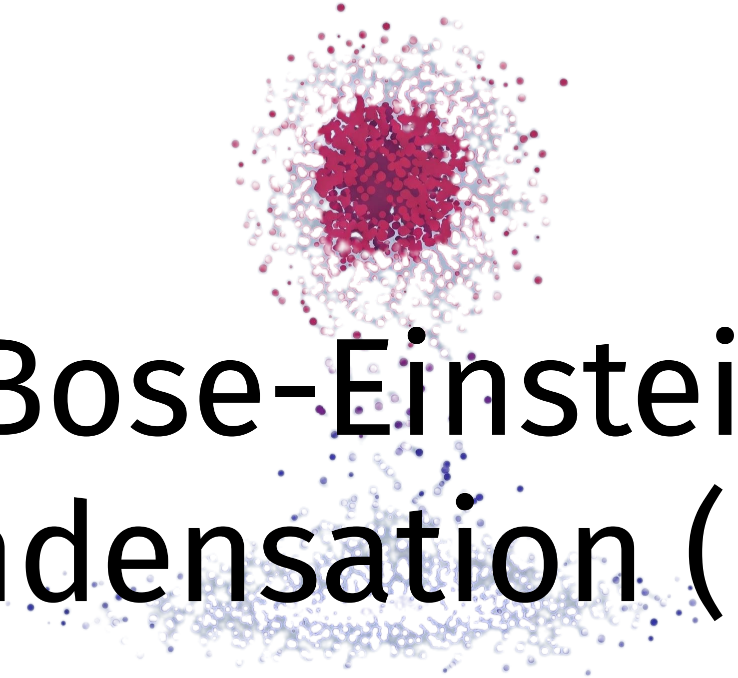

# Bose-Einstein Condensation (BEC)

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# Outline

- Introduction: Bose Einstein Condensate
  - What is Bose-Einstein Condensation?
  - Experimental Evidence
- Code Goals & Design
- Mean Field Approach: GPE Equation
- Stochastic Approach: Metropolis-like algorithm
- Conclusions

# What is Bose-Einstein Condensation?

- Accumulation of atoms in ground state below a critical temperature ($T_C$), forming a “condensate”
- $T_C$ is higher than the “freeze-out temperature” ($T_0$), or the temperature at which only the ground state is accessible)

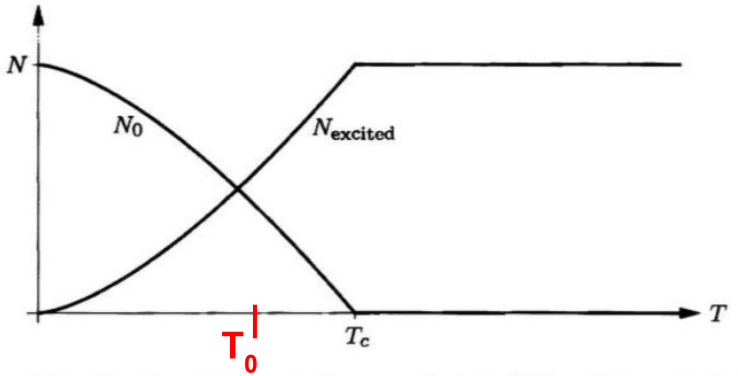



**Figure 7.32.** Number of atoms in the ground state ($N_0$) and in excited states, for an ideal Bose gas in a three-dimensional box. Below $T_c$ the number of atoms in excited states is proportional to $T^{3/2}$.

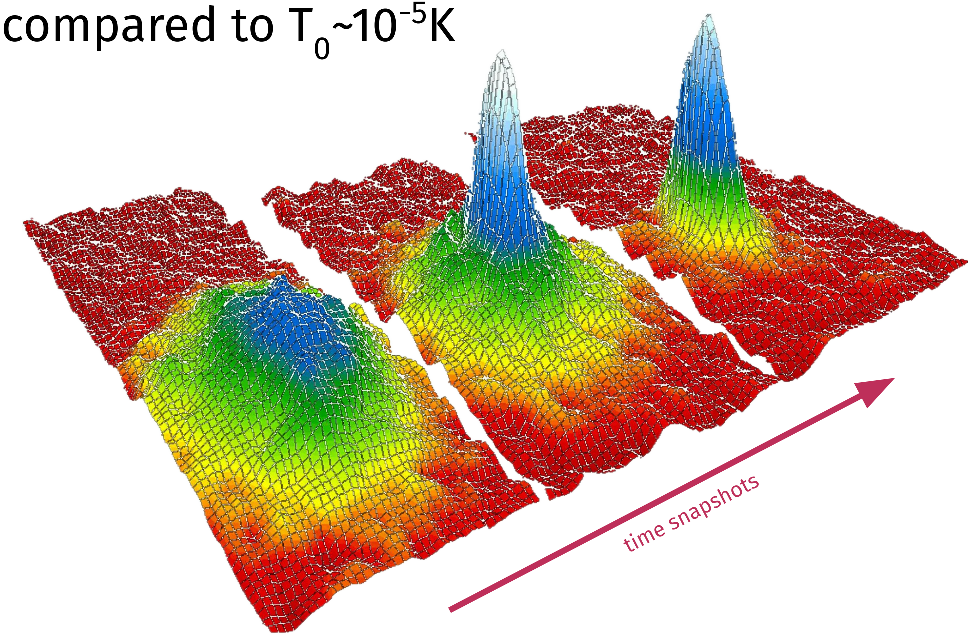
# Experimental Evidence

First observation in 1995 with laser-trapped Rubidium atoms

Condensation occurred at $T_C$~$10^{-7}$K, compared to $T_0$~$10^{-5}$K

The atoms condense from less dense red, yellow and green areas into very dense blue to white areas.

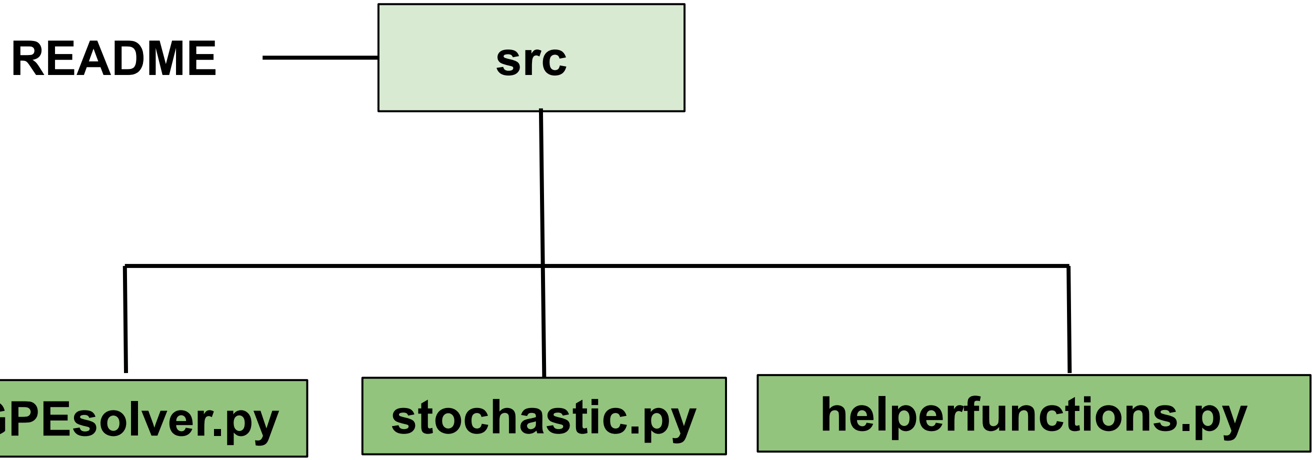
Individual atoms condense into a "superatom" behaving as a single entity.




https://titan.physx.u-szeged.hu/~dpiroska/atmolfiz/bose_einstein_condensation.html

# Code Goals & Design

1. Numerically solve the Schrödinger Equation for the BEC Hamiltonian to model behavior of a condensate
2. Stochastically model behavior of condensate when perturbations to system are applied



# Mean Field Approach: Gross-Pitaevskii Equation (GPE)

The Hamiltonian can be quite difficult to compute exactly ☹

$$\hat{H} = \sum_{i=1}^{N} \left( \frac{\mathbf{p}_i^2}{2m} + V_{\text{ext}}(\mathbf{r}_i) \right) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j \neq i}^{N} V\left( |\mathbf{r}_i - \mathbf{r}_j| \right)$$

We can do “smart” approximations:

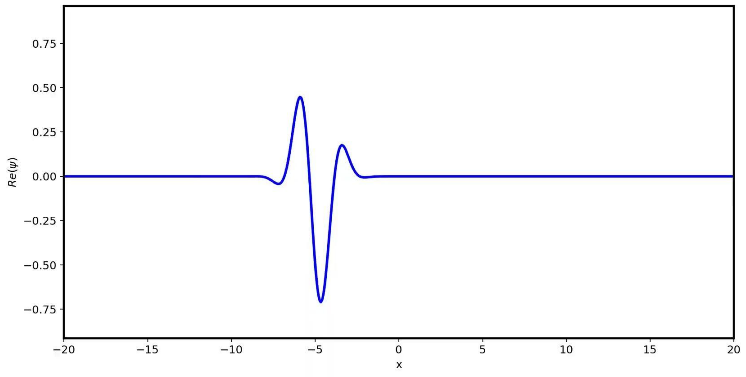
Description by a global wave function

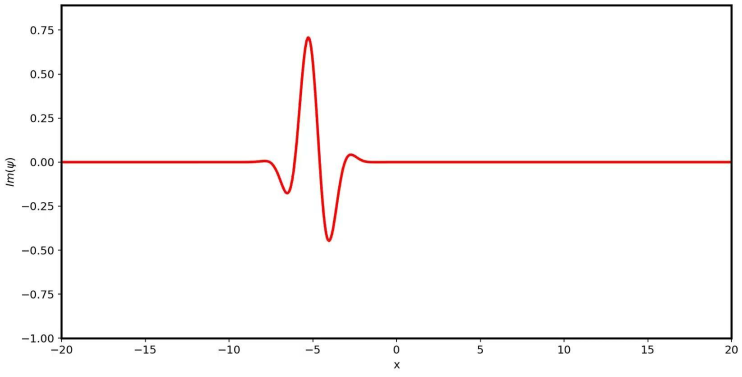
Gross-Pitaevskii Equation (GPE)

$$-i\hbar \frac{\partial \Psi}{\partial t} = \left( -\frac{\hbar^2}{2m} \nabla^2 + V + g|\Psi|^2 \right) \Psi$$

non-linear SE!

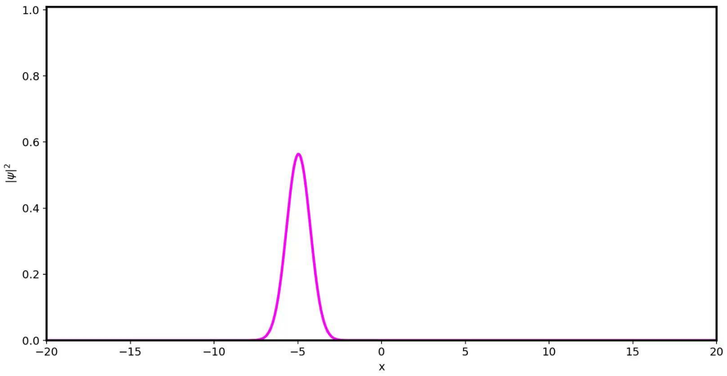
# Wave Function Animation




**Parameters:**

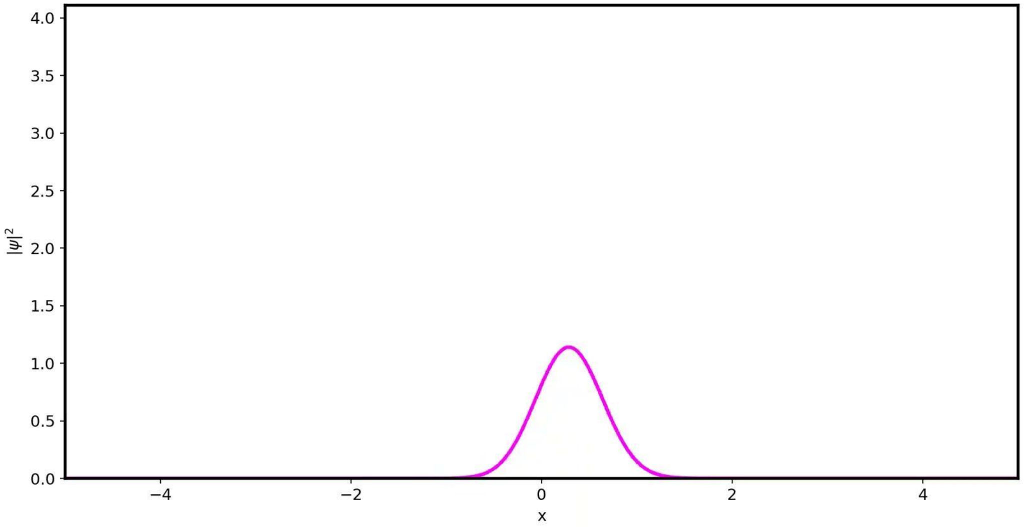
- Gaussian Wave Packet
- Harmonic Trap
- Crank-Nicolson scheme



# Stochastic Method: Metropolis-Hastings for BEC

1. Choose initial wave function (ground state)
2. Calculate “reduced entropy” (quantity that involves the BEC Hamiltonian and the number operator)
3. Apply a density perturbation and a number perturbation to the wave function
4. Re-calculate the “reduced entropy”
5. Accept or reject the new wave function based on the probabilities of the wave function to be in the old and new states

# Evolution of Wave Function with Perturbations




Probability density gradually localizes!

Remaining Questions

- Why does the probability density shift away from 0?
- Can we tune parameters to represent experimental conditions?

# Conclusions

- We’re able to solve the GPE Equation for a simple potential.
- We implemented a stochastic approach.
- We animated our results.
- Results are encouraging from a qualitative perspective.

# Future Work?

- Solve GPE with other potentials and initial conditions and compare result
- Vary parameters for stochastic approach
- Compare results with paper

# References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