Statistical Mechanics

List of results and definitions

Function of state		Statistical mechanical expression
U		$-\frac{\mathrm{d}\ln Z}{\mathrm{d}\beta}$
F		$-k_{\mathrm{B}}T\ln Z$
S	$= -\left(\frac{\partial F}{\partial T}\right)_V = \frac{U-F}{T}$	$k_{\mathrm{B}}\ln Z + k_{\mathrm{B}}T\left(\frac{\partial \ln Z}{\partial T}\right)_V$
p	$= -\left(\frac{\partial F}{\partial V}\right)_T$	$k_{\mathrm{B}}T\left(\frac{\partial \ln Z}{\partial V}\right)_T$
H	$= U + pV$	$k_{\mathrm{B}}T\left[T\left(\frac{\partial \ln Z}{\partial T}\right)_V + V\left(\frac{\partial \ln Z}{\partial V}\right)_T\right]$
G	$= F + pV = H - TS$	$k_{\mathrm{B}}T\left[-\ln Z + V\left(\frac{\partial \ln Z}{\partial V}\right)_T\right]$
C_V	$= \left(\frac{\partial U}{\partial T}\right)_V$	$k_{\mathrm{B}}T\left[2\left(\frac{\partial \ln Z}{\partial T}\right)_V + T\left(\frac{\partial^2 \ln Z}{\partial T^2}\right)_V\right]$

Table 20.1 Thermodynamic quantities derived from the partition function Z.

$$\beta = \frac{1}{k_B T}$$

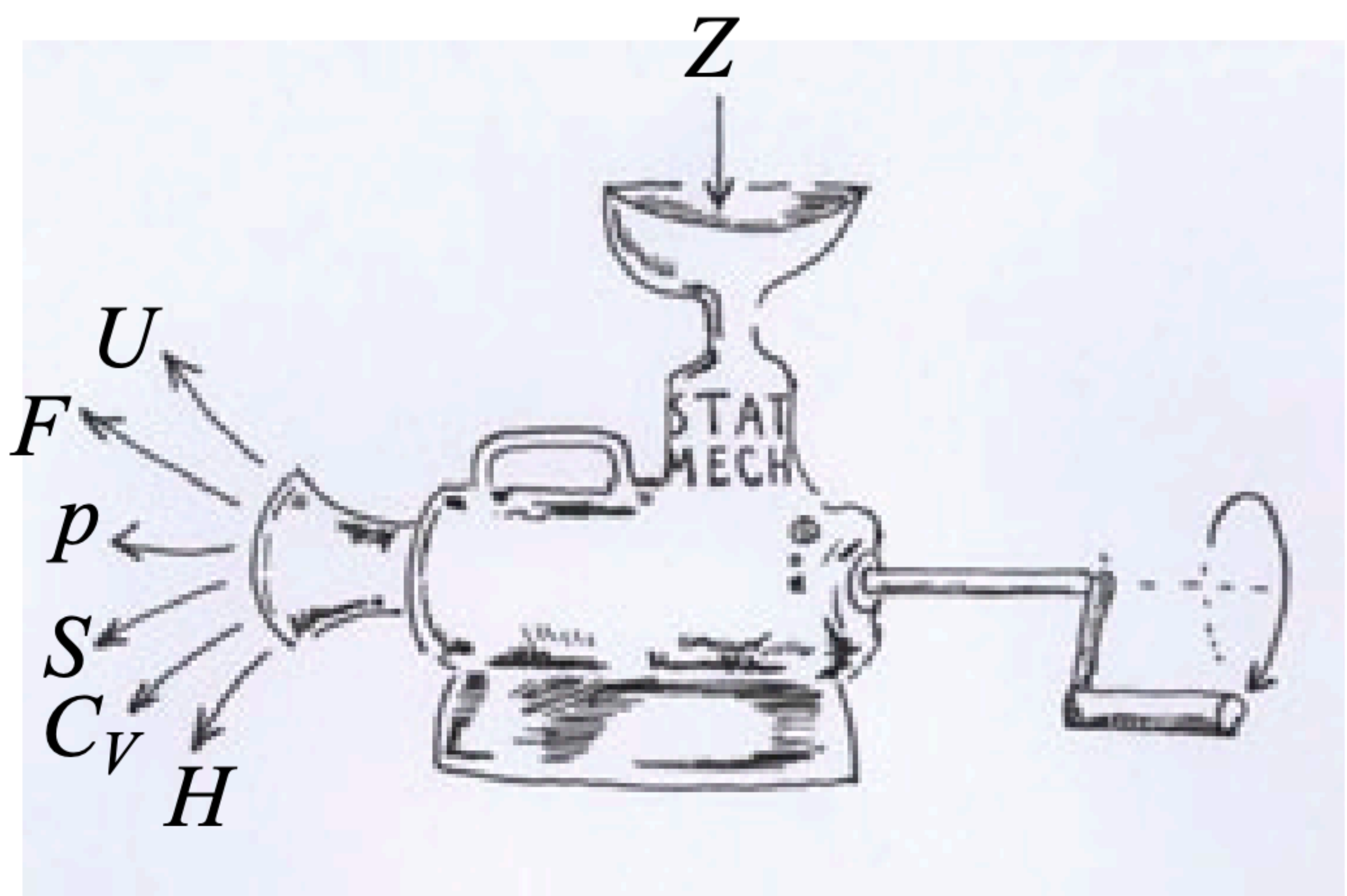


Fig. 20.3 Given Z, it takes only a turn of the handle on our 'sausage machine' to produce other functions of state.

Figures taken from Blundell

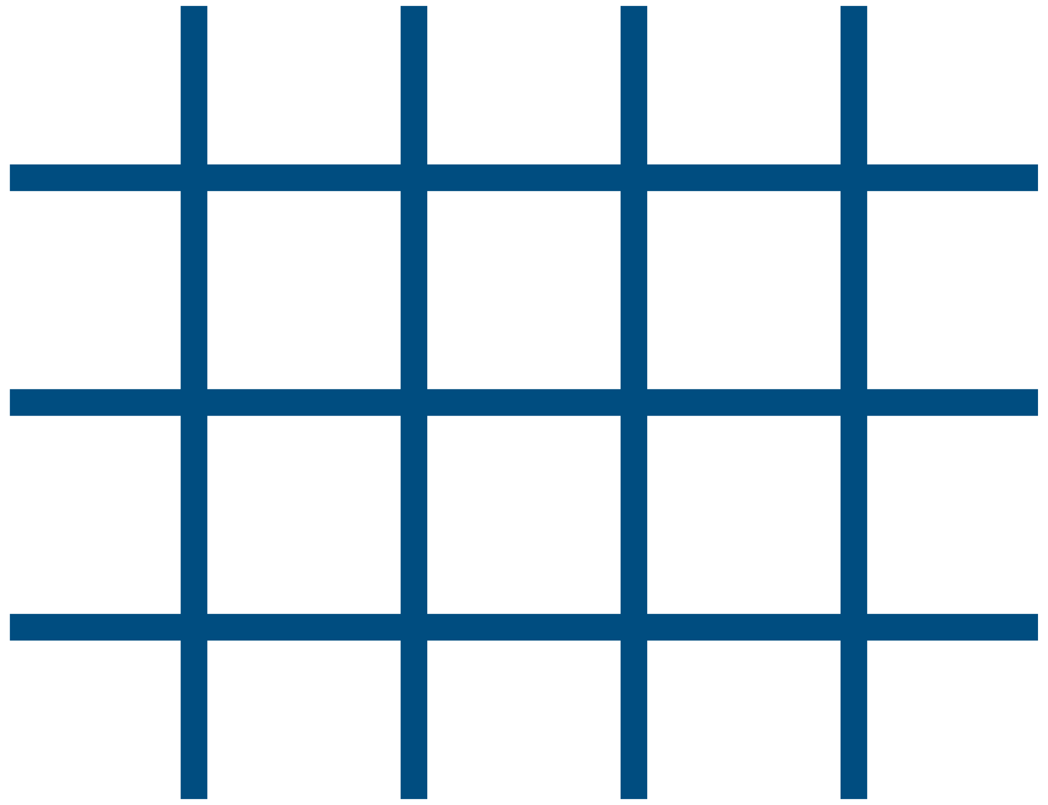
What about the Ising Model?

What is good/used for?

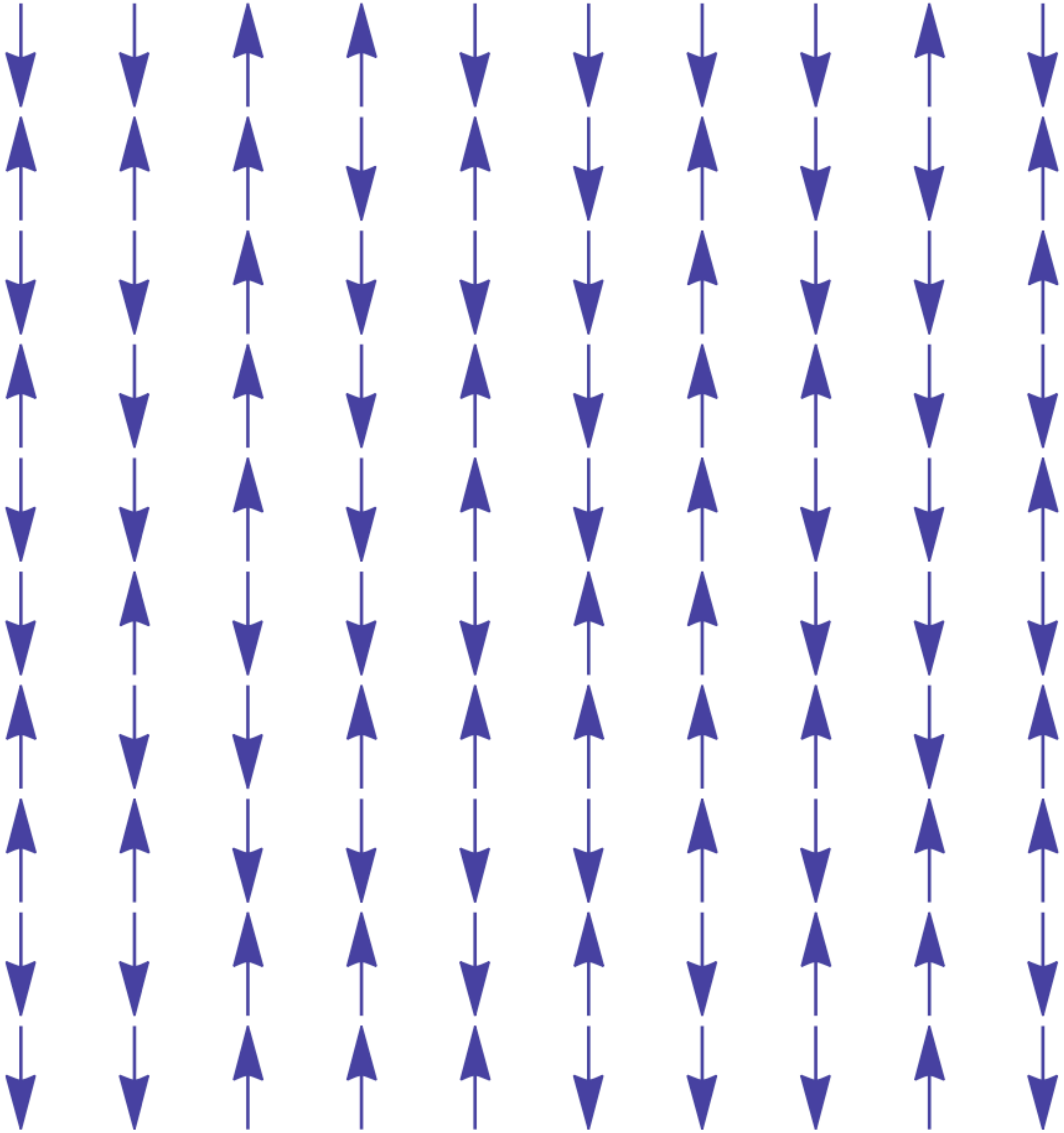
- Ferromagnetism.
- Widely used in the theory of phase transitions.

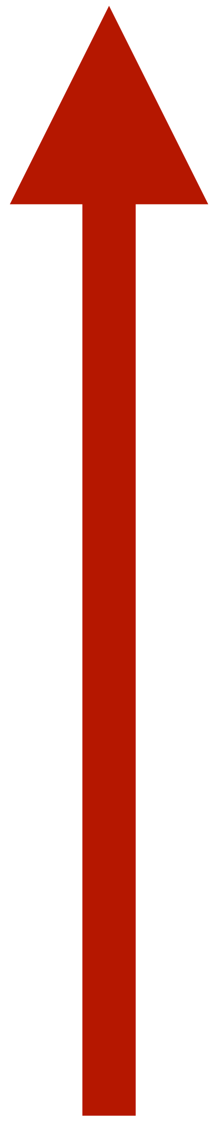
Why is so "famous"?

- Because it's very simple!



Lattice of spins





Magnetic Field

What are the "important" things?

$$E = -J \sum_{<i,j>} \sigma_i \sigma_j - h \sum_i \sigma_i$$

Total Energy Neighbors interaction Strength of the **B** field

Let's define an order parameter…

$$M = \left\langle \frac{N_\uparrow - N_\downarrow}{N} \right\rangle$$

Compute M as function of:
the **interaction strength**, the **magnetic field**, and the **temperature**

First Case: Non-Interacting Spins

Non-Interacting Spins: Entropy

First Case: Interacting Spins

Mean Field Theory Approximation

- We **neglect the correlations** between neighboring spins.
- Assume that they are each **fluctuating independently** with the same statistical distribution.