Electricity and Magnetism
Tufts University
Graduate School of Arts and Sciences
Short Assignment 5

Tufts UNIVERSITY | School of Arts and Sciences

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1. What are the differences and similarities betweenx Coulomb's law and the empirical Ampere's law? Name five of each.

   (a) Similarities

      i. **Functional Form:** Both laws exhibit an inverse square dependence with distance. This means the force or field strength weakens as the square of the distance from the source increases.Therefore, for both, we have the following common feature: the force between charges in Coulomb's law and the magnetic field around a current in Ampere's law decrease as you move farther away. Which is quite amazing because they are quite different in nature.

      ii. **Vector Quantities:** Both laws are written in terms of vector quantities. Coulomb's law describes the vector force between charges, which can point towards or away depending on the charges' signs. Ampere's law relates the vector magnetic field around a current element to the direction and magnitude of the current itself.

      iii. **Superposition Principle:**before elaborate on this point I want to highlight that this principle plays a funcdamental role not only with these laws, but in all the physics, it's a powerful and quite useful principle. Having said that, both laws follow this principle, which mans that the total force or magnetic field due to multiple sources can be calculated by simply summing the individual contributions from each source, and the same follow for the electric fields.

      iv. **Conservation Laws:** again, conservation laws are a huge thing in the physics realm, and in it turns out that both laws are related to fundamental conservation laws in physics. Coulomb's law is closely re;ated to the conservation of charge, which states that the total amount of electric charge in an isolated system remains constant. Ampere's law, along with Faraday's law, contributes to the principle of electromagnetic field energy conservation, which states that the total energy of the electromagnetic field in a closed system remains constant.

      v. **Mathematical Framework:** Both laws can be expressed within the framework of Maxwell's equations, the fundamental set of equations describing electromagnetism. Coulomb's law contributes to

Gauss's law for electricity, which relates the electric field to the distribution of charges. Ampere's law, along with Maxwell's addition, forms the Ampere-Maxwell law, which relates the magnetic field to both currents and changing electric fields.

(b) Differences

i. Nature of Interaction:

A. Coulomb's law: Describes the electrostatic force between electric charges, which can be attractive (opposite charges) or repulsive (like charges). This force acts directly between charges, regardless of their motion.

B. Ampere's law: Describes the magnetic field generated by moving charges (current). This magnetic field doesn't directly affect the charges themselves but interacts with other moving charges or magnetic materials.

ii. 2. Source of Field:

A. Coulomb's law: Static charges are the source of the electric field. The field exists even if the charges are not moving.

B. Ampere's law: It's the flow of charges (current) that generates the magnetic field. A constant flow of charges is necessary to maintain the field.

iii. 3. Field Generation and Propagation:

A. Coulomb's law: The electric field generated by a charge is instantaneous and propagates at the speed of light.

B. Ampere's law: The magnetic field generated by a current element takes time to fully develop and propagates at the speed of light. This delay is due to the finite speed of interaction between moving charges.

iv. 4. Dependence on Medium:

A. Coulomb's law: The force between charges depends only on the permittivity of the medium ϵ(e.g., vacuum, air, water).

B. Ampere's law: The magnetic field generated by a current depends on both the permittivity and permeability of the medium. This introduces additional factors like magnetic susceptibility and material properties.

v. 5. Relationship to Maxwell's Equations:

A. Coulomb's law: Directly contributes to Gauss's law for electricity within Maxwell's equations.

B. Ampere's law: Requires Maxwell's addition to account for the changing electric field contribution to the magnetic field, forming the Ampere-Maxwell law.

2. Compare five of the advantages of the concepts of electric potential V with those of magnetic potential $\mathbf{A}$. Which advantages are possessed by both, and which ones only apply to electric potential?

   (a) Advantages

      i. Simplified Calculations: Both potentials can simplify complex calculations involving forces and fields. Instead of directly calculating the electric or magnetic field, we can use potentials to obtain the fields with simpler mathematical operations like differentiation. This is especially beneficial for problems with complex geometries or intricate source distributions.

      ii. Visualization Tool: Potential functions offer a convenient way to visualize the spatial distribution of electric or magnetic charges/currents. The potential maps allow us to identify regions of high and low intensity, understand field lines and their relationships, and gain a qualitative understanding of the system's behavior.

      iii. Energy and Work Interpretation: Both potentials directly relate to energy and work in their respective domains. For electric potential, it represents the potential energy per unit charge, and the change in potential gives the work done by the electric field on a charge moving between two points. Similarly, the magnetic potential relates to the work done by the magnetic field on a unit magnetic dipole moving along a path.

      iv. Superposition Principle: Both potentials follow the superposition principle. This means the potential due to multiple sources can be obtained by simply adding the individual potentials. This simplifies calculations for problems with multiple charges or currents and allows for easy analysis of combined effects.

      v. Connection to Other Potentials: Both electric and magnetic potentials are part of a larger framework of scalar and vector potentials in physics. They connect to other fields of physics, like quantum mehcanics.

   (b) Unique advantages

      i. Electric Potential:

         A. Direct Relationship to Force: The electric field is directly derivable from the electric potential through taking the gradient. And with this he have an immediate calculation of the force acting on a charge at any point in the potential field.

         B. Scalar Quantity: As a scalar quantity, electric potential is easier to handle mathematically compared to the vector magnetic potential. This simplifies calculations, analysis of symmetries, and boundary conditions. In physics the concept of scalar potentital is used in a wide variety of fields.

         C. Existence in Field-Free Regions: Electric potential is well-defined even in regions where there is no electric field, like inside a conductor. This allows for convenient analysis of potential distributions and energy relationships even in field-free zones.

      ii. Magnetic Potential:

A. Gauge Freedom: Unlike electric potential, the magnetic potential is not unique. It can be shifted by a constant value without affecting the magnetic field. This "gauge freedom" offers additional flexibility in choosing a convenient potential for specific problems.

B. Easier to Calculate in Certain Cases: For some situations, like infinitely long straight wires or solenoids, the magnetic potential can be directly calculated with simpler expressions compared to the full magnetic field calculation.

C. Connection to Quantum Mechanics: Magnetic potential plays a crucial role in quantum mechanics, particularly for describing the behavior of charged particles in magnetic fields which can be relates to the Aharonov-Bohm effect and other quantum phenomena involving magnetic interactions.

Hugo's Solutions

1. Question1, Similarites:

   - These laws have been obtained by systematic empirical studies and were induced from observational and some theoretical accounts.
   - They both started from Coulomb's experiment on magnetism.
   - They have similar empirical conditions limiting the scope of their applicability, namely: static situations at equilibrium, classical description, and applied to extended objects.
   - They can both be generalized by the same approach of defining the force as an infinitesimal quantity, expressing it in terms of a field, and deducing two differential equations using Helmholtz' theorem.
   - They both quantify the force of some charges on some others.
   - They can be measured from mechanical means (force equilibrium).
   - They both satisfy Newton's three laws of physics.
   - They both satisfy the superposition principle (they involve vectors).
   - Both infinitesimal forces are radial.
   - They both varies as the inverse of the distance square
   - Both can be attractive or repulsive.

2. Question 1, Differences:

   - They describe different phenomena: we cannot eliminate one to describe all observations only in terms of the other. Both fields are needed.
   - Different strategies and instruments must be used to measure them.
   - The fields are produced by different sources (circuits are neutral).
   - The effects of matter are different (polarization vs magnetization), due to differences in atomic structure relevant to these effects.
   - Different numerical value, and constants (e0 , µ 0) units.

- Relationships between charge/current elements are different so different sensitivity to the geometries of the systems.
- Satisfy different differential equation and boundary conditions.
- Magnetic forces does no work on charges, but electric forces does.

3. Question 2, Advantages:

   (a) It simplifies a vector problem of finding a field to a scalar problem of finding a potential. That doesn't apply to **A**.

   (b) There is a simple geometrical relationship between V and **E** : gradient and equipotential lines. No such picture relates **A** and **B**.

   (c) V can be interpreted as the potential energy per unit charge for a charge in an electric field, but such picture does apply to **A** since **B** does no work on a charge.

   (d) V is used to formulate other empirical laws, but **A** doesn't (although it will be proved useful in obtaining Neumann formula).

   (e) V is directly controlled in an experiment, but **A** is not.

4. Question 2, Advantages to both:

   (a) Outside regions where there are charges, we can solve one (three) Laplace equation(s) to find out the scalar (vector) potential in situations where we know the potential at boundaries, but not the charge (current) distribution.

   (b) The gauge invariance can be exploited to simplify problems, but most importantly to obtain the fundamental laws of E&M.

   (c) Both are needed to describe and understand the Aharonov-Bohm effects observed experimentally.

   (d) Boundary conditions can be expressed as continuity conditions for both V and **A**.