Aggregation of production functions and optimal environmental regulation

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Introduction

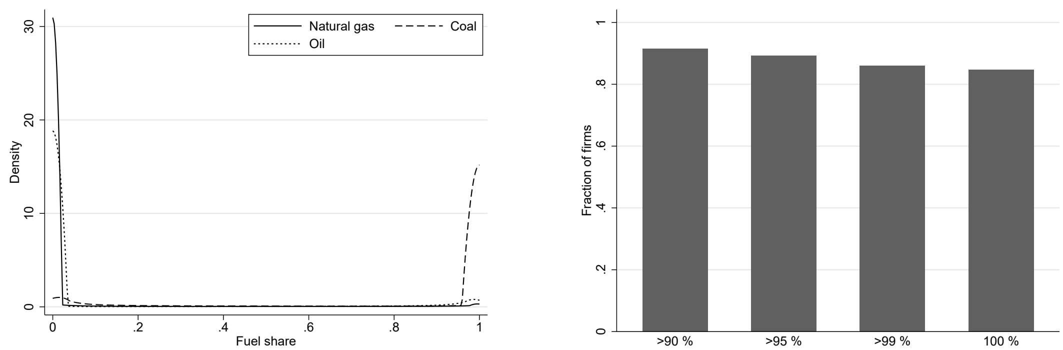
Recent papers in the optimal carbon tax literature (most notably Golosov et al. 2014) find that a Pigouvian carbon tax on fossil fuels (where the tax rate equals the marginal externality damages of pollution) is optimal and leads to the planner's allocation even in a modern model featuring elaborate production structure, dynamic management of nonrenewable resources and general equilibrium. However, one important assumption is made on the existence of an aggregate CES production function with a finite elasticity of substitution between fossil fuels. One of the reasons why this is assumed is because this line of research investigates an environmental externality which affects the entire world, and micro data from all countries is difficult to find and compile in a single database to deal with heterogeneity. This may not be a problem if plants and firms themselves use a CES or a similar homothetic production functions which are known to aggregate quite nicely.

However, these production functions always imply an interior solution in each input, and I provide consistent evidence across multiple data-sets that not only do plants only switch between fuels in an all or nothing and permanent fashion, they almost never mix between fossil fuels at a given time[1]. This is intuitive if one considers fossil fuels as perfect substitutes in providing energy, which is the actual input needed in production. In this context, a limiting CES production function where the elasticity of substitution between fossil fuels goes to infinity with fixed fuel switching costs may be a more appropriate framework to study aggregation and the optimal structure of environmental taxes.

In this proposal I show that relaxing these canonical assumptions is not innocuous. Indeed, I use an example with two fuels to show that a typical pigouvian carbon tax where the relative tax rate reflects the relative pollution intensity of each fuel is not always optimal. This is because perfect substitutability and fix switching costs imply two margins in firms' input choices: the choice of which fuel to use at the extensive margin and the quantity of fuel to use at the intensive margin. In contexts where the price of the dirtier fuel is much lower than the price of the cleaner fuel, a typical carbon tax won't raise the relative price of dirty fuel

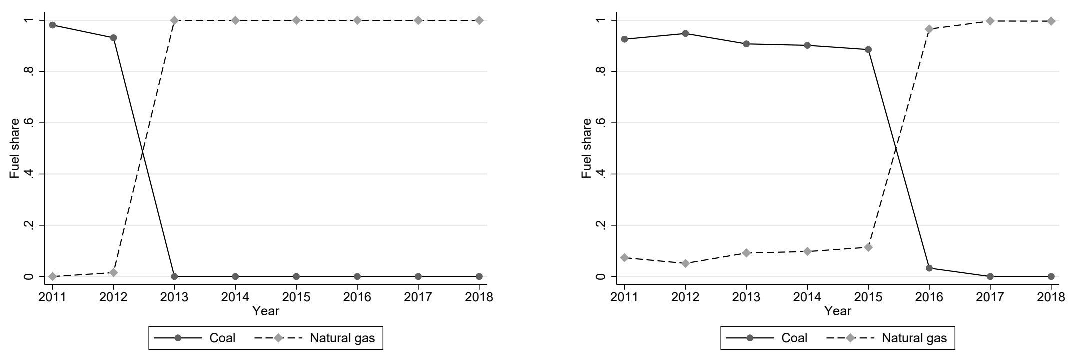
[1] I have cross-checked these patterns across multiple plant-level datasets (NPRI and GHGRP for Canada, GHGRP for the U.S. and ASI for India)

enough to incentivize firms to switch towards the cleaner fuel. In these situations, a larger relative tax rate on the dirty fuel, a tax that only affects the dirty fuel or even a tax on the dirty fuel coupled with a subsidy on the cleaner fuel can increase both aggregate output and profits while achieving the same emission target as the pigouvian carbon tax. This issue does not appear when fuels are imperfect substitutes because any input price variation always features extensive margin variation. I also provide some preliminary evidence on the existence of an aggregate production function which takes aggregate fuel quantities as inputs, such that one can still use aggregate data and proceed with the analysis of optimal carbon taxation in a more elaborate framework in the likes of Golosov et. al (2014).



(a) Kernel density estimate of fuel shares (b) Proportion of single fuel use above given threshold

Figure 1: Evidence of single fuel use (Indian Survey of Industries - ASI)



(a) Example 1 (b) Example 2

Figure 2: Prototypical fuel switching patterns - U.S. plants (GHGRP)

The problem

Typically, a production function that takes capital, labor and energy with hicks-neutral productivity is specified, where i indexes firms:

$$Y_i = A_i f(k_i, l_i, e_i)$$

Energy is then a CES composite of J different energy sources with elasticity of substitution σ:

$$e_i = \Big(\sum_{j=1}^{J} \lambda_j e_{ij}^{(\sigma-1)/\sigma}\Big)^{\sigma/(\sigma-1)}$$

Perfect substitution between energy inputs implies that energy becomes linear in all inputs:

$$\lim_{\sigma\to\infty} e_i = \sum_{j=1}^{J} \lambda_j e_{ij}$$

To expose my argument, I will make some simplifying assumption. First, I will assume away labor and capital such that firm's output is only produced with energy. Second, I will assume that all energy inputs share the same efficiency ($\lambda_j = 1 \forall j$). Third, I will assume that firms face a fixed energy input adjustment cost κ_{ij} which will take the form:

$$\kappa_{ij} = \begin{cases} \kappa & \text{if firm was not initially using input j} \\ 0 & \text{Otherwise} \end{cases}$$

Underlying this fixed cost structure is a unit mass of firms with an initial distribution across energy inputs such that $N_j \in (0,1)$ represents the fraction of firms who are initially using input j, where $\sum_j N_j = 1$. This specification allows me to abstract from dynamic implications. Lastly, I assume a perfectly competitive environment with output price normalized to one and where the production function features decreasing returns to scale in energy, $\alpha \in (0,1)$, which allows me to pin down firms' scale decisions[2]. A firm's problem is then to maximize profits:

$$\pi_i = \max_{e_{ij}} \Big\{ \underbrace{\Big(\sum_j e_{ij}\Big)}_{e_i}^{\alpha} - \sum_j (p_j e_{ij} + \mathbb{1}(e_{ij} > 0)\kappa_j) \Big\}$$

Unless prices inclusive of fixed costs are the same across inputs, a firm will always choose a corner solution in all but one input. If it chooses an interior solution for input j, the intensive

[2]Otherwise, one would typically specific imperfect competition to pin down firms' scale.

margin is given by:

$$e^*_{ij} = \left(\frac{\alpha A_i}{p_j}\right)^{\frac{1}{1-\alpha}}$$

The firm will then choose input j if it yields greater profits than any other inputs. Let $\Gamma(\alpha) = \alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)}$. A firm then chooses input j if $\forall k \neq j$

$$\begin{aligned} &\pi_i(e^*_{ij}) > \pi_i(e^*_{ik}) \\ \iff &\left(\frac{A_i}{p_j}\right)^{1/(1-\alpha)} p_j\Gamma(\alpha) - \kappa_{ij} > \left(\frac{A_i}{p_k}\right)^{1/(1-\alpha)} p_k\Gamma(\alpha) - \kappa_{ik} \end{aligned}$$

This selection into different energy inputs can be rewritten as a productivity threshold:

$$A_i \begin{cases} > \left[\frac{\kappa_{ij}-\kappa_{ik}}{\Gamma(\alpha)} \frac{1}{p_j^{\alpha/(\alpha-1)} - p_k^{\alpha/(\alpha-1)}}\right]^{1-\alpha} \equiv \Omega_{kj} & \text{if } p_k > p_j \\ < \Omega_{jk} & \text{if } p_j > p_k \end{cases} \tag{1}$$

To get a bit of intuition, imagine there is only two inputs, and a firm is considering paying the fixed cost κ to switch to from input k to input j. There are two cases to consider which will have important implications for the optimal environmental tax:

1. If $p_j > p_k$, then $\left(p_j^{\alpha/(\alpha-1)} - p_k^{\alpha/(\alpha-1)}\right) < 0$ and no matter how productive the firm is, equation (1) will never be satisfied because $\Omega_{kj} < 0$ and firms will never switch. This is intuitive because it is more expensive for the firm to use the new input, and it needs to pay a fixed switching cost on top of that, so it will always be better to use the current input (k).

2. If $p_k > p_j$, then $\left(p_j^{\alpha/(\alpha-1)} - p_k^{\alpha/(\alpha-1)}\right) > 0$ and there is a trade-off between paying the switching cost and getting a cheaper input. Intuitively, there will be a productivity cutoff such that firms above that threshold switch because they benefit more from a cheaper input since each input is used more efficiently whereas the adoption cost has the same effect on profits independently of productivity.

The first case is important because if the net-of-tax price of a cleaner fuel (say natural gas) is still higher than the net-of-tax price of dirtier fuel (say coal), then no firm will switch to natural gas. This is particularly relevant in developing countries like India where coal is more than three times cheaper than natural gas:

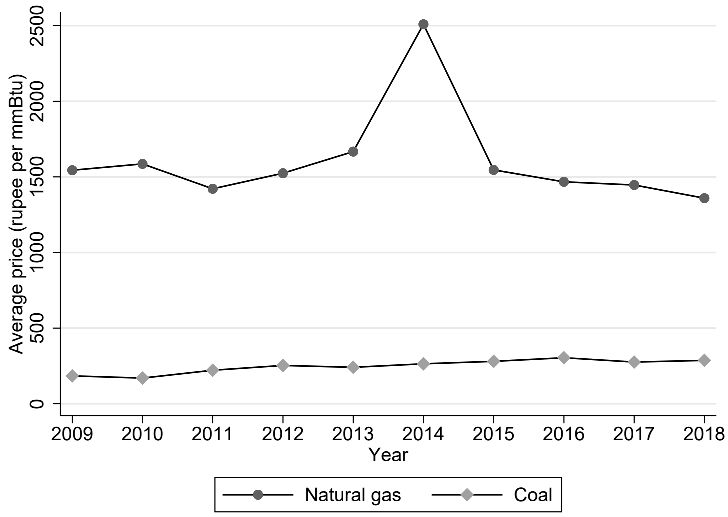


Figure 3: Average price (rupee per mmBtu) of coal and natural gas - ASI plants

In such a context, a typical carbon tax would raise the price of coal relative to natural gas proportionally to the relative emission factor of coal, but not necessarily enough to make natural gas cheaper. To see this, let the emission factors of coal and natural gas (γ_c and γ_n) map units of coal and natural gas (in mmBtu) to ghg emissions:

$$ghg(e_c, e_n) = \gamma_c e_c + \gamma_n e_n$$

Then, a carbon tax would satisfy

$$\tau_n = \tau$$
$$\tau_c = \frac{\gamma_c}{\gamma_n}\tau$$

Since coal is roughly twice as polluting as natural gas (EPA), then $\frac{\gamma_c}{\gamma_n} \approx 2$. If coal is initially 3 times more expensive than natural gas, then there is no carbon tax rate which can make natural gas cheaper. if

$$p_n = 3p_c$$

Then, $\nexists \tau$ such that

$$\begin{aligned}
(1+2\tau)p_c &> (1+\tau)p_n \\
(1+2\tau)p_c &> (1+\tau)3p_c \\
1+2\tau &> 3+3\tau \\
&\Rightarrow\Leftarrow
\end{aligned}$$

In the next section, I discuss the aggregation of production in the case of two inputs only, and I use these results to investigate the welfare implications of different tax structures using a numerical example. I show that in cases where a carbon tax does not induce any firm to switch towards the cleaner input, other tax structures can be welfare improving.

Aggregation with two inputs

In the appendix, I provide a more formal definition of the aggregation problem and I use two canonical examples to motivate the idea. For now, the goal is to find the aggregate quantities of inputs 1 and 2 as a function of prices: $E_1^*(p_1, p_2)$ and $E_2^*(p_1, p_2)$. As a reminder, I assume a fraction of firms N_1 is initially using input 1 and a fraction of firms N_2 is initially using input 2 such that $N_1 + N_2 = 1$. I shall consider both cases ($p_1 > p_2$ and $p_2 > p_1$) separately.

Case 1: $p_1 > p_2$

When the price of input 1 is greater than the price of input 2, the selection equation (1) tells that all firms who are currently using input 2 will still be using input 2, and there will be a fraction of firms currently using input 1 who are productive enough to switch towards input 2. If I denote $N_{j,-j}$ as the fraction of firms going from input j to input -j, then

$$\begin{aligned}
N_{21} &= 0 \\
N_{22} &= N_2 \\
N_{11} &= P\Big(A \leq \Omega_{12}\Big)N_1 \\
N_{12} &= [1 - P\Big(A \leq \Omega_{12}\Big)]N_1
\end{aligned}$$

Let $G(A)$ denote the cumulative distribution function for productivity. The Aggregate quantities of both inputs are then going to be:

$$E_1^*(p_1 > p_2) = \underbrace{N_1 \int_0^{\Omega_1} \left(\frac{\alpha A}{p_1}\right)^{1/(1-\alpha)} dG(A)}_{\text{Firms who keep using input 1}}$$

$$E_2^*(p_1 > p_2) = \underbrace{N_2 \int_0^{\infty} \left(\frac{\alpha A}{p_2}\right)^{1/(1-\alpha)} dG(A)}_{\text{Firms who keep using input 2}} + \underbrace{N_1 \int_{\Omega_1}^{\infty} \left(\frac{\alpha A}{p_2}\right)^{1/(1-\alpha)} dG(A)}_{\text{Firms switching to input 2}}$$

Case 2: $p_1 < p_2$

One can derive similar expressions for the case where $p_2 < p_1$:

$$E_1^*(p_2 > p_1) = \underbrace{N_1 \int_0^{\infty} \left(\frac{\alpha A}{p_1}\right)^{1/(1-\alpha)} dG(A)}_{\text{Firms who keep using input 1}} + \underbrace{N_2 \int_{\Omega_2}^{\infty} \left(\frac{\alpha A}{p_1}\right)^{1/(1-\alpha)} dG(A)}_{\text{Firms switching to input 1}}$$

$$E_2^*(p_2 > p_1) = \underbrace{N_2 \int_0^{\Omega_2} \left(\frac{\alpha A}{p_2}\right)^{1/(1-\alpha)} dG(A)}_{\text{Firms who keep using input 2}}$$

One can see that input prices are going to influence the productivity cutoff that determines which firms switch between inputs, and this productivity cutoff will in turns determine the average quantity of each input that firms will be using. Moreover, one can see that as long as prices don't revert such that we stay within one of the two cases, aggregate input quantities are continuous function of prices. Below, I show that these aggregate quantities are still continuous function of prices as prices revert from one case to another:

Proposition 1. *At the limit where input prices are equal, both cases are equivalent and there is no discontinuity.*

Formally, I show that the following conditions hold:

$$\lim_{(p_1-p_2)\to^{+}0} \{E_1^*(p_1 > p_2), E_2^*(p_1 > p_2)\} = \lim_{(p_1-p_2)\to^{-}0} \{E_1^*(p_2 > p_1), E_2^*(p_2 > p_1)\}$$

Proof. As prices get closer and closer, there is less and less benefits from switching. Hence, at the limit, the cutoff productivity from switching goes to infinity and no one switches:

$$\lim_{(p_1-p_2)\to^+0} \Omega_1 = \lim_{(p_2-p_1)\to^+0} \Omega_2 = \infty$$

It follows for aggregate quantities are the same at the limit:

$$\lim_{(p_1-p_2)\to^+0} E_1^*(p_1 > p_2) = \lim_{(p_1-p_2)\to^-0} E_1^*(p_2 > p_1) = N_1 \int_0^\infty \left(\frac{\alpha A}{p_1}\right)^{1/(\alpha-1)} dG(A)$$
$$\lim_{(p_1-p_2)\to^+0} E_2^*(p_1 > p_2) = \lim_{(p_1-p_2)\to^-0} E_2^*(p_2 > p_1) = N_2 \int_0^\infty \left(\frac{\alpha A}{p_2}\right)^{1/(\alpha-1)} dG(A)$$

□

These aggregate input quantities can then be used to construct aggregate output and profit as function of prices.

Numerical example

The purpose of this investigation is to compare aggregate output and profit loss from different tax structures for a given ghg emission target. To present my argument I will look at two cases. The first one serves as the benchmark carbon tax such as the ones implemented in most real context. The second one will be a tax on coal only[3]. To make it intuitive, I will assume that one input corresponds to natural gas while the other corresponds to coal.

1. Carbon tax: $\tau_n = \tau, \tau_c = \frac{\gamma_c}{\gamma_n}\tau$

2. Coal tax: $\tau_n = 0, \tau_c = \tau$

Formally, the problem can be written as finding the tax rate that maximizes aggregate profits (or aggregate output) subject to a given emission target ($\bar{g}$) and a given tax structure:

[3] I have also experimented with a various other tax structures such as a tax on coal and a subsidy on natural gas which yields similar result.

$$\max_{\tau} \left\{ Y^*(p_c, p_c, \tau_n, \tau_c) - (1+\tau_n)p_n E_n^* - (1+\tau_c)p_c E_c^* \right\}$$
$$s.t.\ \gamma_n E_n^*(p_c, p_n, \tau) + \gamma_c E_c^*(p_c, p_n, \tau) = \bar{g}$$

To investigate the optimal tax structure numerically, I will do a simple simulation based on the following parameter values. The exact value of the parameters do not matter at this point because any situation where the carbon tax leads to a worse outcome contradicts the claim that carbon taxation is always optimal.

Parameter	**Value**
α	0.4
μ	0
σ	2
N_c	0.6
κ	1
γ_n	1
γ_c	2

Table 1: Parameter values

I assume that productivity is distributed log-normal such that I and use the following algorithm to find the optimal tax rate and aggregate output/profit given an emission target:

$$\ln A \sim N(\mu, \sigma^2)$$

Algorithm 1: Simulation of optimal tax rate

1 Given parameter values, input prices and initial distribution of firms, find ghg emissions and aggregate output under no tax regime

$$ghg^* = \gamma_n E_n^*(p_n, p_c) + \gamma_c E_c^*(p_n, p_c)$$

2 Set a target level of ghg emissions lower than emissions under no tax ($\overline{g} < ghg^*$)

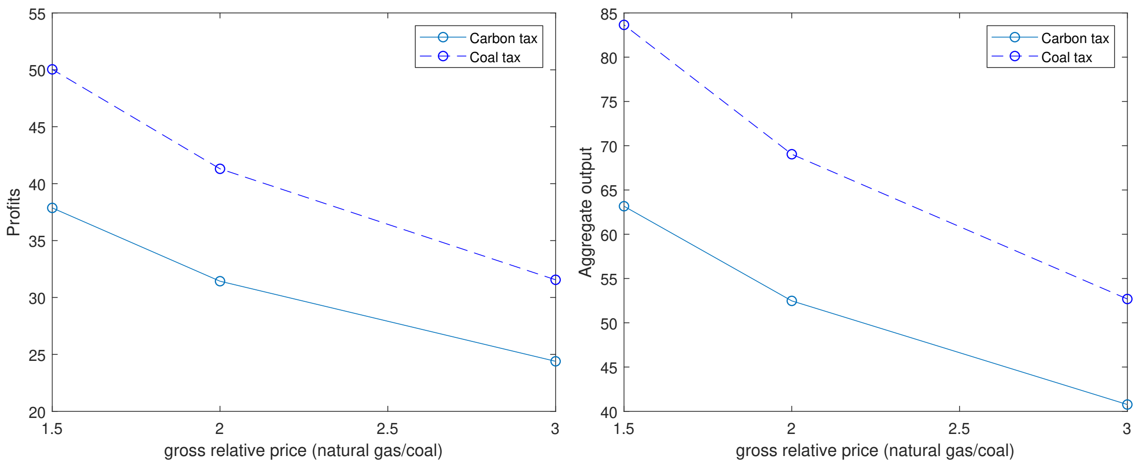
3 Given one of the four tax structures, find the implied tax rate that achieves target emissions by root-finding:

$$\overline{g} - \gamma_n E_n^*(p_n, p_c; \tau) - \gamma_c E_c^*(p_n, p_c; \tau) = 0$$

4 Find aggregate output and profit associated with this tax rate.

Result

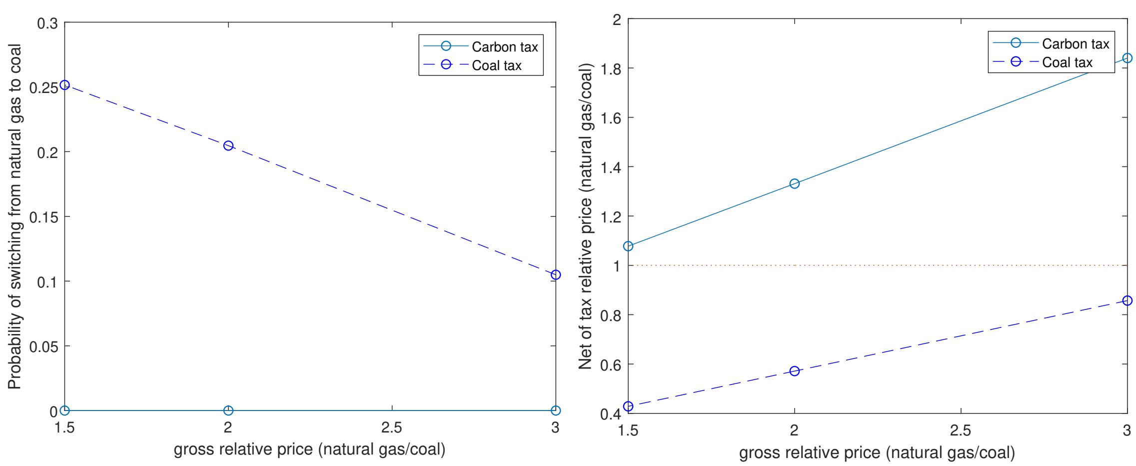
In all cases where the gross price of natural gas is greater than the gross price of coal, there are emission targets where the coal tax leads to larger output and profits, and this is because no firm switches to natural gas when facing the carbon tax.



(a) Aggregate profit

(b) Aggregate output

Figure 4: Effect of different tax structures for a common emission target on output and profit



(a) Fraction of firms who switch from coal to natural gas

(b) Net of tax relative prices

Figure 5: Effect of different tax structures for a common emission target on fuel switching and net relative prices

Appendix A The aggregation problem

Imagine a world populated with a mass of N firms indexed by their Hicks-neutral productivity A_i who produce output using two inputs, call them k_1 and k_2, using an arbitrary decreasing returns to scale production function:

$$Y_i = f(k_{1i}, k_{1i}; A_i)$$

Firms are assumed to maximize profits in a perfectly competitive market where output price is normalized to one and where p_1 and p_2 denote prices of input 1 and 2, respectively:

$$\pi_i = \max_{k_{1i}, k_{2i}} f(k_{1i}, k_{1i}; A_i) - p_1 k_{1i} - p_2 k_{2i}$$

Let $\{k^*_{1i}(p_1, p_2),\, k^*_{2i}(p_1, p_2)\} = \text{argmax}\{\pi_i\}$ be the input choices that solve this problem. The goal is to find an aggregate production function $F(K_1, K_2; A)$ such that:

$$\{K^*_1(p_1, p_2), K^*_2(p_1, p_2)\} = \text{argmax}\{F(K_1, K_2; A) - p_1 K_1 - p_2 K_2\}$$

and

$$\begin{aligned} K^*_1(p_1, p_2) &= \sum_i^N k^*_{1i}(p_1, p_2) \\ K^*_2(p_1, p_2) &= \sum_i^N k^*_{2i}(p_1, p_2) \end{aligned}$$

It is know that if $f(.)$ takes a standard form such as Cobb-Douglas or CES, then the aggregate production function will also take the Cobb-douglas or CES form, making the aggregation quite trivial:

Example 1: Cobb-Douglas

$$\pi_i = \max_{k_{1i},k_{2i}} A_i(k_{1i}^{\alpha}k_{2i}^{1-\alpha})^{\mu} - p_1k_{1i} - p_2k_{2i}$$

Where $\mu < 1$ pins down firm's scale decision. Then,

$$\begin{aligned} k_{1i}^*(p_1, p_2) &= \Big(\frac{A_i\mu}{p_1}\Big(\frac{p_1}{w}\frac{\alpha}{1-\alpha}\Big)^{(1-\alpha)\mu}\Big)^{1/(1-\mu)} \\ &= A_i^{1/(1-\mu)}\Gamma(p_1, p_2) \end{aligned}$$

It follows that the aggregate expenditure on input 1 (and input 2 by analogy) is a function Γ of prices and parameters times firm-specific productivity:

$$K_1^* = \sum_i k_{1i}^* = \sum_i \Big(A_i^{1/(1-\mu)}\Big)\Gamma(p_1, p_2)$$

If one defines $\sum_i \Big(A_i^{1/(1-\mu)}\Big) = A$ as aggregate productivity, then the following Cobb-Douglas aggregate production function yields K_1^* and K_2^* as a result of the aggregate profit maximization:

$$F(K_1, K_2; A) = A^{1-\mu}(K_1^{\alpha}K_2^{1-\alpha})^{\mu}$$

Example 2 (CES)

$$\pi_i = \max_{k_{1i},k_{2i}} A_i\Big[\alpha k_{1i}^{\rho} + (1-\alpha)k_{2i}^{\rho}\Big]^{\mu/\rho} - p_1k_{1i} - p_2k_{2i}$$

In a similar spirit as example one, it is easy to see that the optimal choice of each inputs is also a function Γ of prices and parameters times the firm-specific productivity:

$$k_{1i}^* = A_i^{1/(1-\mu)}\Gamma(p_1, p_2)$$

Then, defining aggregate productivity in the same way as before, the corresponding aggregate production function will also take the CES form:

$$Y = A^{1-\mu}\Big[\alpha K_1^{\rho} + (1-\alpha)K_2^{\rho}\Big]^{\mu/\rho}$$