

# Topology, Assignment 4

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1. Using the standard topology on  $\mathbb{R}^2$ , is the product topology  $\mathbb{R} \times \mathbb{R}$  the same as the standard topology on  $\mathbb{R}^2$ .

*Proof.* □

2. Prove that a topological space  $X$  is Hausdorff if and only if  $\Delta = \{(x, x), x \in X\}$  is closed in the product space  $X \times X$ .

*Proof.* □

3. Let  $(X, \mathcal{T})$  be a topological space and  $Y \subset X$  equipped with the subspace topology. Prove that a subset  $U \subset Y$  is open in  $Y$  if and only if  $U$  is open in  $X$ .

*Proof.* □

4. Prove that a set  $A$  is dense in a topological space  $(X, \mathcal{T})$  if and only if every non-empty open set in  $X$  contains a point in  $A$ .

*Proof.* □

4.1 Find a topology on  $\mathbb{R}$  that is not separable.

5. Suppose that  $(X, \mathcal{T})$  is a topological space that has a countable basis. Prove that  $X$  is separable.

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