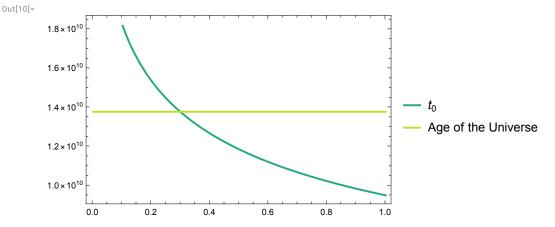
Cosmology: Problem 2

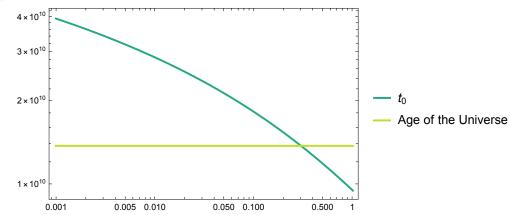
```
In[1]:= params = {H_0 \rightarrow 0.7*^{\Lambda}-10, \Omega_V \rightarrow (1-\Omega_M)};
          ageUniverse = 13.8*^9;
 In[3]:= ClearAll [MPLColorMap];
          << "http://pastebin.com/raw/pFsb4ZBS";
          viridis = MPLColorMap ["Viridis"][0.6];
          viridis1 = MPLColorMap ["Viridis"][0.9];
In[7]:= \ \ t0 \ = \ \frac{1}{H_0} \ Integrate \Big[ \frac{1}{a \ \mathsf{Sqrt}\Big[\frac{\Omega_M}{a^3} + \Omega_V\Big]} \ , \ \{a,\,0,\,1\} \ , \ \mathsf{Assumptions} \ \rightarrow \{\Omega_M > 0 \ , \ \Omega_V > 0\} \Big]
\text{Out[7]=} \quad \frac{\text{2 ArcSinh}\Big[\;\sqrt{\frac{\Omega_{\text{V}}}{\Omega_{\text{M}}}}\;\Big]}{\text{3 H}_{\text{0}}\;\sqrt{\Omega_{\text{V}}}}
 In[8]:= (*Normal Plot*)
          fig1 = Plot[t0 /. params, \{\Omega_M, 0, 1\}, Frame \rightarrow True,
               PlotStyle \rightarrow \{viridis, Thick\}, FrameLabel \rightarrow \{"\Omega_M", "t_0"\}, PlotLegends \rightarrow \{"t_0"\}]
               2.2 \times 10^{10}
               2.0 \times 10^{10}
               1.8 \times 10^{10}
          1.6 × 10<sup>10</sup>
Out[8]=
                                                                                                                  - t<sub>0</sub>
               1.4 \times 10^{10}
               1.2 \times 10^{10}
               1.0 \times 10^{10}
                                                                                        0.8
                          0.0
                                         0.2
                                                                        0.6
                                                                \Omega_M
```

```
In[9]:= (*Log-Log Plot*)
        fig2 = Plot[t0 /. params, \{\Omega_M, 0, 1\},
           ScalingFunctions → {"Log", "Log"},
           Frame → True,
           PlotStyle → {viridis, Thick},
            FrameLabel \rightarrow \{"\Omega_M", "t_0"\},\
           PlotLegends \rightarrow {"t<sub>0</sub>"}]
           4 \times 10^{10}
           3 \times 10^{10}
        _ 2×10<sup>10</sup>
Out[9]=
                                                                                          t_0
           1 \times 10^{10}
                 0.001
                                0.005 0.010
                                                    0.050 0.100
                                                                        0.500
                                                 \Omega_M
```

ln[10]:= (*Evolution of t_{θ} and the age of the universe*) fig3 = Plot[$\{t0 /. params, ageUniverse\}, \{\Omega_M, 0, 1\}, PlotStyle \rightarrow \{viridis, viridis1\},$ Frame \rightarrow True, PlotLegends \rightarrow {"t₀", "Age of the Universe"}]



```
ln[11]:= fig4 = Plot[{t0 /. params, ageUniverse}, {\Omega_M, 0, 1},
          PlotStyle → {viridis, viridis1},
          Frame → True,
          ScalingFunctions → {"Log", "Log"},
          {\tt PlotLegends} \rightarrow \{ {\tt "t_0", "Age of the Universe"} \} ]
Out[11]=
```



ln[12]:= sol = NSolve[(t0 /. params /. $\Omega_M \rightarrow x$) - ageUniverse == 0, x][[1]] Out[12]= $\{x \to 0.297893\}$