Cosmology: Problem Set 5

In[1]:=
```
<< NumericalCalculus`;(*Load Package for Finding Limits Numerically*)
```

In[2]:=
```
vMap = ResourceFunction["ViridisColor"];
```

Problem 1

In[3]:=
```
params = {τ → 1.4*^18, Q → 1.3*^6, G → 6.7087*^-57, g → 10};
```

In[4]:=
$$H = \text{Sqrt}\left[\frac{8\,\pi\,G}{3} \times \frac{\pi\,\text{\^{}}\,2}{30}\,g\right]\frac{Q^2}{x^2};$$

In[5]:=
$$\Gamma = \frac{3060}{\tau}\,\frac{1}{x^5}\left(1 + \frac{x}{2} + \frac{x^2}{12}\right);$$

In[6]:=
$$\text{ode} = \left(D[f[x], x] == -\frac{\Gamma}{H\,x}\,(f[x] - (1 - f[x])\,\text{Exp}[-x])\right) /. \text{params}$$

Out[6]=
$$f'[x] == -\frac{3.00773\left(1 + \frac{x}{2} + \frac{x^2}{12}\right)\,(-e^{-x}\,(1 - f[x]) + f[x])}{x^4}$$

In[7]:=
```
x0 = 0.01;
xF = 100.0;
sol = NDSolveValue[{ode, f[x0] == 0.5}, f, {x, x0, xF}]
```

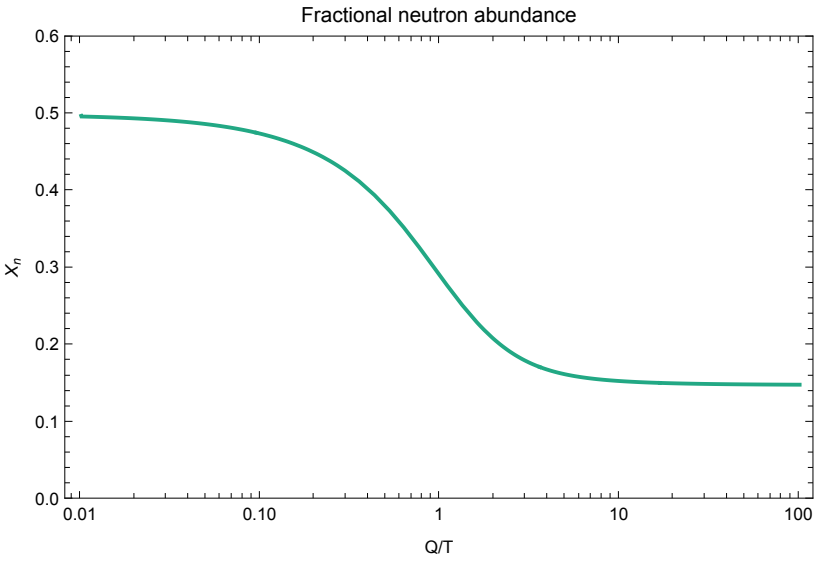
Out[9]=
```
InterpolatingFunction[ 
                       Domain: {{0.01, 100.}}
                       Output: scalar ]
```

In[10]:=
```
ϵ = 1*^-6;
solLim = NLimit[sol[x], x → (xF - ϵ), Direction → xF]
```

Out[11]=
```
0.149533
```

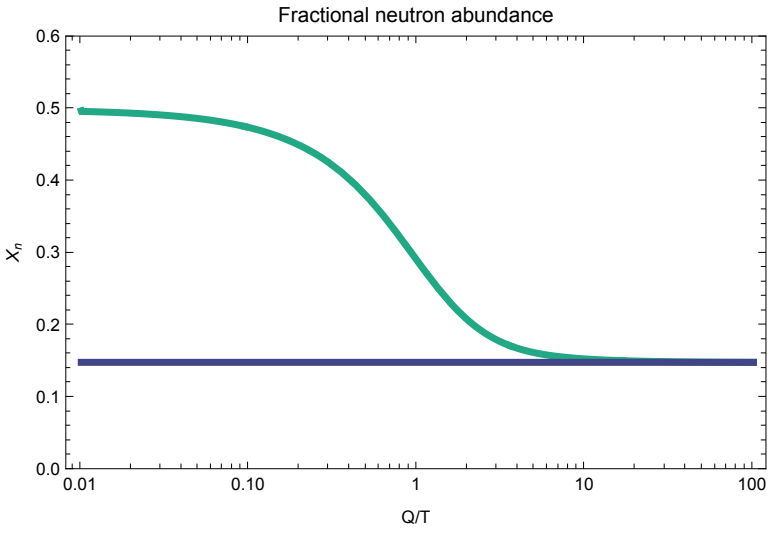
In[12]:=
```
Plot[sol[x], {x, x0, xF},
 ScalingFunctions → {"Log"},
 PlotLabel → "Fractional neutron abundance",
 PlotStyle → vMap[0.6],
 Frame → True,
 FrameLabel → {"Q/T", "Xn"},
 PlotRange → {{0, 0.6}}]
```

Out[12]=



In[13]:=
```
Plot[{sol[x], solLim}, {x, x0, xF},
 ScalingFunctions → {"Log"},
 PlotLabel → "Fractional neutron abundance",
 PlotStyle → {{Thickness[0.01], vMap[0.6]}, {Thickness[0.01], vMap[0.2]}},
 Frame → True,
 FrameLabel → {"Q/T", "Xn"},
 PlotRange → {{0, 0.6}}]
```

Out[13]=



Problem 2:

In[14]:=
```
solX = DSolve[ (1 - χ[T])/χ[T]^2 == χ0 η (2 π T/m_e)^(3/2) Exp[EI / T],
   χ[T], T, Assumptions → {m_e → PositiveReals}]
```

Out[14]=

$$\left\{\left\{\chi[T] \to -\frac{e^{-\frac{EI}{T}}\left(1-\sqrt{1+8\sqrt{2}\,e^{EI/T}\pi^{3/2}\eta\,\chi 0\left(\frac{T}{m_e}\right)^{3/2}}\right)}{4\sqrt{2}\,\pi^{3/2}\eta\,\chi 0\left(\frac{T}{m_e}\right)^{3/2}}\right\},\right.$$

$$\left.\left\{\chi[T] \to -\frac{e^{-\frac{EI}{T}}\left(1+\sqrt{1+8\sqrt{2}\,e^{EI/T}\pi^{3/2}\eta\,\chi 0\left(\frac{T}{m_e}\right)^{3/2}}\right)}{4\sqrt{2}\,\pi^{3/2}\eta\,\chi 0\left(\frac{T}{m_e}\right)^{3/2}}\right\}\right\}$$

In[15]:=
```
χSol1 = χ[T] /. solX[[1]]
```

Out[15]=

$$-\frac{e^{-\frac{EI}{T}}\left(1-\sqrt{1+8\sqrt{2}\,e^{EI/T}\pi^{3/2}\eta\,\chi 0\left(\frac{T}{m_e}\right)^{3/2}}\right)}{4\sqrt{2}\,\pi^{3/2}\eta\,\chi 0\left(\frac{T}{m_e}\right)^{3/2}}$$

In[16]:=
```
χSol1 // TeXForm (*Get String for LaTeX document*)
```

Out[16]//TeXForm=
```
-\frac{e^{-\frac{\text{EI}}{T}} \left(1-\sqrt{8 \sqrt{2} \pi
   ^{3/2} \eta  \text{$\chi $0} e^{\text{EI}/T}
   \left(\frac{T}{m_e}\right){}^{3/2}+1}\right)}{4 \sqrt{2} \pi
   ^{3/2} \eta  \text{$\chi $0}
   \left(\frac{T}{m_e}\right){}^{3/2}}
```

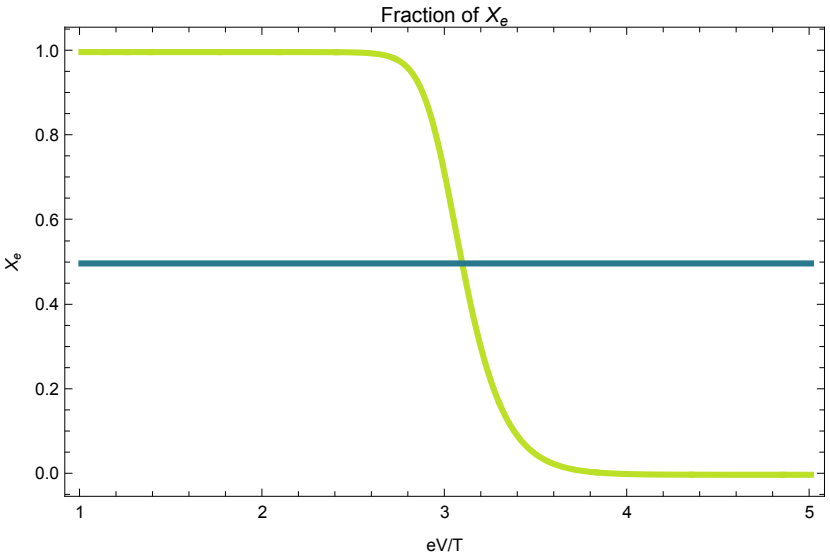
```
paramsP2 = {EI → 13.6, me → 0.511*^6, η → 6*^-10}; (*New list of parameters*)
```

In[23]:=
$$X = \frac{-1 + \text{Sqrt}[1 + 4\, f2]}{2\, f2};$$

In[43]:=
$$f2 = \frac{2\, \text{Zeta}[3]}{\pi \wedge 2}\, \eta \left(\frac{2\, \pi}{\text{me}\, x}\right)^{3/2} \text{Exp}[\text{EI}\, x];$$

In[62]:=
```
Plot[{(X /. paramsP2), 0.5}, {x, 1, 5},
 PlotLabel → "Fraction of Xe",
 PlotStyle → {{Thickness[0.008], vMap[0.9]}, {Thickness[0.008], vMap[0.4]}},
 Frame → True,
 FrameLabel → {"eV/T", "Xe"}]
```

Out[62]=



In[21]:=
```
XEvaluated = X /. paramsP2;
```

In[36]:=
```
xRec = NSolve[XEvaluated == 0.5, x, Reals]
```

Out[36]=
$$\{\{x \to 2.1492 \times 10^{-12}\}, \{x \to 3.08749\}\}$$

In[42]:=
$$TRec = \left(\frac{1}{x}\right) /. xRec[[2]][[1]]$$

Out[42]=
$$0.323887$$

In[51]:=
```
aRec = (T0 / TRec) /. (T0 → 2.3*^-4)
```

Out[51]=
```
0.000710124
```

```
params3 = {H0 → 0.7*^-10, ΩM0 → 0.3, ΩR0 → 8.6*^-5 };(*Third List of Parameters*)
```

In[60]:=
```
t = (1/H0 /. params3) Integrate[(1/(x Sqrt[ΩM0/x^3 + ΩR0/x^4])) /. params3, {x, 0, aRec}]
```

Out[60]=
```
243 884.
```