# Optimal Carbon tax and Energy Substitution

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May 2021

#### Abstract

In this paper I expose novel empirical facts on plants' use of fossil fuels that suggest non-standard production processes. These facts relate to plants mixing between fuels and switching across fuels over time, and are reminiscent of no complementarity and perfect substitutability but costly switching. I then develop a stylized model with two fuels and a climate externality that feature these facts to show that this granular structure does not aggregate to a standard production function. In this context, a typical pigouvian carbon tax where the relative tax rate on each fuel is equal to its relative emission intensity is suboptimal. Instead, a government can do better with a relative tax rate that is higher on the most polluting fuel to take advantage of costly fuel switching possibilities.

## 1 introduction

CO2 and related greenhouse gases (GHG) emissions that contribute to climate variation is a problem of interest to economists because it creates an externality. Standard theory going back to Pigou (1920) proposes market intervention through a carbon tax that reflects marginal externality damages. However, putting a price directly on emissions would require putting a monitoring device on each polluting capital which is too costly to be feasible. Policymakers around the world have instead implemented carbon taxes by taxing fossil fuels because they are the main polluting inputs that contribute to CO2e emissions, where relative tax rates reflects relative CO2e contribution of each fuel.<sup>1</sup>

In this context, recent literature suggests that a uniform carbon tax implemented on fossil fuels is the optimal solution to solve the climate externality using arguments that include an integrated assessment of the global economy (Golosov et al. 2014, 2020). While this literature's main contribution is to analytically characterize the externality from the emission of greenhouse gases and provide a solution for the optimal *level* of a global carbon tax, it also implies that the relative tax rate on different fossil fuels should be equal to their relative emission intensity. For the last point to hold, it is required that the economy feature a standard aggregate production function where aggregate energy consumption is a composite of different fuels with possibilities to substitute among them.

However, in this paper I document novel empirical evidence on plant's usage of fossil fuels

 $<sup>^1</sup>$ For example, 1 mmBtu of Coal produces on average 99 kg of CO2e versus 53 kg of CO2e for 1 mmBtu of Natural Gas.

suggestive of non-standard production processes which raises doubt on canonical aggregate technology assumptions and requisite a more careful look at the aggregation process. I provide this evidence from Indian plants which are known to consume large amounts of coal relative to natural gas in comparison to plants in the same industries located in developed countries. More specifically, I observe that plants rarely mix between fossil fuels in a given period but often switch between fuels over time in a way that is systematically related to their size/productivity. Overall, I picture a story where there is little to no complementarity between fossil fuels and high but costly substitutability at the plant level.

I then develop a stylized heterogeneous firms model that rationalizes these facts in a simple two fuels framework where firms are exogenously endowed a technology but can pay a fixed cost to switch between technologies and use a different fuel, where fuel consumption contributes to an externality through GHG emissions. To this end, I solve an optimal tax problem in the Diamond and Mirrlees (1971) tradition and show two important results by comparing the optimal tax when I allow for fuel switching and when I don't:

- 1. The no-switching model is isomorphic to a model with an aggregate constant elasticity of substitution (CES) production function that takes aggregate fuel quantities as inputs, and the optimal tax is a carbon tax where the relative tax rate reflects relative contribution of each fuel to CO2e.
- 2. When I introduce fuel switching in the model, the optimal tax is not a carbon tax anymore; rather it features a greater relative tax on the most polluting fuel than in the previous case. This is because a marginal increase in the price of the most polluting fuel decreases the productivity threshold required for firms to change technology, thus incentivizing a larger portion of firms to switch towards a cleaner fuel, discretely reducing their emissions without reducing output. This means that there is a larger marginal gain from increasing the relative price of the most dirty fuel than the ratio of CO2e emission intensity of each fuel.

Since my result is so far qualitative, I then plan to develop a more serious empirical model that can give realistic policy prescriptions to quantify welfare gains. Many interesting counterfactuals can be considered in a quantitative setting. First, I can compare the optimal tax with a pigouvian carbon tax and a command & control approach which bans the use of highly polluting fuels like coal. Second, I can compare the optimal policy when allowing for perfect price discrimination with a more realistic uniform tax rate on each fuel and a tax rate that partially discriminates based on observable such as plants' location. Third, I can align the optimal tax rate with existing plans to increase the natural gas distribution network through high pressure pipelines that have a wider coverage of the Indian territory.

# 2 Empirical Evidence

#### 2.1 Data

The dataset used to document novel facts on fossil fuels is the Indian Survey of industries (ASI), which features all indian manufacturing plants with more than 100 workers and a sample of plants with less than 100 workers. I look at plants operating between 2009 and 2018 in one of four manufacturing industries: steel & iron, cement, aluminium and pulp & paper. I specifically target these industries because they are very energy intensive (Aldy and Pizer 2015, IPCC 2018), produce fairly homogeneous outputs which allows for greater

comparability of plants within each industries, and they use fossil fuels in specific ways. Indeed, plants in these industries use fossil fuels to generate high amounts of heat required to transform crude resources into refined products, and different fuels can be used in this combustion process. For example, steel plants require heat for casting metal which is done in large furnaces. Traditionally, such furnaces would be powered by coal but recent Electric Arc furnaces are made to use a combination of natural gas and recycled materials, making them less polluting and more energy efficient (Cavaliere 2019). However, I plan to generalize these findings to all manufacturing industries in the future.

Moreover, I specifically target Indian plants because Coal is much cheaper than natural gas in India and both aggregate spending & quantity shares are heavily skewed towards coal. At the same time, plants in the same industries located in a developed economy like Canada massively use natural gas instead of coal, making fuel switching policy-relevant in the indian context.<sup>2</sup>

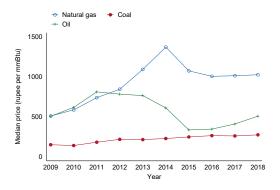


Figure 1: Median Fossil Fuel Prices (rupee/mmBtu) - ASI plants

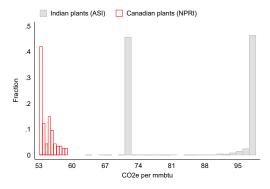


Figure 2: Energy intensity of 1 mmBtu of energy - Cement manufacturing

<sup>&</sup>lt;sup>2</sup>In Figure 2, plants in Canada are from the National Pollutant Inventory Survey (NPRI), and the three clusters in the distribution represent Natural Gas, Oil and Coal, respectively.

# 2.2 Stylized facts

I then document four stylized facts on how indian plants use fossil fuels. One of the stylized fact is on single fuel consumption/mixing and the other three relate to plants switching across fuels between years. These facts are meant to shift the story on fossil fuel consumption from a story about technological complementarity to a story about price and potentially access.

#### 2.2.1 70% of plants use a single fuel in a given year

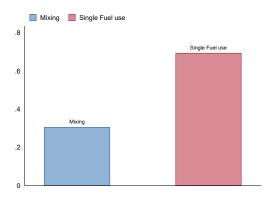


Figure 3: Proportion of single fuel usage vs. mixing

## 2.2.2 Half of the plants switch between fuels at least once

To document evidence on fuel switching, I define switching between two years in four distinct categories:

- 1. Single to Single (other fuel)
- 2. Single to Mixing
- 3. Mixing to single
- 4. Mixing to Mixing (other mix)

It is important to note that all these categories are related to some form of corner solution/discrete choice and would not be captured by standard production functions. I then show that 55% of plants switching in one of those categories at least once in their lifetime:

	Never Switch	Switch	Total
Number	526	643	1,169
Fraction	0.45	0.55	1

Table 1: Proportion of unique plants who switch at least once

Moreover, conditional on switching, plants tend to switch one average two times. Investigating this further reveals that roughly 30% of plants in the sample switch from a single fuel to mixing and then from mixing to another single fuel. This behavior has two implications.

First, it suggests that switching may entail fixed costs that plants want to expunge over multiple periods by switching to a new fuel gradually. Second, it implies that the 30% of plants who mix in a given period are not necessarily doing so due to technological complementarity but possibly due to a transition phase.

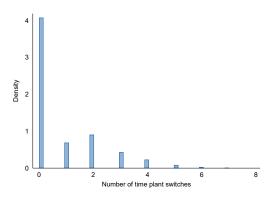


Figure 4: Distribution of switching frequency

Single to Mix	Mix to Single	No	Yes	Total
No		576	113	689
Yes		114	366	480
Total		690	479	1169

Table 2: Mixing as transition phase

### 2.2.3 Switching matters to explain variation in carbon content of energy

Next, I show that switching is a quantitatively important driver of aggregate variation in the carbon content of energy, which is a proxy for emission intensity. To do so, I normalize plants' energy level to their energy consumption in the first period of observation, and I compute plant-level GHG emissions. For a plant i at time t, ghg emissions are defined as

$$g_{it} = \sum_{s}^{n} \gamma_s e_{ist}$$

Where

$$\sum_{s}^{n} e_{ist} = \bar{e}_{i,2009}$$

The emission mapping takes n different fuels in equivalent energy units (mmBtu) and transforms them into kg of CO2 equivalent through fuel-specific emission intensity,  $\gamma_s$ , which

are computed from the United States Environmental Protection Agency (EPA). As such, any variation in energy-normalized emissions within a firm across years is due to variation in relative fuel shares that compose the plant's energy bundle. I then look at year-to-year variation in ghg emissions, and decompose this change into two groups: switching if the plant switched between fuels between t-1 and t, and non-switching if the plant didn't switch between t-1 and t:

$$\Delta g_{it} = \sum_{s}^{n} \gamma_{s} (e_{ist} - e_{ist-1})$$

$$= \Delta g_{it} \mathbb{I}(\text{switch}) + \Delta g_{it} \mathbb{I}(\text{Does not switch})$$

I then aggregate this decomposition across all plants and plot the aggregate change in emission intensity over time:



Figure 5: Decomposition of aggregate change in emission intensity

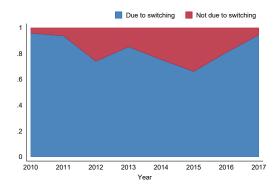


Figure 6: Proportion of aggregate change in emission intensity due to switching and non-switching

In Figure 6, I show that an average of 80-90% of aggregate changes in emission intensity is

due to plants switching across fuels, which suggest that switching is paramount to understanding aggregate fuel consumption patterns.

### 2.2.4 Fuel switching is positively correlated with productivity

Lastly, I show that plants who switch between fuels tend to be more productive, strengthening the assumption of fixed switching costs where more productive plants have more to gain from paying fixed costs and reducing variable costs, which is analogous to productivity-efficiency arguments standard in the trade literature.

To do so, I estimate plant-level productivity using the Ackerberg, Caves, and Frazer (ACF) method with industry dummies  $\lambda_i$ 

$$logY_{it} = \alpha_l logL_{it} + \alpha_k logK_{it} + \alpha_m logM_{it} + \alpha_e logE_{it} + \lambda_i + \omega_{it} + \epsilon_{it}$$

Since this is a simple reduced-form exercise, I posit a Cobb-Douglas revenue production function in 4 inputs: labor, capital, intermediates and energy. The goal is to estimate  $\omega_{it} + \epsilon_{it}$  where  $\omega_{it}$  is the persistent component of productivity which may be correlated with production inputs and  $\epsilon_{it}$  is an iid productivity shock. The novelty in my approach is that I treat energy as fixed within period rather than being part of the intermediate bundle because costly switching has dynamic implications.

I then test for the difference in mean between plants who switch and plants who don't switch in three distinct set-up: plants who switch in any period vs. plants who never switch, plants who switch in the current period and plants who switch next period. In all three cases, there is a statistically significant productivity premium for switching:

Table 3: T-test for Difference in mean productivity (switchers minus non-switchers)

	(1)	(2)	(3)
	Any period	Year of switching	Year before switching
Difference in mean productivity	0.0827***	0.0473*	0.0690***
	(5.09)	(2.35)	(3.38)
Observations	6810	6810	5688

 $<sup>\</sup>boldsymbol{t}$  statistics in parentheses

<sup>\*</sup> p < 0.05, \*\* p < 0.01, \*\*\* p < 0.001

# 3 Model

To rationalize these facts, I develop a stylized model with heterogeneous firms that use a linear technology in a dirty (d) and a clean (c) fuel, and that can pay a fixed cost to switch between fuels where fuel consumption contributes to an externality. The equilibrium features an endogenous productivity cutoff for firms to switch based on a productivity-efficiency argument, and I show that the resulting switching rule has important implications for an optimal externality tax, especially on the relative tax rate between the dirty and the clean fuel.

## 3.1 Aggregation:

Fraction  $N_d^* \in (0,1)$  of firms who use dirty fuel and  $N_c^* \in (0,1)$  who use clean fuel. Continuum of firms indexed by productivity  $\varphi$  who produce differentiated goods substitutable at rate  $\rho > 1$  to form final good Y which is consumed by a representative consumer:

$$Y = \left( \int_{\Phi} y_d(\varphi)^{\frac{\rho-1}{\rho}} dF(\varphi) + \int_{\Phi} y_c(\varphi)^{\frac{\rho-1}{\rho}} dF(\varphi) \right)^{\frac{\rho}{\rho-1}}$$

The representative consumer chooses the utility-maximizing bundle of each differentiated good that forms Y, taking as given prices and purchasing each good with some exogenous income I that I normalize to 1. for  $s = \{c, d\}$  This leads to the following inverse demand function:

$$p_s(\varphi) = \left(\frac{Y}{y_s(\varphi)}\right)^{\frac{1}{\rho}} P \tag{1}$$

Where P is the price index of the final good:

$$P = \left( \int_{\Phi_d} p_d(\varphi)^{1-\rho} dF(\varphi) + \int_{\Phi_c} p_c(\varphi)^{1-\rho} dF(\varphi) \right)^{\frac{1}{1-\rho}}$$

## **3.2** Firms:

The continuum of firms is initially distributed into two groups: dirty input users:  $N_d$  and clean input users:  $N_c$ . Firms only use fuels  $(e_d, e_c)$  in production, can switch between fuels by paying a fixed cost  $\kappa$  and engage in monopolistic competition by internalizing their inverse demand. The profit function of dirty fuel users is as follows:

$$\pi_d(\varphi) = \max \left\{ \max_{e_d} \left\{ p(\varphi)\varphi e_d - \tilde{p}_d e_d \right\}, \max_{e_c} \left\{ p(\varphi)\varphi e_c - \tilde{p}_c e_c - \kappa \right\} \right\}$$
 (2)

Where  $\tilde{p}_d = p_d + \tau_d$  is the price fuel d inclusive of its potential tax. An analogous profit function can be derived for firms initially using the clean fuel. Note that the solution to

problem (2) is mathematically equivalent to a formulation where both fuels are perfect substitutes but where the firm needs to pay a fixed cost for switching between the two:

$$f(\varphi) = \varphi(e_d + e_c)$$

$$\pi_d(\varphi) \equiv \max_{e_d, e_c} \left\{ p(\varphi) \varphi(e_d + e_c) - \tilde{p}_d e_d - \tilde{p}_c e_c - \mathbb{1}(e_c > 0) \kappa \right\}$$

This will have important implications to understand the intuition underlying optimal fossil fuel taxation in this context.

$$\pi_d = \max_{e_d, e_c} \left\{ Y^{1/\rho} P(\varphi(e_d + e_c))^{\frac{\rho - 1}{\rho}} - \tilde{p}_d e_d - \tilde{p}_c e_c - \mathbb{1}(e_c > 0) \kappa \right\}$$

### 3.3 Solution

Demand for both fuels:

$$e_d(\varphi) = \left(\frac{\rho - 1}{\rho} \frac{P}{\tilde{p}_d}\right)^{\rho} Y \varphi^{\rho - 1}$$

$$e_c(\varphi) = \left(\frac{\rho - 1}{\rho} \frac{P}{\tilde{p}_c}\right)^{\rho} Y \varphi^{\rho - 1}$$

Profit for not switching:

$$\pi_{dd}(\varphi) = \frac{Y P^{\rho} \varphi^{\rho - 1}}{p_d^{\rho - 1}} \underbrace{\left[ \left( \frac{\rho - 1}{\rho} \right)^{\rho - 1} - \left( \frac{\rho - 1}{\rho} \right)^{\rho} \right]}_{\Gamma(\rho) > 0}$$

Profits for switching:

$$\pi_{dc}(\varphi) = \frac{Y P^{\rho} \varphi^{\rho - 1}}{p_c^{\rho - 1}} \Gamma(\rho) - \kappa$$

Fuel switching rule:

By comparing profits for using both fuels if the firms was initially using dirty fuel will lead to the following decision rule a the extensive margin based on a productivity threshold:

Let  $\Omega$  define a threshold for productivity:

$$\Omega = \left[ \frac{\kappa}{P^{\rho} Y \Gamma(\rho)} \frac{1}{p_c^{1-\rho} - p_d^{1-\rho}} \right]^{\frac{1}{\rho-1}}$$

Where 
$$\Gamma(\rho) = \left(\frac{\rho-1}{\rho}\right)^{\rho-1} - \left(\frac{\rho-1}{\rho}\right)^{\rho}$$

Let  $N_{dc}$  denote the switching for dirty fuel users switching to clean fuel  $(d \to c)$ , and likewise for  $N_{dd}$ ,  $N_{cd}$ ,  $N_{cc}$ , there are two cases to consider:

Case 1:  $p_d > p_c, \Omega > 0$ 

New fuel			
		d	c
Original fuel	d	$\varphi < \Omega$	$\varphi > \Omega$
	c	$\varphi < -\Omega$	$\varphi > -\Omega$

$$E_{d} = \underbrace{\left[N_{d} \int_{0}^{\Omega} \varphi^{\rho-1} dF(\varphi) + N_{c} \int_{0}^{-\Omega} \varphi^{\rho-1} dF(\varphi)\right]}_{\tilde{\varphi}_{d}^{\rho-1}} \left(\frac{\rho - 1}{\rho} \frac{P}{\tilde{p}_{d}}\right)^{\rho} Y$$

$$E_{c} = \underbrace{\left[N_{d} \int_{\Omega}^{\infty} \varphi^{\rho-1} dF(\varphi) + N_{c} \int_{-\Omega}^{\infty} \varphi^{\rho-1} dF(\varphi)\right]}_{\tilde{\varphi}_{c}^{\rho-1}} \left(\frac{\rho - 1}{\rho} \frac{P}{\tilde{p}_{c}}\right)^{\rho} Y$$

Case 2:  $p_c > p_d, \Omega < 0$ 

New fuel			
		d	c
Original fuel	d	$\varphi > \Omega$	$\varphi < \Omega$
	c	$\varphi > -\Omega$	$\varphi < -\Omega$

$$E_{d} = \underbrace{\left[N_{d} \int_{\Omega}^{\infty} \varphi^{\rho-1} dF(\varphi) + N_{c} \int_{-\Omega}^{\infty} \varphi^{\rho-1} dF(\varphi)\right]}_{\tilde{\varphi}_{d}^{\rho-1}} \left(\frac{\rho - 1}{\rho} \frac{P}{\tilde{p}_{d}}\right)^{\rho} Y$$

$$E_{c} = \underbrace{\left[N_{d} \int_{0}^{\Omega} \varphi^{\rho-1} dF(\varphi) + N_{c} \int_{0}^{-\Omega} \varphi^{\rho-1} dF(\varphi)\right]}_{\tilde{\varphi}_{c}^{\rho-1}} \left(\frac{\rho - 1}{\rho} \frac{P}{\tilde{p}_{c}}\right)^{\rho} Y$$

**Proposition 1.** There is no two-way switching (There is always an integration region that is empty and one that has full support)

Indeed, in case 1 where  $p_d > p_c$ ,  $\Omega > 0$  and  $-\Omega < 0$  such that no firms switches from the clean fuel to the dirty fuel. This is intuitive because they face a higher unit cost to use the dirty fuel while having to pay a fixed cost on top of that, hence it is never profitable for them to switch. An analogous argument can be made for case 2.

Aggregate price index

Regardless of which fuel firms use, individual pricing decisions are always a constant markup over marginal costs:

$$p_s(\varphi) = \frac{\rho}{\rho - 1} \frac{\tilde{p}_d}{\varphi}$$

Let  $\Phi_d$  and  $\Phi_c$  denote the integration regions for using fuels d and c, respectively, then the aggregate price index in this economy is:

$$P = \frac{\rho}{\rho - 1} \left( \tilde{p}_d^{1-\rho} \int_{\Phi_d} \varphi^{\rho - 1} dF(\varphi) + \tilde{p}_c^{1-\rho} \int_{\Phi_c} \varphi^{\rho - 1} dF(\varphi) \right)^{\frac{1}{1-\rho}}$$
$$= \frac{\rho}{\rho - 1} \left( \tilde{p}_d^{1-\rho} \tilde{\varphi}_d^{\rho - 1} + \tilde{p}_c^{1-\rho} \tilde{\varphi}_c^{\rho - 1} \right)^{\frac{1}{1-\rho}}$$

# 3.4 GHG emission and externality

For a given firm, GHG emissions are defined as follows, where  $\gamma_d > \gamma_c$  defines the emission intensity of one unit (in equivalent energy units) of the dirty and clean fuel, respectively:

$$ghg(\varphi) = \gamma_d e_d(\varphi) + \gamma_c e_c(\varphi)$$

Which aggregates to:

$$ghg = \gamma_d \int_{\Phi_d} e_d(\varphi) dF(\varphi) + \gamma_c \int_{\Phi_c} e_c(\varphi) dF(\varphi)$$
$$= \gamma_d E_d + \gamma_c E_c$$

To get some intuition on how variation in fuel prices affect ghg emissions, let  $ghg_d$  be part of aggregate emissions from using fossil fuel d, where  $ghg = ghg_d + ghg_c$ . Then,

$$\frac{d \log ghg_d}{d \log \tilde{p}_d} = \underbrace{\frac{1}{1-\rho} \frac{d \log \tilde{\varphi}_d}{d \log \tilde{p}_d}}_{\text{Agg. prod. ("switching") channel } < 0} + \underbrace{\rho \frac{d \log P}{d \log \tilde{p}_d}}_{\text{Agg price ("Competitive") channel } > 0} - \underbrace{\rho \frac{d \log \tilde{p}_d}{d \log \tilde{p}_d}}_{\text{own price channel } > 0}$$

$$\frac{d \log ghg_c}{d \log \tilde{p}_d} = \underbrace{\frac{1}{1-\rho} \frac{d \log \tilde{\varphi}_c}{d \log \tilde{p}_d}}_{\text{Agg. prod. ("switching") channel } > 0} + \underbrace{\rho \frac{d \log P}{d \log \tilde{p}_d}}_{\text{Agg price ("Competitive") channel } > 0}$$

I then define the externality as multiplicative damages following the work of Nordhaus (2008) and Golosov et. al (2014), where the function  $D(ghg) \in (0,1)$  maps damages from CO2e emissions in units of the final good. The specific functional form of D(ghg) is not important for the main result of this paper.

$$\tilde{Y} = (1 - D(ghg))Y$$

# 3.5 Optimal tax on fossil fuels

In the spirit of Diamond and Mirrlees (1971) I study optimal taxation by defining the problem of a government to choose a tax on both fuels to maximize the representative worker's utility, taking into account the externality and all decisions made in the monopolistic competition equilibrium. I first study optimal taxation in partial equilibrium, which means that the government effectively chooses the price of each fuel  $\tilde{p}_s$  which pins down the tax rate  $\tau_s$ . This is because fuel prices are exogenous from the model and any price variation that does not reflect the externality can be corrected for by the government. General equilibrium will be considered later. Moreover, the interest here is on the relative tax rates between both fuels rather than the level of the tax rates. The government then solves the following problem, taking into account the externality:

$$\begin{aligned} \max_{\tilde{p}_d, \tilde{p}_c} \{\tilde{Y}\} \\ s.t. \ PY &= I \\ \tilde{Y} &= \left(1 - D(ghg)\right) Y \end{aligned}$$

Where the representative worker's income is normalized to 1. Moreover, I assume that the representative worker consumes the proceeds Hence, this problem becomes:

$$\max_{\tilde{p}_d, \tilde{p}_c} \left\{ \left( 1 - D(ghg) \right) \frac{1}{P} \right\}$$

Where the fundamental economic trade-off from the government's perspective is that increasing taxes on fossil fuels increases real output by decreasing the externality but also decreases real output due to the increase in the aggregate price index.

To learn about the optimal relative tax rate, most of the intuition can be derived from the ratio of FOCs for the price of both fuels. The government's objective is to find relative fuel prices that equate relative marginal losses (by increasing the aggregate price index which decreases real income) to relative marginal gains (by decreasing aggregate GHG emissions):

$$\frac{\partial P/\partial \tilde{p}_d}{\partial P/\partial \tilde{p}_c} = \underbrace{\frac{\partial ghg/\partial \tilde{p}_d}{\partial ghg/\partial \tilde{p}_c}}_{\text{rel. marginal losses}} = \underbrace{\frac{\partial ghg/\partial \tilde{p}_d}{\partial ghg/\partial \tilde{p}_c}}_{\text{rel. marginal gains}}$$

#### A useful benchmark

The first case to consider is the case where there is no switching in the model, that is where firms always use the technology they start with. In such a case, there is no selection into different fuels based productivity, and the economy is proportionally equivalent to an economy with an aggregate CES production that takes two inputs (the dirty and the clean fuel) which can be substituted at rate  $\rho$ :<sup>3</sup>

$$\begin{split} Y &= \tilde{\varphi} \Big( N_d^{\frac{1}{\rho}} E_d^{\frac{\rho-1}{\rho}} + N_c^{\frac{1}{\rho}} E_c^{\frac{\rho-1}{\rho}} \Big)^{\frac{\rho}{\rho-1}} \\ N_d + N_c &= 1 \end{split}$$

Where

$$\tilde{\varphi} = \left( \int_0^\infty \varphi^{\rho - 1} dF(\varphi) \right)^{\frac{1}{\rho - 1}}$$

This is important because this is the type of aggregate production functions that Golosov et al. (2014) use to study optimal carbon taxation, and this allows me to highlight how the presence of fuel switching departs from canonical assumptions in the literature. Under the no switching assumptions, the solution to the optimal tax problem is standard, and the relative difference in prices reflects the relative emission intensity of both fuels, which can be seen in the following graph:

$$\frac{\tilde{p}_d}{\tilde{p}_c} = \frac{\gamma_d}{\gamma_c}$$

<sup>&</sup>lt;sup>3</sup>That is, conditional on some aggregate output level Y, the aggregate fuel demand  $(E_d(Y))$  and  $E_c(Y)$  are the same in both models. However, aggregate output may differ due to the presence of markups in the model with monopolistic competition and heterogeneous firms. See appendix for proof.

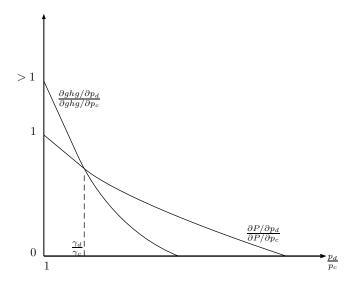


Figure 7: Optimal relative fuel prices (taxes) - no switching benchmark

Note that the marginal relative gains in reducing emissions are greater than one when prices are equal to one due to the difference in the emission intensity of both fuels  $\gamma_d > \gamma_c$ , whereas marginal effects on the aggregate price index is the same when prices are equal.

#### Case with switching

In the presence of fuel switching, it is still optimal for the government to choose fuel prices (tax rates) in the range  $p_d > p_c$  because  $\gamma_d > \gamma_c$ . However, conditional on  $p_d > p_c$ , there are two new effects on the relative marginal gains/losses from increasing  $p_d$  relative to increasing  $p_c$  that are going to shift the optimal relative prices.

First, increasing the price of the dirty fuel *increases* the output price of firms that are not productive enough to switch to the clean fuel, but *decreases* the output price of firms that do switch because they are now facing lower input costs. Overall, increasing  $p_d$  increases P but the magnitude of this change is smaller than in the no switching benchmark whereas the opposite is true for increasing  $p_c$ . This essentially shift down the  $\frac{\partial P/\partial p_d}{\partial P/\partial p_c}$  in figure 1:

$$\frac{\partial P}{\partial p_d} = \underbrace{\text{benchmark effect}}_{>0} + \underbrace{\frac{1}{1-\rho} \Big[ (p_d^{1-\rho} - p_c^{1-\rho}) N_d \Omega^{\rho-1} \frac{\partial \Omega}{\partial p_d} \Big]}_{\text{Selection effect due to switching } < 0}$$

Second, increasing the price of the dirty fuel decreases ghg emissions for firms that don't switch due to reduction in scale of production (as in the benchmark), but decreases even more ghg emissions for firms that switch to the clean fuel and now pollute at rate  $\gamma_c$  rather than  $\gamma_d$ :

$$\frac{\partial ghg}{\partial p_d} = \underbrace{\text{benchmark effect}}_{<0} + \underbrace{\left(\frac{\rho-1}{\rho}\right)^{\rho} \left[N_d \Omega^{\rho-1} \frac{\partial \Omega}{p_d} \left(\frac{1}{1-\rho} (p_d^{1-\rho} - p_c^{1-\rho}) + (\gamma_d - \gamma_c)\right)\right]}_{\text{Selection effect due to switching} < 0}$$

Overall, the net effect of introducing switching is that relative marginal losses are smaller when increasing  $p_d$  relative to  $p_c$  while relative marginal gains are larger when increasing  $p_d$  relative to  $p_c$ . This effectively leads to optimal relative fuel prices that are much larger than with a carbon tax:

$$\begin{split} \frac{\partial P/\partial \tilde{p}_d}{\partial P/\partial \tilde{p}_c} \Big|_{\text{switching}} &< \frac{\partial P/\partial \tilde{p}_d}{\partial P/\partial \tilde{p}_c} \Big|_{\text{no switching}} \\ \frac{\partial ghg/\partial \tilde{p}_d}{\partial ghg/\partial \tilde{p}_c} \Big|_{\text{switching}} &> \frac{\partial ghg/\partial \tilde{p}_d}{\partial ghg/\partial \tilde{p}_c} \Big|_{\text{no switching}} \end{split}$$

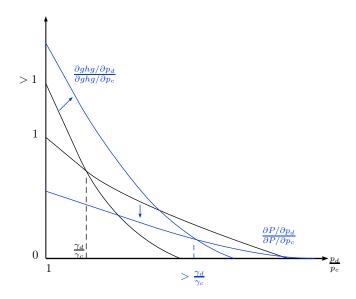


Figure 8: Optimal relative fuel prices (taxes) - switching

Moreover, it can be seen that as the fixed switching cost increases, the optimal relative prices converge to the no-switching benchmark of  $\frac{\gamma_d}{\gamma_c}$ :

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