (4)$$

Defining V_T as the total velocity, we find the following relations:

$$\begin{aligned} V_T &= \sqrt{u^2 + v^2 + w^2} \\ \alpha &= \arctan \frac{w}{u} \\ \beta &= \arcsin \frac{v}{V_T} \end{aligned} \quad (5)$$

Furthermore, the flight path angle γ can be defined as

$$\gamma = \arcsin \left(\frac{\dot{H}}{V_T} \right) \quad (6)$$

1.3 Equation of Motion for a Rigid Body Aircraft

The equation of motion for the aircraft can be derived from Newton's Second Law of motion, which states that the summation of all external forces acting on a body must be equal to the time rate of change its momentum, and the summation of the external moments acting on a body must be equal to the time rate of change of its angular momentum. The equation of motion for rigid aircraft is as following:

$$\begin{aligned} \dot{V}_T &= \frac{1}{m} (-D + F_E \cos \alpha \cos \beta + mg_1) \\ \dot{\alpha} &= q - (p \cos \alpha + r \sin \alpha) \tan \beta + \frac{1}{m V_T \cos \beta} (-L - F_E \sin \alpha + mg_3) \end{aligned} \quad (7)$$

$$\dot{\beta} = p \sin \alpha - r \cos \alpha + \frac{1}{m V_T} (Y - F_E \cos \alpha \sin \beta + mg_2)$$

$$\begin{aligned} \dot{p} &= (c_1 r + c_2 p) q + c_3 \bar{L} + c_4 \bar{N} \\ \dot{q} &= c_5 p r - c_6 (p^2 + r^2) + c_7 (\bar{M} + F_E z_T) \\ \dot{r} &= (c_8 p - c_2 r) q + c_4 \bar{L} + c_9 \bar{N} \end{aligned} \quad (8)$$

$$\begin{aligned}
\dot{\phi} &= p + \tan \theta (q \sin \phi + r \cos \phi) \\
\dot{\theta} &= q \cos \phi - r \sin \phi \\
\dot{\psi} &= \frac{q \sin \phi + r \cos \phi}{\cos \theta}
\end{aligned} \tag{9}$$

$$\begin{aligned}
\dot{x}_E &= u \cos \psi \cos \theta + v(\cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi) \\
&\quad + w(\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) \\
\dot{y}_E &= u \sin \psi \cos \theta + v(\sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi) \\
&\quad + w(\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi) \\
\dot{z}_E &= -u \sin \theta + v \cos \theta \sin \phi + w \cos \theta \cos \phi
\end{aligned} \tag{10}$$

where the gravity components g_1, g_2, g_3 are given by

$$\begin{aligned}
g_1 &= g(-\cos \alpha \cos \beta \sin \theta + \sin \beta \sin \phi \cos \theta + \sin \alpha \cos \beta \cos \phi \cos \theta) \\
g_2 &= g(\cos \alpha \sin \beta \sin \theta + \cos \beta \sin \phi \cos \theta - \sin \alpha \sin \beta \cos \phi \cos \theta) \\
g_3 &= g(\sin \alpha \sin \theta + \cos \alpha \cos \phi \cos \theta).
\end{aligned} \tag{11}$$

the moment of inertia components are given by

$$\begin{aligned}
\Gamma c_1 &= (I_y - I_z)I_z - I_{xz}^2 & \Gamma c_4 &= I_{xz} & c_7 &= \frac{1}{I_y} \\
\Gamma c_2 &= (I_x - I_y + I_z)I_{xz} & c_5 &= \frac{I_x - I_y}{I_y} & \Gamma c_8 &= I_x(I_x - I_y) + I_{xz}^2 \\
& & \Gamma c_3 &= I_z & c_6 &= \frac{I_{xz}}{I_y} & \Gamma c_9 &= I_x
\end{aligned} \tag{12}$$

with $\Gamma = I_x I_z - I_{xz}^2$

The relation between \dot{h} and \dot{z}_E are given by

$$\dot{h} = -\dot{z}_E \tag{13}$$

1.4 ISA Atmospheric Model

For the atmospheric data an approximation of the International Standard Atmosphere (ISA) is used

$$\begin{aligned}
g &= 9.80665 * \left(\frac{6371020}{6371020 + h} \right)^2 \\
T &= T_0 - 0.0065h \\
\rho &= \rho_0 e^{-\frac{g}{287.05T}h} \\
a &= \sqrt{1.4 \times 287.05T}
\end{aligned}$$

where $T_0 = 288.15$ is the temperature at the sea level and $\rho_0 = 1.225$ is the air density at sea level. This atmospheric model is only valid in the troposphere ($h < 11000$ m). Given the aircraft's altitude (h in meters) it returns the current temperature (T in Kelvin), the current air density (ρ in kg/m^3) and the speed of sound (a in m/s) and the local gravity (g in m/s^2).

2 Control Variables

The aircraft model allows for control over thrust, elevator, ailerons, rudder, and flap. The thrust is measured in Newtons. All deflections are defined positive in the conventional way, i.e. positive thrust causes an increase in acceleration along the body x -axis, a positive elevator deflection results in a decrease in pitch rate, a positive aileron deflection gives a decrease in roll rate and a positive rudder deflection decreases the yaw rate. The aircraft also has a flap, which helps to fly the aircraft at high angles of attack. The deflection of the flap δ_f is switched by the pilot, reflecting different configuration of the aircraft.

The maximum values and units for all control variables are given in Table 1.

Table 1 The control input units and maximum values

Control	Units	MIN	MAX	rate limit
Elevator	deg	?	?	?
Ailerons	deg	?	?	?
Rudder	deg	?	?	?
Flap	deg	?	?	?

3 Engine Model

The thrust F_T generated by engine and fuel consumption rate is given by engine model.

The model is not available yet.

4 Propeller Effects

The rotation of propellers affect the fluid field.

The model is not available yet.

5 Wind Effects

If aircraft is in an environment with wind parameter defined as:

$$[u_w, v_w, \omega_w, \dot{u}_w, \dot{v}_w, \dot{\omega}_w]$$

The force components F_{wx_b} , F_{wy_b} , F_{wz_b} generated by wind in body-fixed reference frame are given by:

$$\begin{aligned} F_{wx_b} &= -m(\dot{u}_w + q\omega_w - rv_w) \\ F_{wy_b} &= -m(\dot{v}_w - p\omega_w + ru_w) \end{aligned} \quad (14)$$

$$F_{wzb} = -m(\dot{\omega}_w + p v_w - q u_w)$$

Then the force components F_{wx} , F_{wy} , F_{wz} generated by wind in wind reference frame are given by:

$$\begin{bmatrix} F_{wx} \\ F_{wy} \\ F_{wz} \end{bmatrix} = T_{w/b} \begin{bmatrix} F_{wxb} \\ F_{wyb} \\ F_{wzb} \end{bmatrix} \quad (15)$$

The formula (7) can be modified as following:

$$\begin{aligned} \dot{V}_T &= \frac{1}{m} (-D + F_E \cos \alpha \cos \beta + mg_1 + F_{wx}) \\ \dot{\alpha} &= q - (p \cos \alpha + r \sin \alpha) \tan \beta + \frac{1}{m V_T \cos \beta} (-L - F_E \sin \alpha + mg_3 + F_{wz}) \\ \dot{\beta} &= p \sin \alpha - r \cos \alpha + \frac{1}{m V_T} (Y - F_E \cos \alpha \sin \beta + mg_2 + F_{wy}) \end{aligned} \quad (16)$$

And formula (10) can be modified as following:

$$\begin{aligned} \dot{x}_E &= (u + u_w) \cos \psi \cos \theta + (v + v_w) (\cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi) \\ &\quad + (\omega + \omega_w) (\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) \\ \dot{y}_E &= (u + u_w) \sin \psi \cos \theta + (v + v_w) (\sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi) \\ &\quad + (\omega + \omega_w) (\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi) \\ \dot{z}_E &= -(u + u_w) \sin \theta + (v + v_w) \cos \theta \sin \phi + (\omega + \omega_w) \cos \theta \cos \phi \end{aligned}$$

6 Aerodynamic Coefficients

Total coefficient equation have been used to sum the various aerodynamic contributions to a given force (in stability-axes reference frame) or moment (in body-fixed reference frame) coefficient as follows:

For the total lift force coefficient:

$$C_{LT} = C_{L\alpha} \alpha + C_{L\delta_e} \delta_e + C_{L0} + C_{L\delta_f} \delta_f \quad (18)$$

For the total drag force coefficient:

$$C_{DT} = C_{D0} + C_{D\alpha} \alpha + C_{D\delta_f} \delta_f \quad (19)$$

For the total side force coefficient:

$$C_{YT} = C_{Y\beta} \beta + C_{Y\delta_r} \delta_r \quad (20)$$

For the total pitching moment coefficient:

$$C_{mT} = C_{m\alpha} \alpha + C_{m\delta_e} \delta_e + C_{mq} \frac{q\bar{c}}{2V} + C_{m0} \quad (21)$$

For the total rolling moment coefficient:

$$C_{lT} = C_{l\beta} \beta + C_{lp} p + C_{lr} r + C_{l\delta_a} \delta_a + C_{l\delta_r} \delta_r \quad (22)$$

For the total yawing moment coefficient:

$$C_{nT} = C_{n\beta} \beta + C_{np} p + C_{nr} r + C_{n\delta_a} \delta_a + C_{n\delta_r} \delta_r \quad (23)$$

7 Matlab/Simulink Files

The model is established using Simulink, comprising different modules for different functions. The model is shown as below:

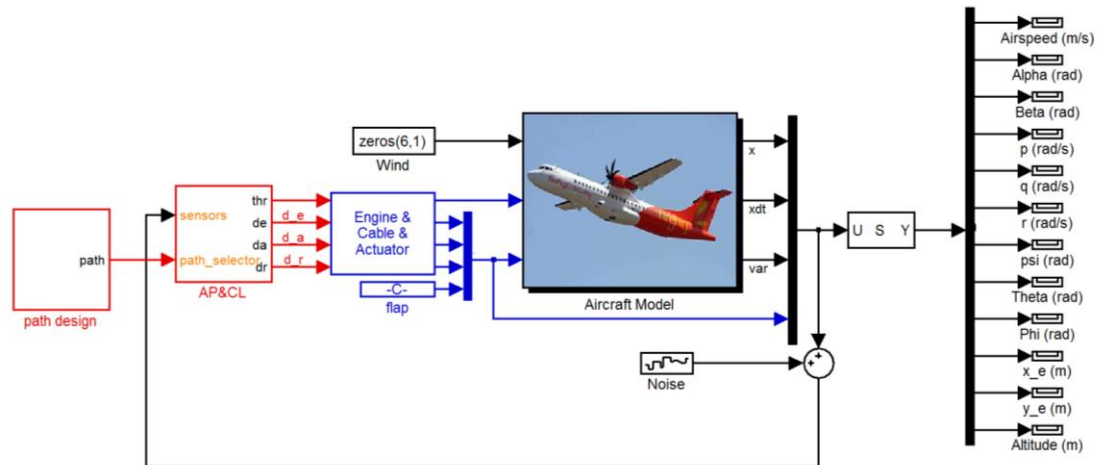


Figure 2 Structure of simulator

The module path design is left for path design.

The module AP&CL is left for autopilot and control law design.

The module Engine&Cable&Actuator is left for engine model, cable model and actuator model.

The module wind is for wind perturbation. The vector is given by:

$$[u_w, v_w, \omega_w, \dot{u}_w, \dot{v}_w, \dot{\omega}_w]$$

The module Aircraft Model is the plane simulator. Double clicking the block, user can define the aircraft parameter as shown below:



Figure 3 User-defined aircraft parameter

To modify the structure of the simulator, user can right click the block and choose “look under mask”. The structure of the simulator is indicated in Figure 4.

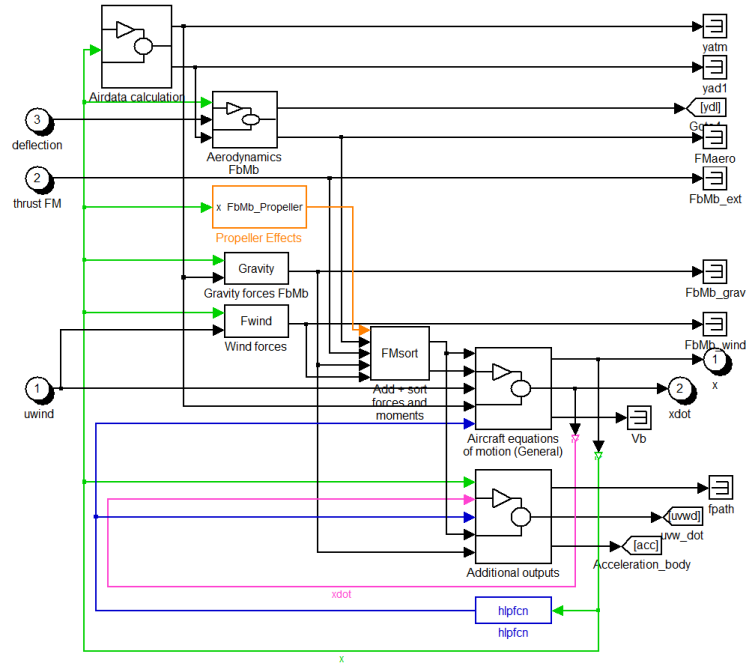


Figure 4 Structure of simulator

The plane simulator include 7 blocks. The function of each block is given in Table 2.

Table 2 Function of blocks in the simulator

Blocks	Inputs	Outputs	Function of block
Airdata	$x = \begin{bmatrix} V_T \\ \alpha \\ \beta \\ p \\ q \\ r \\ \psi \\ \theta \\ \phi \\ x_E \\ y_E \\ h \end{bmatrix}$	$yatm = \begin{bmatrix} \rho \\ p_s \\ T \\ \mu \\ g \end{bmatrix}$	Calculate airdata according to aircraft states.
		$yad1 = \begin{bmatrix} a \\ M \\ \bar{q} \end{bmatrix}$	
Aerodynamics FbMb	$x = \begin{bmatrix} V_T \\ \alpha \\ \beta \\ p \\ q \\ r \\ \psi \\ \theta \\ \phi \\ x_E \\ y_E \\ h \end{bmatrix}$	$FMaero = \begin{bmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \\ \bar{L} \\ \bar{M} \\ \bar{N} \end{bmatrix}$	Calculate components of aerodynamic forces and moments in fixed-body reference frame
	$\delta = \begin{bmatrix} \delta_e \\ \delta_a \\ \delta_r \\ \delta_f \end{bmatrix}$		
	$yad1 = \begin{bmatrix} a \\ M \\ \bar{q} \end{bmatrix}$		

Blocks	Inputs	Outputs	Function of block
Propeller Effects	$\mathbf{x} = \begin{bmatrix} V_T \\ \alpha \\ \beta \\ p \\ q \\ r \\ \psi \\ \theta \\ \phi \\ x_E \\ y_E \\ h \end{bmatrix}$	$\mathbf{FM}_{\text{propeller}} = \begin{bmatrix} F_{pxb} \\ F_{pyb} \\ F_{pzb} \\ M_{pxb} \\ M_{pyb} \\ M_{pzb} \end{bmatrix}$	Calculate propeller effects on components of forces and moments in fixed-body reference frame
Gravity	$\mathbf{x} = \begin{bmatrix} V_T \\ \alpha \\ \beta \\ p \\ q \\ r \\ \psi \\ \theta \\ \phi \\ x_E \\ y_E \\ h \end{bmatrix}$	$\mathbf{F}_{\text{grav}} = m \begin{bmatrix} g_{1b} \\ g_{2b} \\ g_{3b} \end{bmatrix}$	Calculate components of force of gravity in fixed-body reference frame
	$\mathbf{y}_{\text{atm}} = \begin{bmatrix} \rho \\ p_s \\ T \\ \mu \\ g \end{bmatrix}$		
Wind forces	$\mathbf{x} = \begin{bmatrix} V_T \\ \alpha \\ \beta \\ p \\ q \\ r \\ \psi \\ \theta \\ \phi \\ x_E \\ y_E \\ h \end{bmatrix}$	$\mathbf{F}_{\text{wind}} = \begin{bmatrix} F_{wxb} \\ F_{wyb} \\ F_{wzb} \end{bmatrix}$	Calculate components of force generated by wind in fixed-body reference frame
	$\mathbf{V}_{\text{wind}} = \begin{bmatrix} u_w \\ v_w \\ \omega_w \\ \dot{u}_w \\ \dot{v}_w \\ \dot{\omega}_w \end{bmatrix}$		
FMsort	$\mathbf{FM}_{\text{propeller}} = \begin{bmatrix} F_{pxb} \\ F_{pyb} \\ F_{pzb} \\ M_{pxb} \\ M_{pyb} \\ M_{pzb} \end{bmatrix}$	$\mathbf{FM}_T = \begin{bmatrix} F_{Txb} \\ F_{Tyb} \\ F_{Tzb} \\ M_{Txb} \\ M_{Tyb} \\ M_{Tzb} \end{bmatrix}$	Calculate components of total force and moment in fixed-body reference frame
	$\mathbf{FM}_{\text{aero}} = \begin{bmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \\ \bar{L} \\ \bar{M} \\ \bar{N} \end{bmatrix}$		

Blocks	Inputs	Outputs	Function of block
	$FM_E = \begin{bmatrix} F_{Exb} \\ F_{Eyb} \\ F_{Ezb} \\ M_{Exb} \\ M_{Eyb} \\ M_{Ezb} \end{bmatrix}$		
	$F_{grav} = m \begin{bmatrix} g_{1b} \\ g_{2b} \\ g_{3b} \end{bmatrix}$		
	$F_{wind} = \begin{bmatrix} F_{wx b} \\ F_{wy b} \\ F_{wz b} \end{bmatrix}$		
Equation of Motion	$F_T = \begin{bmatrix} F_{Tx b} \\ F_{Ty b} \\ F_{Tz b} \end{bmatrix}$	$x = \begin{bmatrix} V_T \\ \alpha \\ \beta \\ p \\ q \\ r \\ \psi \\ \theta \\ \phi \\ x_E \\ y_E \\ h \end{bmatrix}$	Iterative computation of equation of motion
	$M_T = \begin{bmatrix} M_{Tx b} \\ M_{Ty b} \\ M_{Tz b} \end{bmatrix}$		
	$V_{wind} = \begin{bmatrix} u_w \\ v_w \\ \omega_w \\ \dot{u}_w \\ \dot{v}_w \\ \dot{\omega}_w \end{bmatrix}$	$\dot{x} = \begin{bmatrix} \dot{V}_T \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \\ \dot{x}_E \\ \dot{y}_E \\ \dot{h} \end{bmatrix}$	
	$y_{atm} = \begin{bmatrix} \rho \\ p_s \\ T \\ \mu \\ g \end{bmatrix}$		
	$hlpfcn = \begin{bmatrix} \cos\alpha \\ \sin\alpha \\ \cos\beta \\ \sin\beta \\ \tan\beta \\ \sin\psi \\ \cos\psi \\ \sin\theta \\ \cos\theta \\ \sin\phi \\ \cos\phi \end{bmatrix}$	$V_b = \begin{bmatrix} u \\ v \\ \omega \end{bmatrix}$	