CLASSWORK 8

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Let $(X, \mathcal{T})$ be a topological space. Show that the set of connected components of X forms a partition of X.

Proof. The definition of connected components is given by

- A connected component is a maximal connected subset.
- Each point belong to exactly one connected component.

Now, to show, that the connected components of X form a partition, we need to show that they are pairwise disjoint and their union is the whole X. Pairwise disjoint: Let C_1 and C_2 be two connected components of X, and by contradiction let's assume they have a commont point p, now let's look at $C = C_1 \cup C_2$, which is a connected subset of X, but whis contradicts the maximality of both C_1 and C_2, it follows that distinct connected components must be disjoint. Union is all X: every point in X belongs to at least one connected component, otherwise it would contradict the definition of connected components.

$\therefore$ the set of connected components forms a partition of X. $\square$

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