Problem 1: Evolution of Universe with general Energy of Particles

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The Friedmann equation is given by

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho + \frac{2E}{mR^2},$$

where

$$\rho = \frac{\rho_0 R_0^3}{R^3}.$$

1. Rewrite the Friedmann equation by replacing derivatives with respect to the time variable t by derivatives with respect to the conformal time variable η defined by

$$dt = Rd\eta$$

Sol. By the chain rule, we have

$$\frac{dR}{dt} = \frac{dR}{d\eta}\frac{d\eta}{dt},$$

thus, we have

$$\frac{dR}{dt} = \frac{dR}{d\eta}\frac{1}{R},$$

and from this, the Friedmann equation is rewriten as

$$\left(\frac{dR}{d\eta}\frac{1}{R^2}\right)^2 = \frac{8\pi G}{3}\rho + \frac{2E}{mR^2},$$

thus

$$\left(\frac{dR}{d\eta}\right)^2 = \frac{8\pi GR^4}{3}\rho + \frac{2ER^4}{mR^2},$$

but ρ is a function of R, thus

$$\left(\frac{dR}{d\eta}\right)^2 = \frac{8\pi\rho_0 R_0^3 G}{3} R + \frac{2E}{m} R^2.$$

Now, let's make

$$\alpha = \frac{8\pi\rho_0 R_0^3 G}{3}, \beta = \frac{2E}{m},$$

thus, we have

$$\frac{dR}{d\eta} = \sqrt{(\alpha + \beta R) R},$$

and by separation of variables, we have

$$\int dR \frac{1}{} = \int d\eta$$