Numerical Techniques in Cosmology

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Cosmololgy Final Presentation,
Tufts University

Outline

Motivation

The more tricks you have up your sleeve, the easier it is to handle stuff.

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The formal way to proceed:

- Is to find criteria under which the solution exists (well posed problem), and prove that under some discretization (on desired function spaces) the approximate solution is bounded from below and it converges

Standard Techniques in Cosmology

We can separate cosmo observables into: deterministic and stochastic

In general we can separate

$$g_{\mu\nu} = g^{0}_{\mu\nu}$$

arXiv:1908.00116v1(Selected Topics in Numerical Methods for Cosmology)

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Initial conditions in the universe are given in terms of initial perturbations

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We can also separate numerical cosmology into two groups

1. Deterministic: Background observables (H_0 for example) $\rightarrow$ ODE theory
2. Stochastic: Inhomoenoeus part $\rightarrow$ linear perturbation theory and beyond
    - CMB radiation
    - Large Scale Structure Observables: galaxy spatial correlations, galaxy cluster count, gravitational lensing, etc.
    - We want to describe the evolution of the universe from initial primordial fluctuations to the structure formation $\rightarrow$ observables we can measure

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which lead us to

$$h_{00} = 2\phi, h_{0i} = -aD_iB, h_{ij} = 2a^2(\psi\gamma_{ij} - D_iD_jE)$$

where D_i is the covariant derivative, (ψ, ϕ, E, B) are scalar field and γ is the spatial projection of the FLRW metric.

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where D_i is the covariant derivative, (ψ, ϕ, E, B) are scalar field and γ is the spatial projection of the FLRW metric. And from here, the idea is to obtain Einstein's equations and solve them...

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Background dynamics and Machine Learning

For some function spaces, a neural network is an universal approximator

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Universal Approximation Theorem

Given a family of neural networks $\forall f \in \mathcal{F}$ where $\mathcal{F}$ is some function space, there exist a family of functions $\{\phi_n\}$, such that $\phi_n \to f$. We can also say that $\{\phi_n\}$ is dense in $\mathcal{F}$.

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Thus it makes sense to try to use ML with ODE's: Physics Informed Neural Networks

Cosmology-Informed Neural Networks

Starting with the FLRW metric

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + +sin^2\theta d\phi^2) \right],$$

and assuming the universe is a perfect fluid, we have

$$\dot{\rho} + 3H(\rho + p) = 0$$

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Models: ΛCDM, parametric dark energy, quintessence and $f(R)$ gravity

ΛCMD: the background cosmological evolution considering an T^{μ}_{ν} with only nonrelativistic matter

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$$\frac{dx}{dz} = \frac{3x}{1+z}, \;\; x(z)|_{z=0} = \frac{\kappa\rho_{m,0}}{3H_0^2} = \Omega_{m,0}$$

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Parametric Dark Matter: incorporation of new component of T^{μ}_{ν}, whose equation of state is that of a fluid and a function of redshift(DM)

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$$\frac{dx}{dz} = \frac{3x}{1+z}\left(1+\omega_0+\frac{\omega_1 z}{1+z}\right), \quad x(z)|_{z=0} = \frac{\kappa\rho_{DE,0}}{3H_0^2} = 1-\Omega_{m,0}$$

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$$\frac{dx}{dN} = -3x + \frac{\sqrt{6}}{2}\lambda y^2 + \frac{3}{2}x\left(1 + x^2 - y^2\right),$$

$$\frac{dy}{dN} = -\frac{\sqrt{6}}{2}xy\lambda + \frac{3}{2}y\left(1 + x^2 - y^2\right)$$

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$f(R)$ gravity: GR modifications

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$f(R)$ gravity: GR modifications

$$\frac{dx}{dz} = \frac{1}{1+z}\left(-\Omega + 2v + x + 4y + xv + x^2\right),$$

$$\frac{dy}{dz} = -\frac{1}{1+z}\left(vx\Gamma - xy + 4y - 2vy\right),$$

$$\frac{dv}{dz} = -\frac{v}{1+z}\left(x\Gamma + 4 - 2v\right), \quad \frac{d\Omega}{dz} = \frac{\Omega}{1+z}\left(-1 + 2v + x\right),$$

$$\frac{dr}{dz} = -\frac{r\Gamma x}{1+z}$$

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Core Methodology and Trainig Details

- Unsupervised learning: the training process does not rely on pre-computed analytical or numerical solutions of the differential equations

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- Minimize loss on batches of points until convergence

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Validation Methodology

Having trained their models, they validate with

- Cosmic Chronometers (CC)

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- MCMC to explore the parameter space of each cosmological model

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2. Successful implementation of bundle solution: NN can output solutions across a continuous landscape of parameters.
3. Parameter constraints were found to be consistent with those obtained in previous studies that used numerical solvers
4. In some cases can be more efficient than traditional numerical solvers after the initial training phase, especially with the $f(R)$ model

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Thanks!