ISING MODEL: TEACHING PLAN

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The plan of this design is to cover a 75-minute class on the Ising model, and we plan to cover fundamental concepts, examples, and encouraging interactive discussions. The following is an skeleton of the class, we're still working on the actual notes for the class nd also the "problem set".

(1) Get the Basics of the Ising Model: By the end of the class, you should understand what the Ising model is all about—how it represents spins on a grid, the interactions between them, and why it's a big deal in statistical mechanics. You'll see how it helps us study phase transitions and critical behavior in materials.
(2) Learn to Solve Ising-like Problems: We want to provide our classmates with the skills to tackle simple problems related to the Ising model. This means using formulas like Hamiltonians and partition functions to figure out stuff like magnetization and critical temperatures.
(3) Think Critically about the Model and Possible Generalizations: Finally, we want to provide our classmates with an overview or the things that we can study with the Ising Model or Spin Systems, i.e., some generalizations and cool stuff that people are doing in the research community.

(1) Introduction to Statistical Mechanics **(10 minutes, Zahra & Emmanuel)**: Briefly review the principles of statistical mechanics relevant to the Ising model, we want to present useful results and set some notation in for the rest of the class. We want to discuss the importance of models in understanding physical systems, with an special emphasis on the fact that the model is designed to capture some specific phenomena (ferromagnetism) and sometimes a full description of the system under study is not possible, at least not with just one framework. - Introduce the Ising model as a key model in statistical mechanics.

   (a) Here we want to start with a nice "warm-up" and ask questions such as:

      (i) Please write the definition of the partition function, and write some thermodynamical quantities that you can derive knowing that function.

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(ii) Then we want to give them around 3 minutes to write something on the importante of the partition function in statistical mechanics.
(iii) With this in mind we're going to proceed with the discussion.

(2) Ising Model Basics **(15 minutes, Emmanuel)**:
  (a) Define the Ising model and its historical significance: in a "pictorial" way.
  (b) Define the Hamiltonian of the Ising model: nearest-neighbor interactions and external magnetic field interaction.

(3) Two-Dimensional Ising Model for Non-interacting spins **(20 minutes, Zahra)**: Instead of focusing on the 1D version and then switch to the 2D version we decide to focus from the start on the 2D version, but first we want to start with non-interacting spins because it leads to a very nice solution.
  (a) Define "magnetic order parameter".
  (b) Obtain the parition function:
    (i) Show how we can separate the partition function as a product of partition functions for a two-level system.
    (ii) **Class activity:** compute the internal energy of the two level system.
  (c) Obtain the Entropy of the system: this part is quite important because we are going to make use of this result for the interacting case.
  (d) Calculate magnetization.

(4) Two-Dimensional Ising Model with interaction **(25 minutes, Emmanuel)**:
  (a) **Class activity:** say the class that now, they're ready to solve the full problem and give them 2-3 minutes to give it a try.
    (i) We decided to start this way because this will make our classmates start thinking in the difficulties when working with the intercting spins. Rather than say: oh look this is "difficult" we want to first them try and see if they can find where is the "big" problem.
  (b) Solution using mean field approach:
    (i) Here we're going to explore the meann field approach as an approximation which will allow us to compute the free energy.
    (ii) Explore the graph of the free energy as a function of the parameters and show them how to recognize a phase transition by looking at the minima of the free energy.
    (iii) **Class activity:** try to answer the "ok, but now, what can we compute from this?", and give them some hints on the homework.

(5) Highlitgh the importance of the Ising Model **(5 minutes, Zahra and Emmanuel)**.
   (a) We want to quickly talk about universality classes and say that the Ising model is so imporant that defines a whole universality class. We know that this topic quite advanced but we are going to present it to the class in very easy terms.

- After your great suggestions we dedided to include several activities during the class, such as easy derivations. And during some of the “live” derivations we want to ask question as: "ok, what could be the next step?", and what’s the interpretation of the computed results.
- We plan to use some diagrams which think are going to help understant the computed results: we plan to emphasize that sometimes it’s quite helfun to know how to explore computationaly some systems, that could lead you to a better understaning of the phenomena.

- Compute transition temperature, T_c for the phase transition.