FOAMS: NOTES

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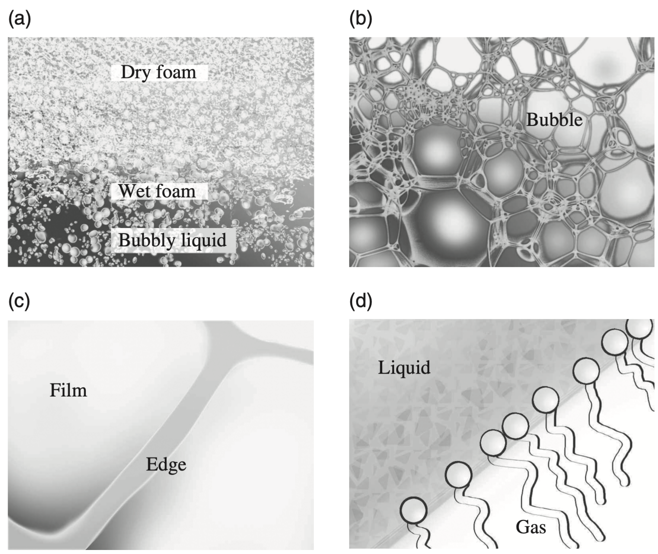
Foams are some kind of materials formed by trapping pockets of gas in a liquid or solid matrix, thus broadly speaking, a foam consists of bubbles packed together.

We can classify foams by the volume ratio of the liquid's volume and the foam's volume:

- Dry Foams: almost zero liquid in the films.
- Wet Foam: we say a foam is wet if both the Plateau borders (edges) and vertices are inflated with liquid.
- Bubbly liquid: bubbles dispersed in a liquid.



0.1. **Classification:**

- At the scale of a gas/liquid interface:
  - Surface tension and the Young–Laplace law: in general, we determine the shape from the Young–Laplace law, which states that the pressure difference between the two sides of an interface is equal to the mean curvature of the interface multiplied by its surface tension
- At the scale of a film
  - Minimal surfaces
  - From film to bubble: when a group of bubbles come together the conservation of volumes and the minimization of surface area leads to some simple laws which govern the local shape of the bubbles known as Plateau's laws.

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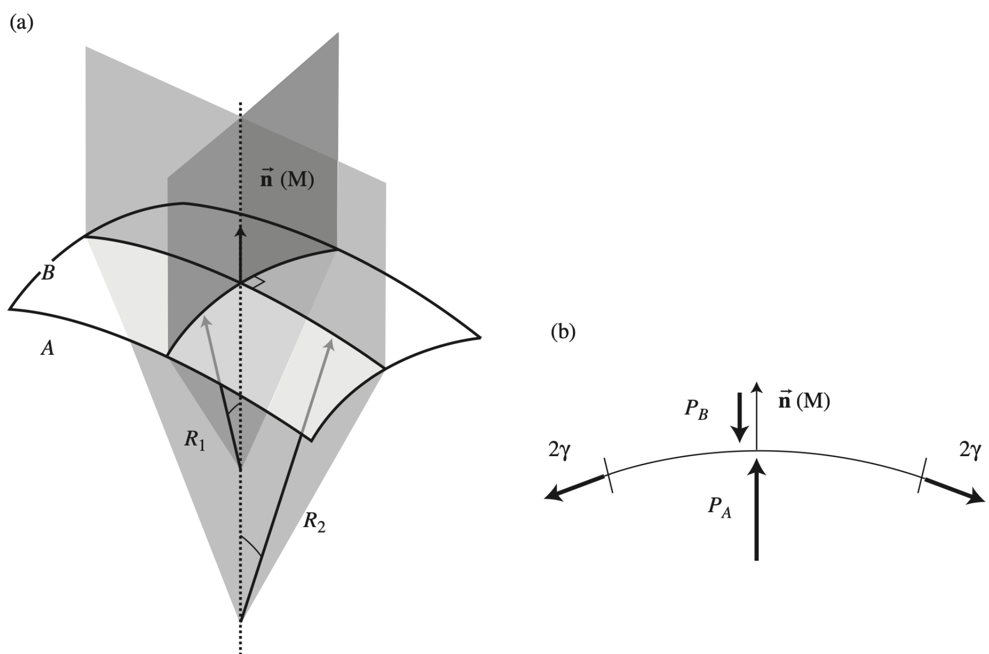
- At the scale of a bubble
  - Bubbles, films, and Plateau borders: a bubble in the centre of a dry foam is polyhedral in shape because of its neighbors. Its faces are thin films that are gently curved either because of the pressure differences between the bubbles, or simply because its perimeter does not lie in one plane. The films intersect in threes along the edges, which are liquid-carrying channels known as **Plateau borders**.
  - From bubble to foam: the amount of liquid contained in a foam is defined by the liquid volume fraction, $\phi_l = V_l/V_f$, the ratio of the volume of liquid to the total volume of the foam. This quantity is linked to the density ρ of the foam. Now, based on this, different types of structure are obtained depending on the liquid fraction
    * $\phi_l > \phi_l^*$: the bubbles are spherical and do not touch (**bubbly liquid**).
    * $0.05 \lesssim \phi_l < \phi_l^*$: the bubbles touch and take the shape of a squashed sphere at each bubble/bubble contact (**wet foam**).
    * $0.05 \lesssim \phi_l$: the bubbles are polyhedral and the Plateau borders have a negligible cross-section (**dry foam**).
    * The variable ϕ^* is associated to the transition from bubbly liquid to wet foam, and it occurs when the osmotic pressure vanishes, which corresponds to the numerical value $\phi^* = 0.3$.

0.2. **The Young-Laplace Law.** If an interface is not flat then the surface tension induces normal forces which are compensated, at equilibrium, by the pressure in each side.



$$\Delta P = P_A - P_B = \gamma H = \gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right),$$

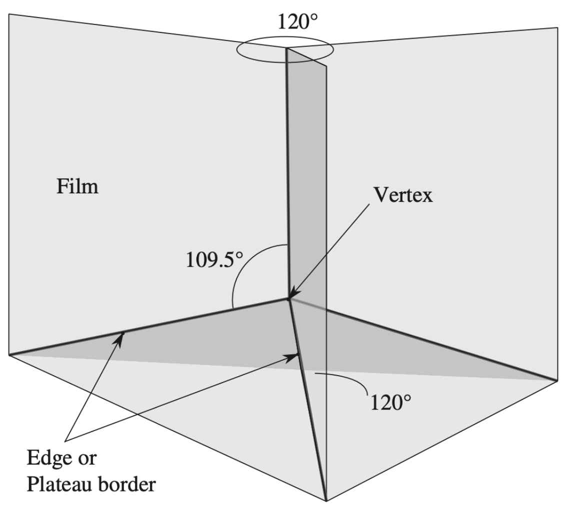
this law indicates that surface tension tends to reduce the curvature of an interface, making it more planar, and that is counterbalanced by a pressure difference, which tends to bend the interface.

0.3. **Definition of an ideal foam.** Here I list the properties that define an ideal foam.

- The foam is very dry.
- The foam is at mechanical equilibrium and is thus static.
- The foam has an energy proportional to the surface area of its bubbles.
- The foam is incompressible.

0.4. **Plateau's laws.**

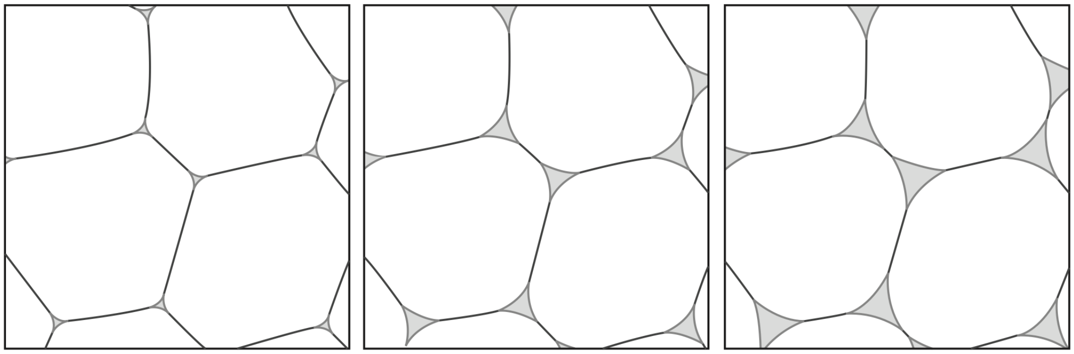
- Equilibrium of faces: the soap films are smooth and have a constant mean curvature which is determined by the Young-Laplace Law.
- Equilibrium of edges: the films always meet in threes along edges, forming angles of 120.
- Equilibrium of vertices: the edges meet at four-fold at vertices, forming angles of $\theta_a \approx 109.5$



These laws are a necessary and sufficient condition to ensure mechanical equilibrium of an ideal foam.

Now, in 2D for dry foams, the Plateau's Laws are given by:

- Edges are arcs of circles
- Edges meet in threes at vertices
- Angles at vertices are 120
- Edges meet solid walls al 90



thus, following these laws bubbles are polygons with curved edges and vertices are all triconnected.

0.5. **Time dependence of the bubble's area.** For ideally dry purely 2D foams, a bubble's area A changes at a rate that depends only on its number n of sides according to von Neumann's law

$$\frac{dA}{dt} = K_0 \, (n - 6) \, .$$

but there've been some efforts to generalize this equation to

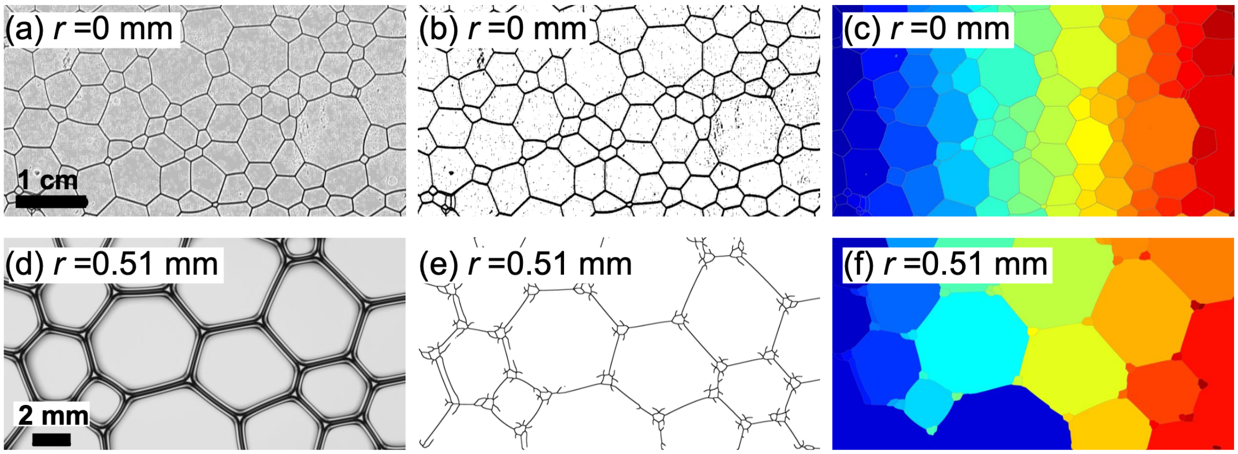
$$\frac{dA}{dt} = K_0 \left(1 - \frac{2r}{H} + \frac{\pi \sqrt{rl}}{H} \right) \left[(n-6) + \frac{6nCr}{\sqrt{3\pi A}} \right] .$$

0.6. **Image Analysis for Wet Foams.** In order to analyze images of wet foams, one should take a little more care with the process the main reason is this: because the wet foam has bright vertices surrounded by dark surface Plateau borders the images do not skeletonize in a way that is representative of the foams. Therefore, we can not simply binarize, skeletonize and watershed the images of the wet foams in order determine bubble areas like we can for the dry foams, the major reason is because the wet foams have features of varying brightness that make the skeletonized images poor representations of the foam.

Here's an approach for this analysis:

- To find the vertex positions we start by binarizing, skeletonizing, and watershedding the images.
- The locations determined from the watershedding basins are close enough to the vertices that they make excellent seeding locations for the structuring elements of a Monte Carlo like reconstruction method.

In the figure below, we can appreciate the things that we've just discussed: the first row refers to a dry foam, whereas the second refers to a wet foam.



Remark 1. All figures are taken from the references and the goal of this document is just academic.

References

[Cantat] Cantat, Isabelle, Sylvie Cohen-Addad, Florence Elias, François Graner, Reinhard Höhler, Olivier Pitois, Florence Rouyer, and Arnaud Saint-Jalmes. Foams: structure and dynamics. OUP Oxford, 2013.

[Chieco] Chieco, A. T., and D. J. Durian. "Experimentally Testing a Generalized Coarsening Model for Individual Bubbles in Quasi-Two-Dimensional Wet Foams." Physical Review E 103, no. 1 (January 20, 2021): 012610. https://doi.org/10.1103/PhysRevE.103.012610.