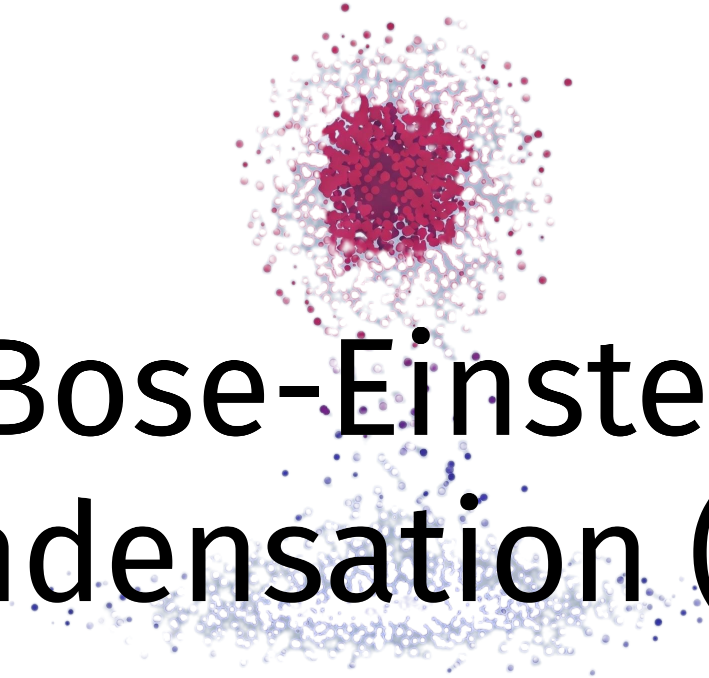


Bose-Einstein Condensation (BEC)

Sam, Emmanuel

Outline

- Introduction: Bose Einstein Condensate
  - What is Bose-Einstein Condensation?
  - Experimental Evidence
- Code Goals & Design
- Mean Field Approach: GPE Equation
- Stochastic Approach: Metropolis-like algorithm
- Conclusions

What is Bose-Einstein Condensation?

- Accumulation of atoms in ground state below a critical temperature (T_C), forming a “condensate”
- T_C is higher than the “freeze-out temperature” (T_0), or the temperature at which only the ground state is accessible)

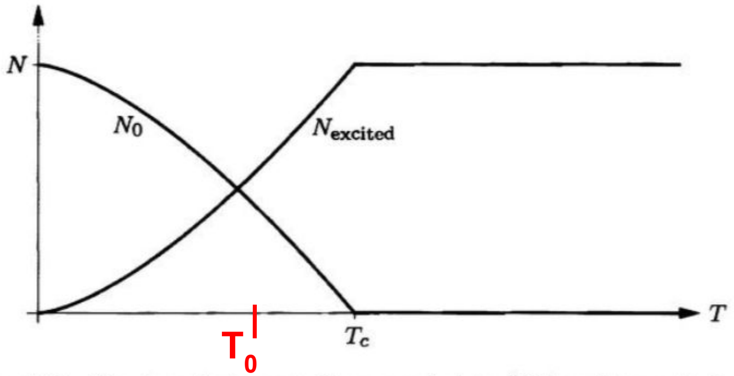


Figure 7.32. Number of atoms in the ground state (N_0) and in excited states, for an ideal Bose gas in a three-dimensional box. Below T_c the number of atoms in excited states is proportional to $T^{3/2}$.

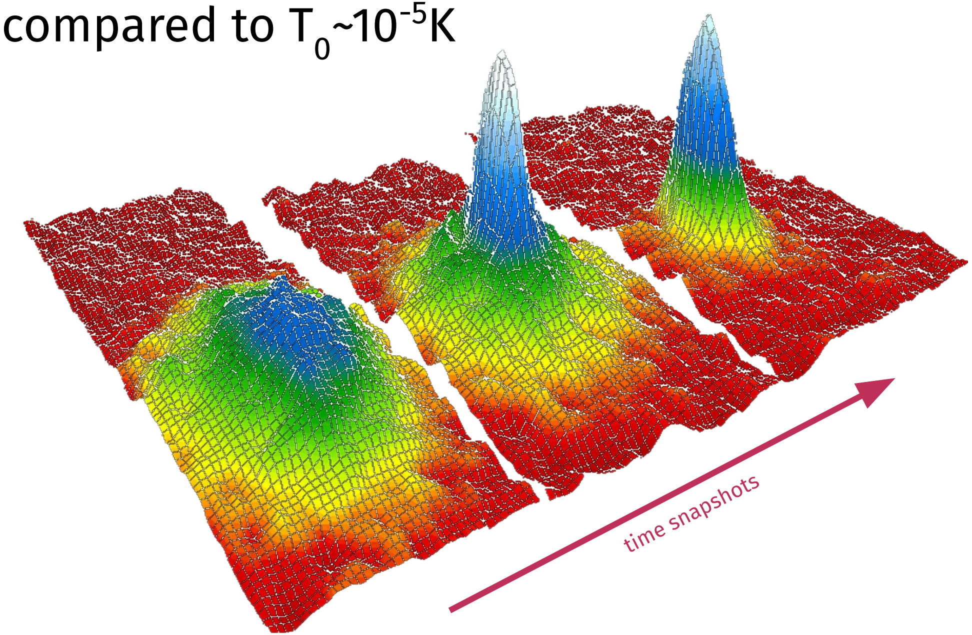
Experimental Evidence

First observation in 1995 with laser-trapped Rubidium atoms

Condensation occurred at T_C~10^{-7}K, compared to T_0~10^{-5}K

The atoms condense from less dense red, yellow and green areas into very dense blue to white areas.

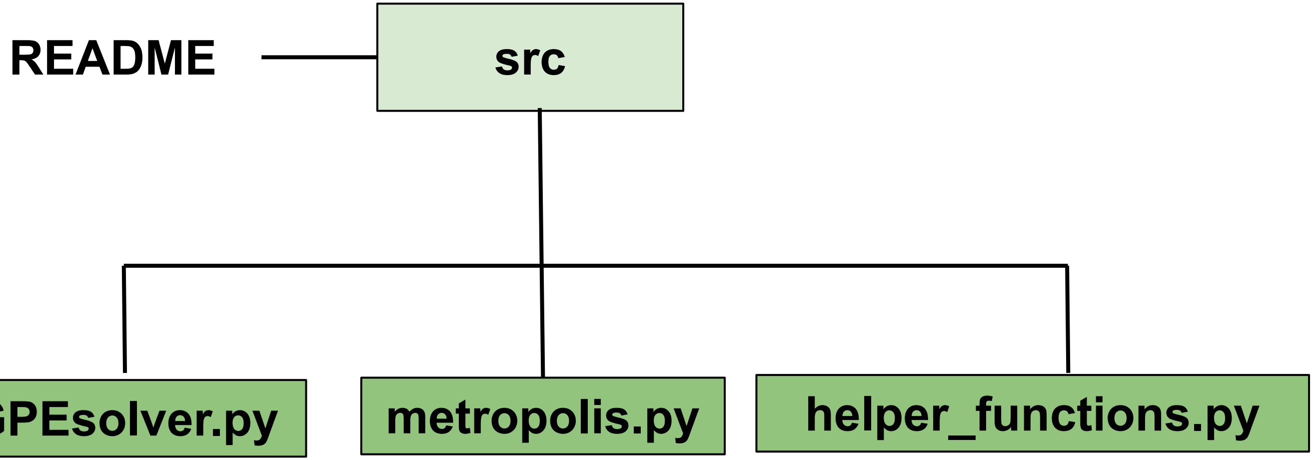
Individual atoms condense into a "superatom" behaving as a single entity.



https://titan.physx.u-szeged.hu/~dpiroska/atmolfiz/bose_einstein_condensation.html

Code Goals & Design

1. Numerically solve the Schrödinger Equation for the BEC Hamiltonian to model behavior of a condensate
2. Stochastically model behavior of condensate when perturbations to system are applied
3. Code stored in: https://github.com/emmanuellfc/Gross-Pitaevskii



Mean Field Approach: Gross-Pitaevskii Equation (GPE)

The Hamiltonian can be quite difficult to compute exactly ☹

$$\hat{H} = \sum_{i=1}^{N} \left(\frac{\mathbf{p}_i^2}{2m} + V_{\text{ext}}(\mathbf{r}_i) \right) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j \neq i}^{N} V\left(|\mathbf{r}_i - \mathbf{r}_j| \right)$$

We can do “smart” approximations:

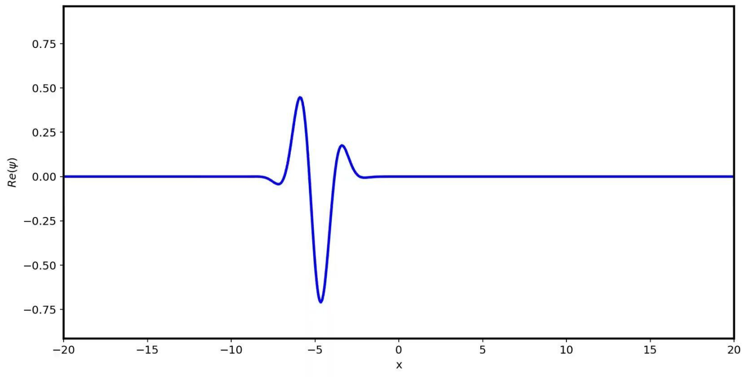
Description by a global wave function

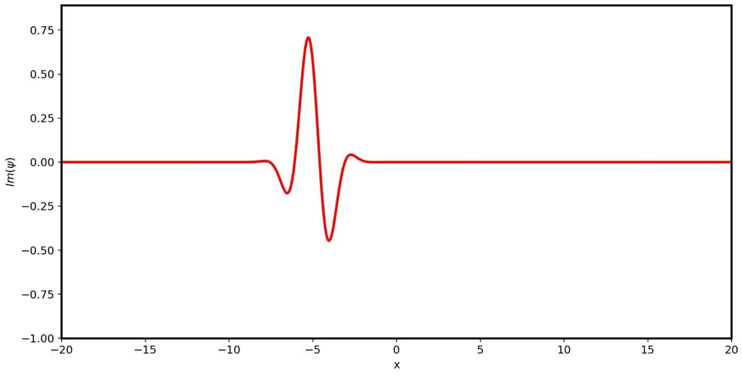
Gross-Pitaevskii Equation (GPE)

$$-i\hbar \frac{\partial \Psi}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V + g|\Psi|^2 \right) \Psi$$

non-linear SE!

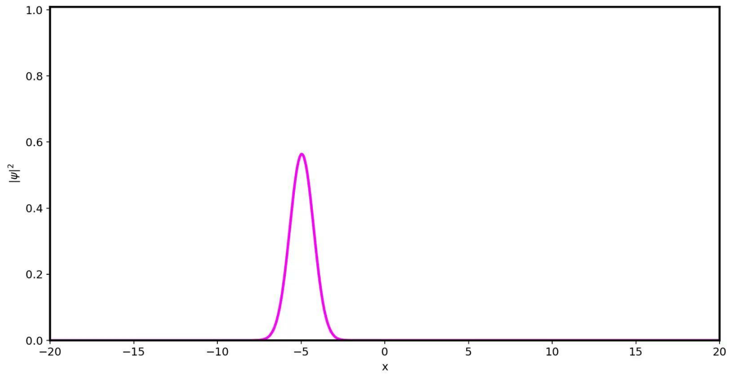
Wave Function Animation





Parameters:

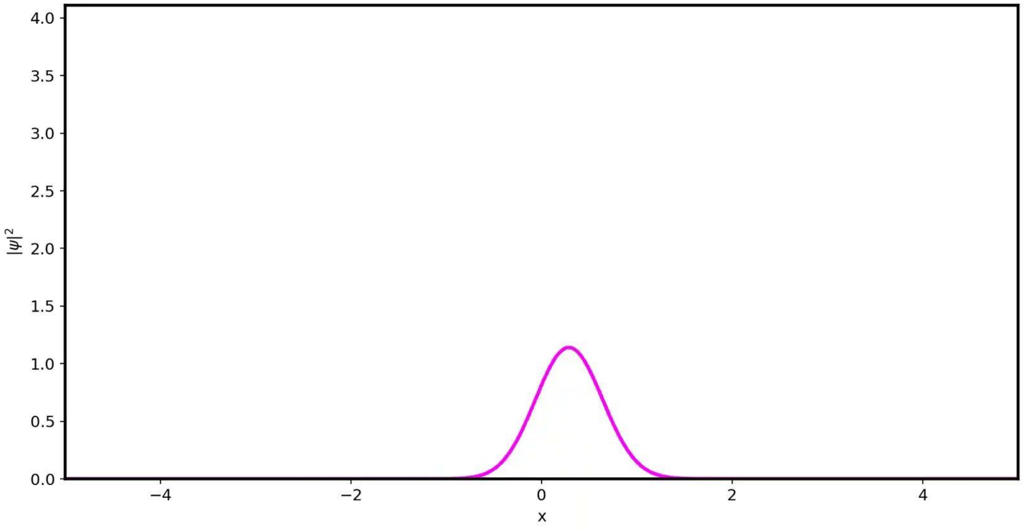
- Gaussian Wave Packet
- Harmonic Trap
- Crank-Nicolson scheme



Stochastic Method: Metropolis-Hastings for BEC

1. Choose initial wave function (ground state)
2. Calculate “reduced entropy” (quantity that involves the BEC Hamiltonian and the number operator)
3. Apply a density perturbation and a number perturbation to the wave function
4. Re-calculate the “reduced entropy”
5. Accept or reject the new wave function based on the probabilities of the wave function to be in the old and new states

Evolution of Wave Function with Perturbations



Probability density gradually localizes!

Remaining Questions

- Why does the probability density shift away from 0?
- Can we tune parameters to represent experimental conditions?

Conclusions

- We’re able to solve the GPE Equation for a simple potential.
- We implemented a stochastic approach.
- We animated our results.
- Results are encouraging from a qualitative perspective.

Future Work?

- Solve GPE with other potentials and initial conditions and compare result
- Vary parameters for stochastic approach
- Compare results with paper

References

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