Numerical Techniques in Cosmology

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Tufts University

Outline

Motivation

The more tricks you have up your sleeve, the easier it is to handle stuff.

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Why should we care about numerical techniques?

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The formal way to proceed:

- Is to find criteria under which the solution exists (well posed problem), and prove that under some discretization (on desired function spaces) the approximate solution is bounded from below and it converges

Some Numerical Techniques in Cosmology

We can separate observables into: deterministic and stochastic

In general we can separate

$$g_{\mu\nu} = g^0_{\mu\nu}$$

arXiv:1908.00116v1(Selected Topics in Numerical Methods for Cosmology)

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Initial conditions in the universe are given in terms of initial perturbations

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1. Deterministic: background observables $\rightarrow$ ODE theory
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1. Deterministic: background observables $\rightarrow$ ODE theory
   - H_0 determination for example
2. Stochastic: inhomogeneous part $\rightarrow$ linear perturbation theory and beyond
   - CMB radiation
   - Large Scale Structure Observables: galaxy spatial correlations, galaxy cluster count, gravitational lensing, etc.
   - We want to describe the evolution of the universe from initial primordial fluctuations to the structure formation $\rightarrow$ observables we can measure

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We take the FLRW metric with small perturbations and do GR

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which lead us to

$$h_{00} = 2\phi, h_{0i} = -aD_i B, h_{ij} = 2a^2(\psi\gamma_{ij} - D_i D_j E)$$

where D_i is the covariant derivative, (ψ, ϕ, E, B) are scalar field and γ is the spatial projection of the FLRW metric.

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where D_i is the covariant derivative, (ψ, ϕ, E, B) are scalar field and γ is the spatial projection of the FLRW metric. And from here, the idea is to obtain Einstein's equations and solve them...

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Background Dynamics and Machine Learning

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Universal Approximation Theorem

Given a family of neural networks $\forall f \in \mathcal{F}$ where $\mathcal{F}$ is some function space, there exist a family of functions $\{\phi_n\}$, such that $\phi_n \to f$. We can also say that $\{\phi_n\}$ is dense in $\mathcal{F}$.

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Thus it makes sense to try to use ML with ODE's: Physics Informed Neural Networks (PINN's)

Starting with the FLRW metric

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + +sin^2\theta d\phi^2)\right],$$

and assuming the universe is a perfect fluid, we have

$$\dot{\rho} + 3H(\rho + p) = 0$$

Phys. Rev. D 107, 063523 (Cosmology-informed neural networks to solve the background dynamics of the Universe)

Models: ΛCDM, parametric dark energy, quintessence and $f(R)$ gravity

ΛCMD: the background cosmological evolution considering an T^{μ}_{ν} with only nonrelativistic matter

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$$\frac{dx}{dz} = \frac{3x}{1+z}, \quad x(z)|_{z=0} = \frac{\kappa\rho_{m,0}}{3H_0^2} = \Omega_{m,0}$$

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Parametric Dark Matter: incorporation of new component of T^{μ}_{ν}, whose equation of state is that of a fluid and a function of redshift (DM)

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$$\frac{dx}{dz} = \frac{3x}{1+z}\left(1+\omega_0+\frac{\omega_1 z}{1+z}\right), \quad x(z)|_{z=0} = \frac{\kappa\rho_{DE,0}}{3H_0^2} = 1-\Omega_{m,0}$$

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$$\frac{dx}{dN} = -3x + \frac{\sqrt{6}}{2}\lambda y^2 + \frac{3}{2}x\left(1 + x^2 - y^2\right),$$

$$\frac{dy}{dN} = -\frac{\sqrt{6}}{2}xy\lambda + \frac{3}{2}y\left(1 + x^2 - y^2\right)$$

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$f(R)$ gravity: GR modifications

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$f(R)$ gravity: GR modifications

$$\frac{dx}{dz} = \frac{1}{1+z}\left(-\Omega + 2v + x + 4y + xv + x^2\right),$$

$$\frac{dy}{dz} = -\frac{1}{1+z}\left(vx\Gamma - xy + 4y - 2vy\right),$$

$$\frac{dv}{dz} = -\frac{v}{1+z}\left(x\Gamma + 4 - 2v\right), \quad \frac{d\Omega}{dz} = \frac{\Omega}{1+z}\left(-1 + 2v + x\right),$$

$$\frac{dr}{dz} = -\frac{r\Gamma x}{1+z}$$

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- Minimize loss on batches of points until convergence

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Validation Methodology

Having trained their models, they validate with

- Cosmic Chronometers (CC)

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- MCMC to explore the parameter space of each cosmological model

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2. Successful implementation of bundle solution: NN can output solutions across a continuous landscape of parameters.
3. Parameter constraints were found to be consistent with those obtained in previous studies that used numerical solvers
4. In some cases can be more efficient than traditional numerical solvers after the initial training phase, especially with the $f(R)$ model

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Thanks!