Topology: Class Notes

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1 General Defintions

2 Bases and Subspaces

In what follows I'm going to use TS for Topological Space. Given a topological space we define a basis for this TS as follows:

Definition 1 (Basis). *Given a TS $(X, \mathcal{T})$, we say that $\beta \subset \mathcal{T}$ is a basis for the TS if all open subsets of X can be written as the union of elements of β*

Theorem 1. *Let $(X, \mathcal{T})$ be a TS, and let β be a collection of subsets of X. Then β is a basis for $\mathcal{T}$ if and only if*

- *$\beta \subset \mathcal{T}$,*
- *for each open set U in $\mathcal{T}$ and a point $p \ni U$ there is a set $V \in \beta$ such that $p \in V \subset U$*

  Proof. Let's assume that β is a basis, thus it follows that $\beta \subset \mathcal{T}$ by definition. On the other hand, let $U \in \mathcal{T}$ such that $p \in U$, because β is basis it follows that U can be written as the union of elements of β, and from this it follows that there exists $V \in \beta$ such that$p \in V \subset U$.

  Now let's β as given $\square$

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