

Topology, Assignment 4

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1. Using the standard topology on \mathbb{R}^2 , is the product topology $\mathbb{R} \times \mathbb{R}$ the same as the standard topology on \mathbb{R}^2 .

Proof. □

2. Prove that a topological space X is Hausdorff if and only if $\Delta = \{(x, x), x \in X\}$ is closed in the product space $X \times X$.

Proof. □

3. Let (X, \mathcal{T}) be a topological space and $Y \subset X$ equipped with the subspace topology. Prove that a subset $U \subset Y$ is open in X if and only if U is open in Y .

Proof. □

4. Prove that a set A is dense in a topological space (X, \mathcal{T}) if and only if every non-empty open set in X contains a point in A .

Proof. □

4.1 Find a topology on \mathbb{R} that is not separable.

5. Suppose that (X, \mathcal{T}) is a topological space that has a countable basis. Prove that X is separable.

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