Cosmology PSET 6:

Problem 1: Dark Matter and Baryon Density Growth

In[2]:=

```
vMap = ResourceFunction["ViridisColor"];
```

In[3]:=

```
ode = D[ϵ[t], {t, 2}] + (4 / (3 t)) D[ϵ[t], t] == 0;
ϵSol = DSolveValue[ode, ϵ[t], t]
```

Out[4]=

$$-\frac{3\,\mathbb{c}_1}{t^{1/3}} + \mathbb{c}_2$$

In[5]:=

$$\text{params} = \left\{\Omega d \to \frac{5}{6},\ \Omega b \to \frac{1}{6}\right\};$$

In[6]:=

$$\text{odeD} = D[\delta_d[t], \{t, 2\}] + \frac{4}{3\,t} D[\delta_d[t], t] - \frac{2}{3\,t^2}(\Omega d\, \delta_d[t] + \Omega b\, \delta_b[t]) == 0$$

Out[6]=

$$-\frac{2\,(\Omega b\, \delta_b[t] + \Omega d\, \delta_d[t])}{3\,t^2} + \frac{4\,\delta_d{}'[t]}{3\,t} + \delta_d{}''[t] == 0$$

In[7]:=

```
odeD /. params
```

Out[7]=

$$-\frac{2\left(\frac{\delta_b[t]}{6} + \frac{5\,\delta_d[t]}{6}\right)}{3\,t^2} + \frac{4\,\delta_d{}'[t]}{3\,t} + \delta_d{}''[t] == 0$$

In[8]:=

$$\text{odeB} = D[\delta_b[t], \{t, 2\}] + \frac{4}{3\,t} D[\delta_b[t], t] - \frac{2}{3\,t^2}(\Omega d\, \delta_d[t] + \Omega b\, \delta_b[t]) == 0$$

Out[8]=

$$-\frac{2\,(\Omega b\, \delta_b[t] + \Omega d\, \delta_d[t])}{3\,t^2} + \frac{4\,\delta_b{}'[t]}{3\,t} + \delta_b{}''[t] == 0$$

In[9]:=

```
odeB /. params
```

Out[9]=

$$-\frac{2\left(\frac{\delta_b[t]}{6} + \frac{5\,\delta_d[t]}{6}\right)}{3\,t^2} + \frac{4\,\delta_b{}'[t]}{3\,t} + \delta_b{}''[t] == 0$$

In[10]:=

```
sysSolution = NDSolveValue[
  {odeD, odeB, δd[1] == 1*^-3, δb[1] == 1*^-4, δd'[1] == 0, δb'[1] == 0} /. params,
  {δd, δb}, {t, 1, 500}]
```

Out[10]=

{InterpolatingFunction[Domain: {{1., 500.}} Output: scalar],

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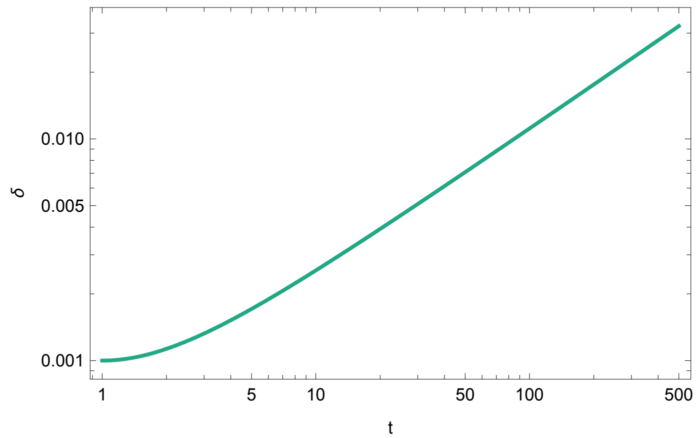
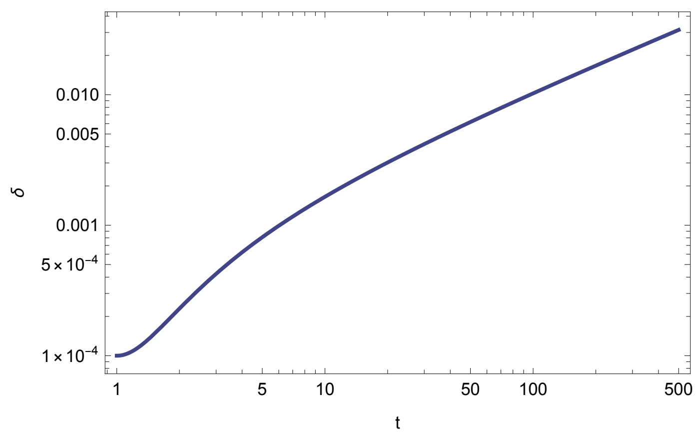
In[11]:=
```
δDPlot = Plot[sysSolution[[1]][t], {t, 1, 500}, ScalingFunctions → {"Log", "Log"},
  PlotStyle → vMap[0.6], Frame → True, FrameLabel → {"t", "δ"}]
```

Out[11]=



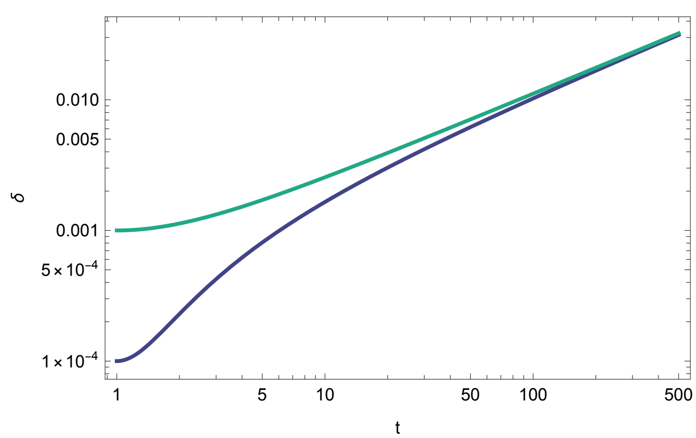
In[12]:=
```
δBPlot = Plot[sysSolution[[2]][t], {t, 1, 500}, ScalingFunctions → {"Log", "Log"},
  PlotStyle → vMap[0.2], Frame → True, FrameLabel → {"t", "δ"}]
```

Out[12]=



In[13]:=
```
Show[δBPlot , δDPlot]
```

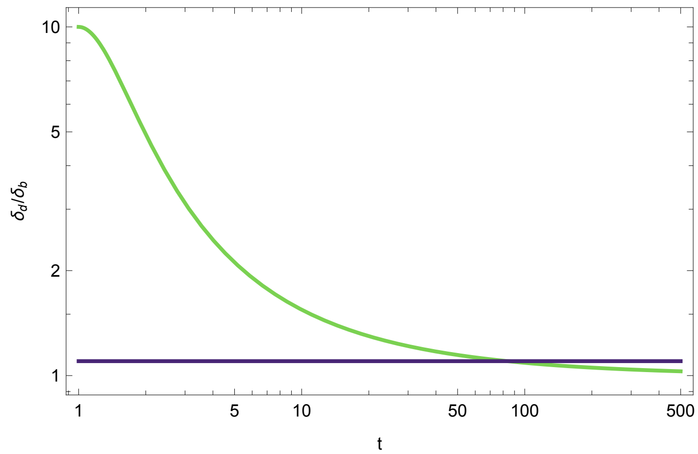
Out[13]=



In[14]:=
```
ratio = sysSolution[[1]][t] / sysSolution[[2]][t];
```

In[15]:=
```
ratioPlot = Plot[{ratio, 1.1}, {t, 1, 500},
  ScalingFunctions → {"Log", "Log"}, PlotStyle → {vMap[0.8], vMap[0.1]} ,
  PlotRange → All, Frame → True, FrameLabel → {"t", "δd/δb"}]
```

Out[15]=



In[16]:=
```
tPerturbation = (t /. FindRoot[ratio == 1.1, {t, 1, 500}]) × 380000
```

Out[16]=

3.17432×10^7

Problem 2: Matter Growth with Dark Energy

In[17]:=
```
ode = a^2 (1 - ΩM0 + ΩM0 / (a^3)) f''[a] + (3 a / 2)
  (2 - 2 ΩM0 + ΩM0 / (a^3)) f'[a]  - (3 / 2) (ΩM0 / (a^3)) f[a] == 0
```

Out[17]=

$$-\frac{3\,\Omega M0\, f[a]}{2\,a^3} + \frac{3}{2}\, a \left(2 - 2\,\Omega M0 + \frac{\Omega M0}{a^3}\right) f'[a] + a^2 \left(1 - \Omega M0 + \frac{\Omega M0}{a^3}\right) f''[a] == 0$$

In[18]:=
```
params2 = {ΩM0 → 0.3};
ai = 1 / 3600;
```

In[20]:=
```
sol = NDSolveValue[{ode /. params2, f[ai] == 2*^-3, f'[ai] == f[ai] / ai}, f, {a, ai, 5}]
```

Out[20]=

InterpolatingFunction[Domain: {{0.000278, 5. }} Output: scalar]

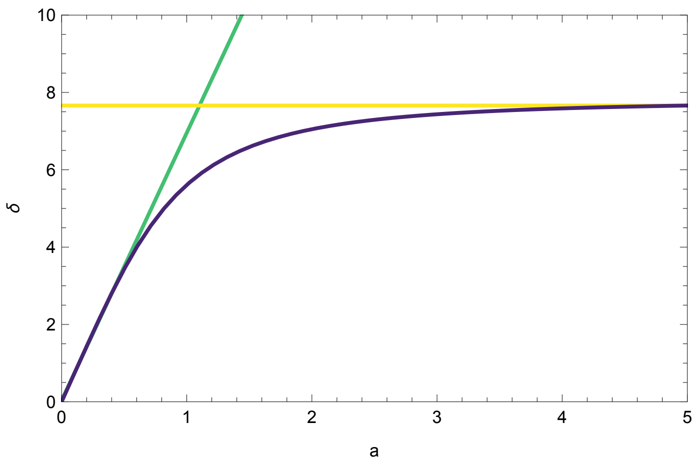
In[21]:=
```
data = Table[{a, sol[a]}, {a, ai, 0.5, 0.001}];
model = NonlinearModelFit[data, x0 + α a, {x0, α}, a]["BestFit"]
```

Out[22]=

$0.034241 + 6.922\, a$

In[23]:=
```
Plot[{model, sol[5], sol[a]}, {a, ai, 5},
 Frame → True, PlotStyle → {vMap[0.7], vMap[1], vMap[0.1]},
 PlotRange → {{0, 5}, {0, 10}}, FrameLabel → {"a", "δ"}]
```

Out[23]=



Problem 3: Scalar Field (Inflaton) in Expanding Universe

In[24]:=
```
odeInflaton = D[ϕ[t], {t, 2}] + 3 H D[ϕ[t], t] + m^2 ϕ[t] == 0;
```

In[25]:=
```
solInflaton = DSolveValue[{odeInflaton}, ϕ[t], t, Assumptions → {H > 0, m > 0}]
```

Out[25]=

$$e^{\frac{1}{2}\left(-3 H-\sqrt{9 H^2-4 m^2}\right) t} \mathbb{C}_1+e^{\frac{1}{2}\left(-3 H+\sqrt{9 H^2-4 m^2}\right) t} \mathbb{C}_2$$