Computational Macro - Problem set 3

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Question 1.

For each of the utility functions, I use the L2 norm for interpolation accuracy: $||V(x)-\tilde{V}(x)||_2 = \left(\sum_i (V(x_i)-\tilde{V}(x_i))^2\right)^{1/2}$.

For the cubic interpolation, I use the natural assumption $y_1'' = y_N'' = 0$ and the tridiagonal system to solve for second derivatives has symmetric lower and upper diagonals:

$$\begin{bmatrix} d_2 & c_2 & & & \\ c_2 & d_3 & c_3 & & \\ & & \ddots & & \\ & & c_{N-3} & d_{N-2} & c_{N-2} \\ & & & c_{N-2} & d_{N-1} \end{bmatrix} = \begin{bmatrix} y_2'' \\ y_3'' \\ \vdots \\ y_{N-2}'' \\ y_{N-2}'' \end{bmatrix} \begin{bmatrix} s_2 - s_1 \\ s_3 - s_2 \\ \vdots \\ s_{N-2} - s_{N-3} \\ s_{N-1} - s_{N-2} \end{bmatrix}$$

Log utility: $u(c) = log(c)$

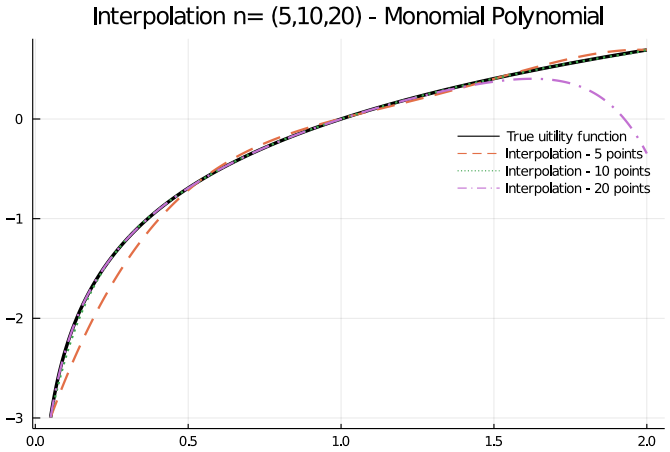

Figure 1: Log utility

Interpolation method	5 grid points	10 grid points	20 grid points
Global monomial	3.624	0.549	6.621
Linear spline	7.52	2.61	0.833
cubic spline	7.375	2.578	0.826

Table 1: Interpolation accuracy - log utility

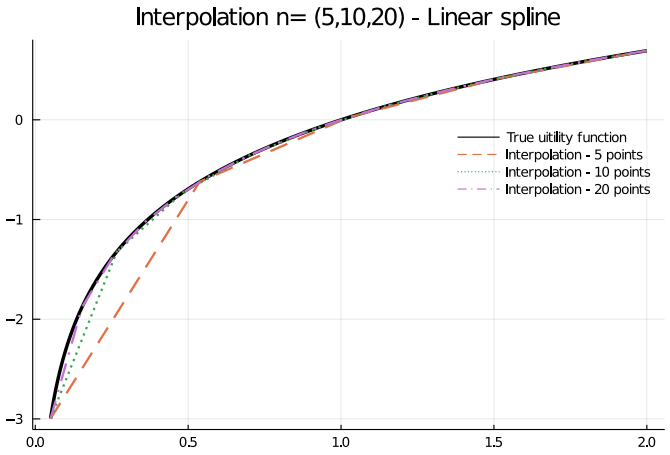

Figure 2: Log utility

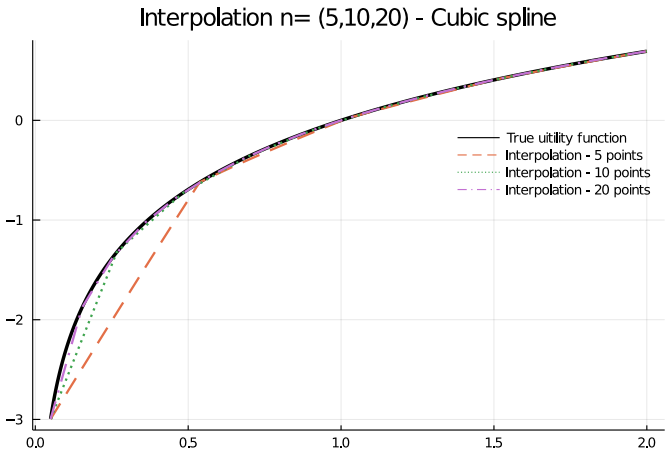

Figure 3: Log utility

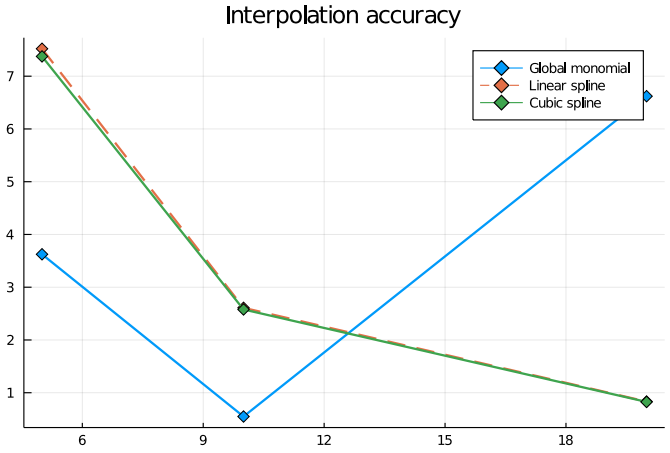

Figure 4: Interpolation accuracy - log utility (x-axis: number of grid points)

Square root utility: $u(c) = \sqrt{c}$

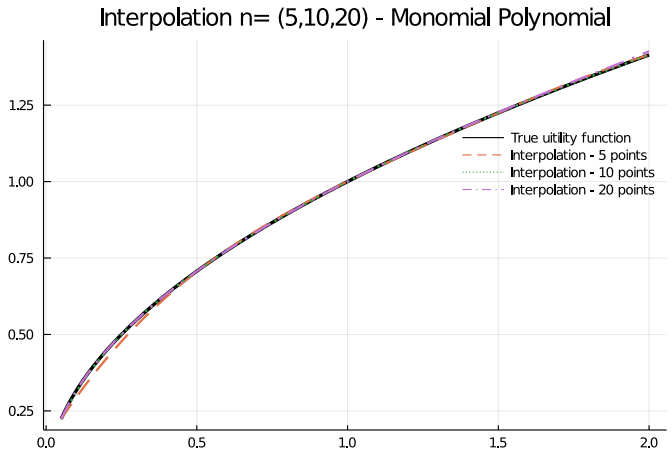

Figure 5: Square root utility

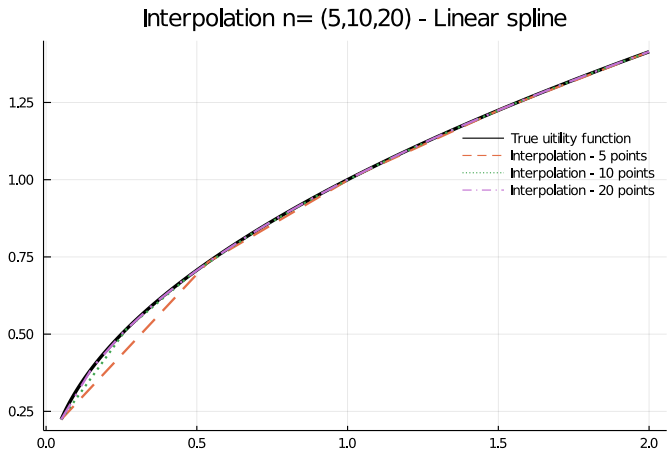

Figure 6: Square root utility

Interpolation method	5 grid points	10 grid points	20 grid points
Global monomial	0.292	0.031	0.096
Linear spline	0.797	0.231	0.065
cubic spline	0.644	8.337	0.887

Table 2: Interpolation accuracy - square root utility

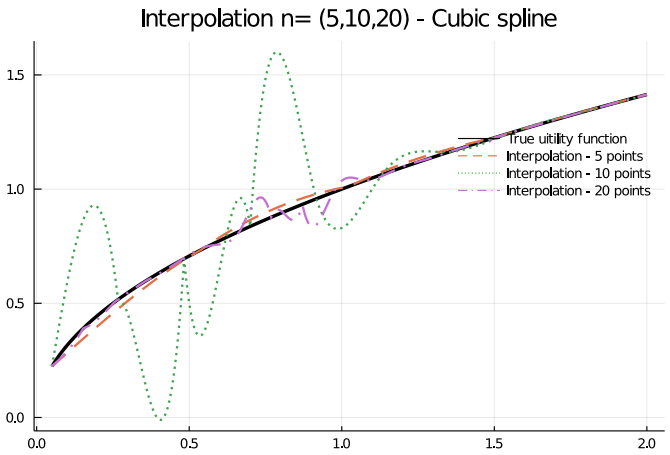

Figure 7: Square root utility

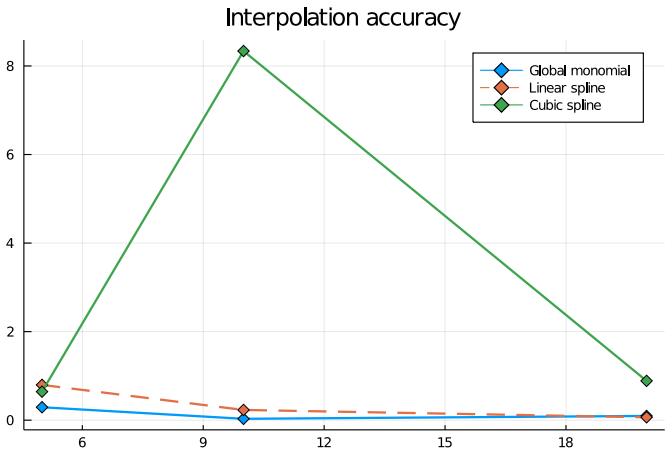

Figure 8: Interpolation accuracy - square root utility (x-axis: number of grid points)

CRRA utility with $\sigma = 2$: $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$

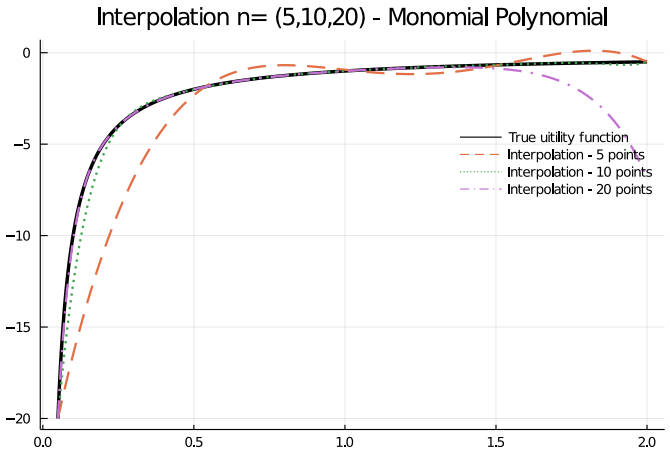

Figure 9: CRRA utility with $\sigma = 2$

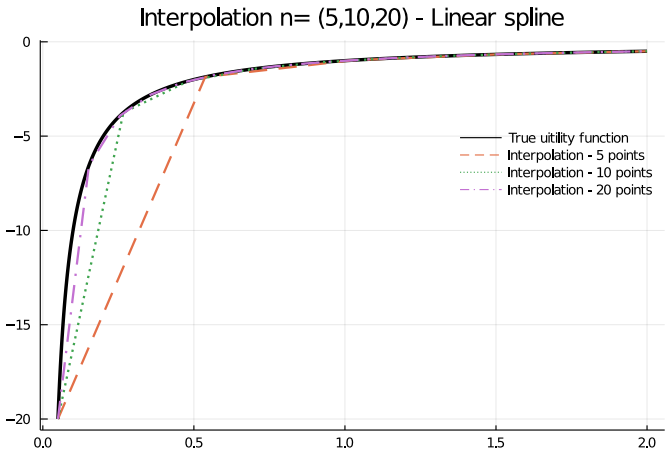

Figure 10: CRRA utility with $\sigma = 2$

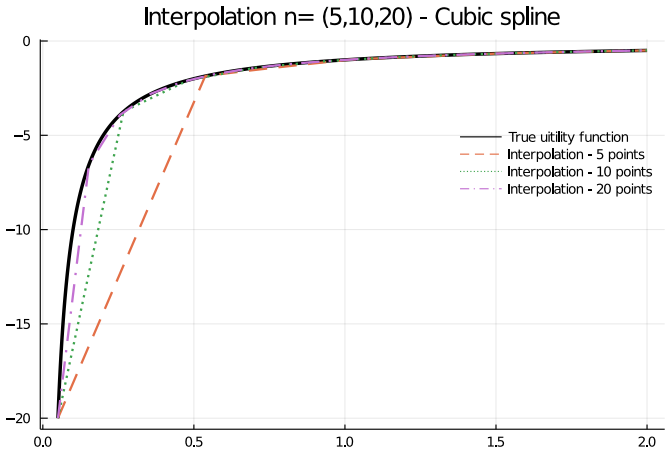

Figure 11: CRRA utility with $\sigma = 2$

Interpolation method	**5 grid points**	**10 grid points**	**20 grid points**
Global monomial	67.231	17.127	40.658
Linear spline	106.645	48.359	19.251
cubic spline	106.376	48.3	19.237

Table 3: Interpolation accuracy - CRRA utiltiy with $\sigma = 2$

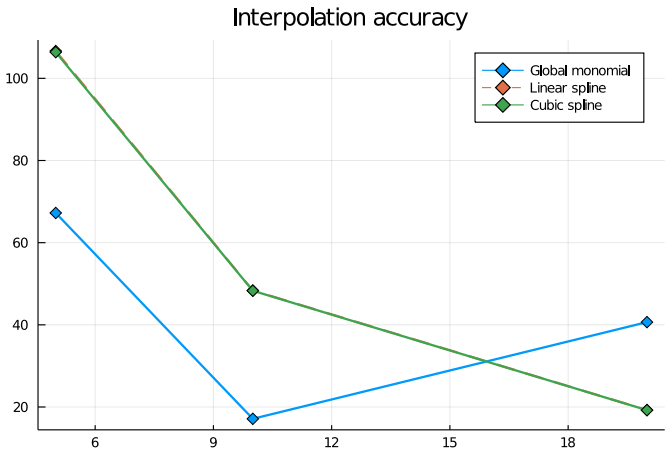

Figure 12: Interpolation accuracy - CRRA utility $\sigma = 2$ (x-axis: number of grid points)

CRRA utility with $\sigma = 5$: $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$

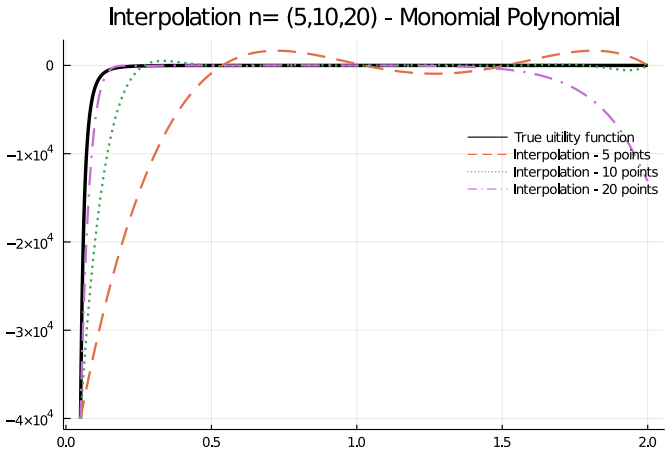

Figure 13: CRRA utility with $\sigma = 5$

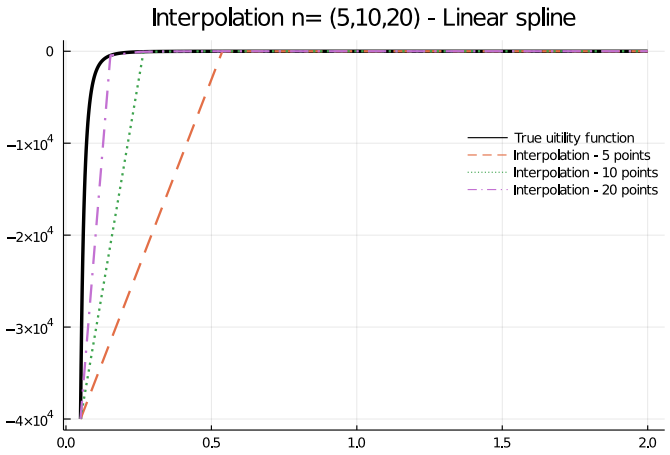

Figure 14: CRRA utility with $\sigma = 5$

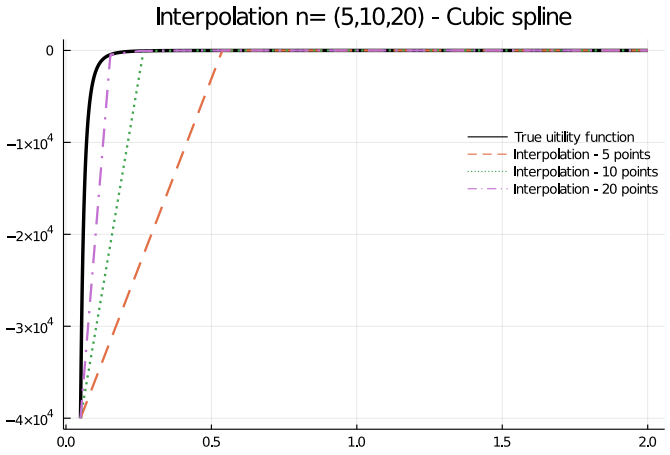

Figure 15: CRRA utility with $\sigma = 5$

Interpolation method	5 grid points	10 grid points	20 grid points
Global monomial	251884.258	115697.137	93647.925
Linear spline	336349.131	202097.314	113060.359
cubic spline	336349.059	202097.306	113060.355

Table 4: Interpolation accuracy - CRRA utiltiy with $\sigma = 5$

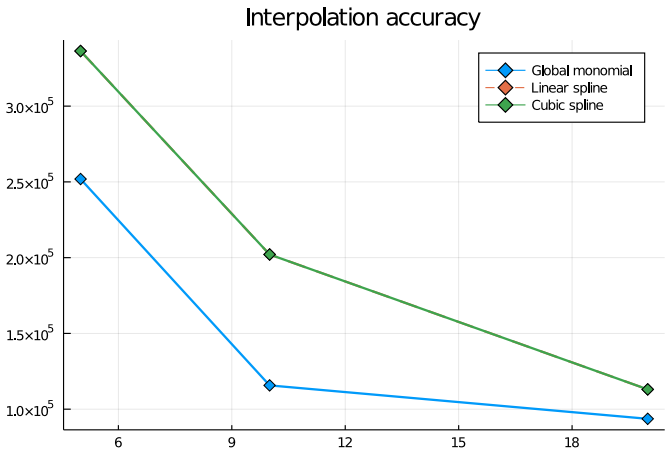

Figure 16: Interpolation accuracy - CRRA utility $\sigma = 5$ (x-axis: number of grid points)

CRRA utility with $\sigma = 10$: $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$

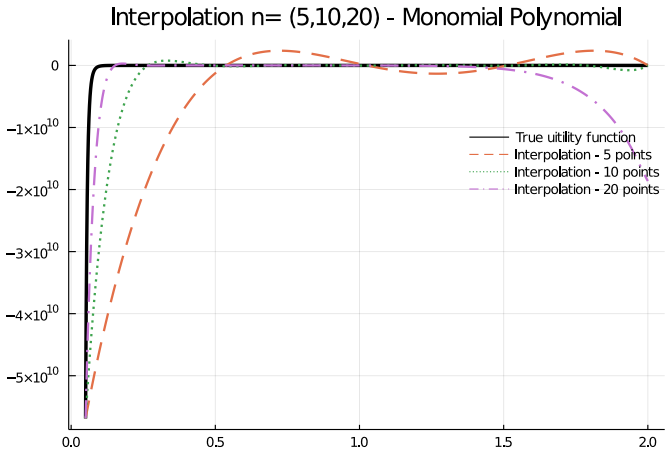

Figure 17: CRRA utility with $\sigma = 10$

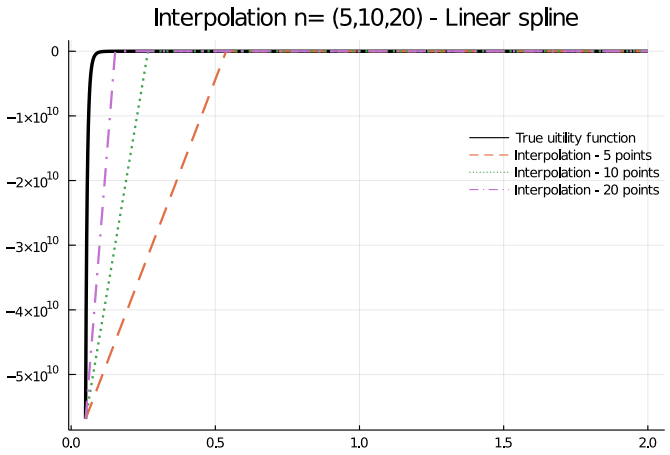

Figure 18: CRRA utility with $\sigma = 10$

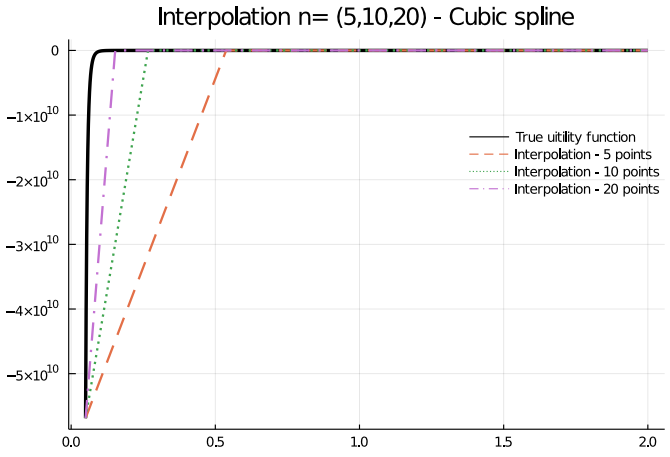

Figure 19: CRRA utility with $\sigma = 10$

Interpolation method	5 grid points	10 grid points	20 grid points
Global monomial	3.88934724074138e11	2.08387581221338e11	1.59177864232513e11
Linear spline	5.03864673743989e11	3.2338560205199e11	2.05707722591662e11
cubic spline	5.03864673744076e11	3.23385602051989e11	2.05707722591662e11

Table 5: Interpolation accuracy - CRRA utility with $\sigma = 10$

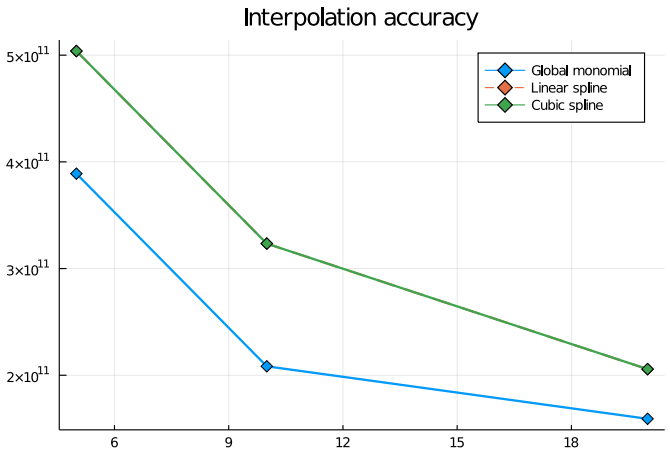

Figure 20: Interpolation accuracy - CRRA utility $\sigma = 10$ (x-axis: number of grid points)

Question 2.

I did the exercise to find the grid curvature θ that minimizes the difference between the true function and the interpolated one using a polynomial grid spacing.

$\hat{\theta}$	**Sup norm**	**Absolute differences in iterated** $\hat{\theta}$	**Number of iterations**
2.742	116.784	7.525e-6	30

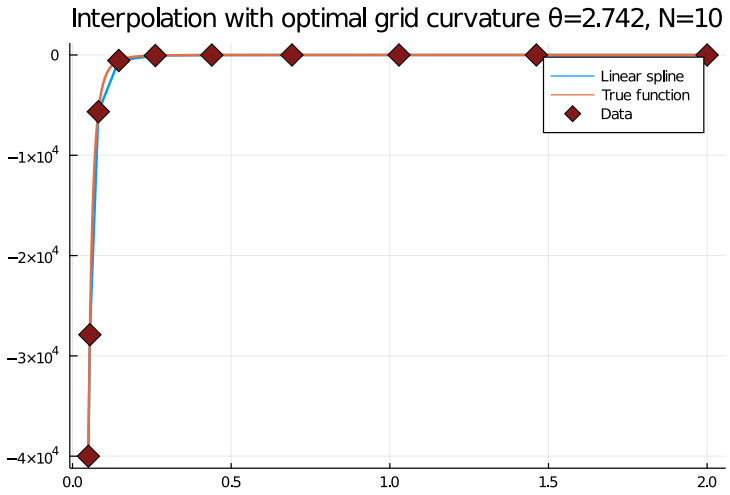

Figure 21: CRRA utility with $\sigma = 5$

Question 3.

Since I do not know how to extrapolated from local interpolation methods (splines), I only did the exercise with global interpolation using monomial basis. Here are the results for each utility function:

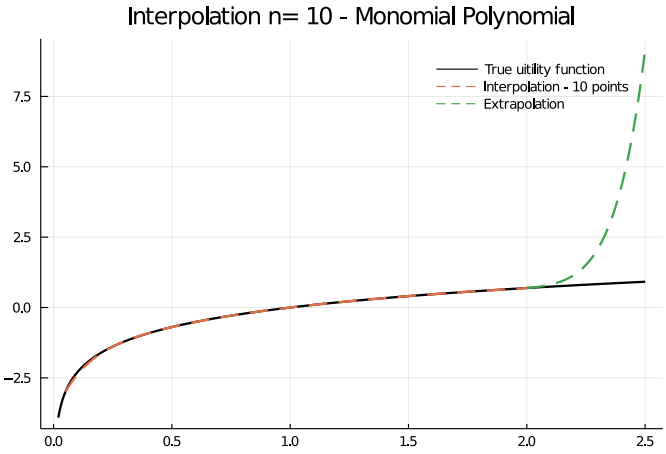

Figure 22: Extrapolation - log utility

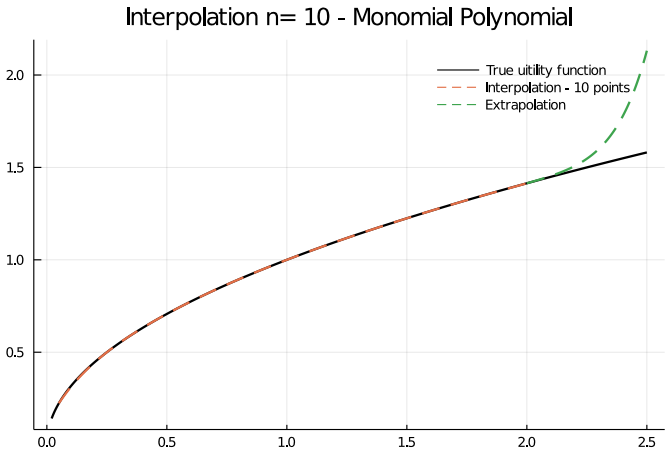

Figure 23: Extrapolation - square root utility

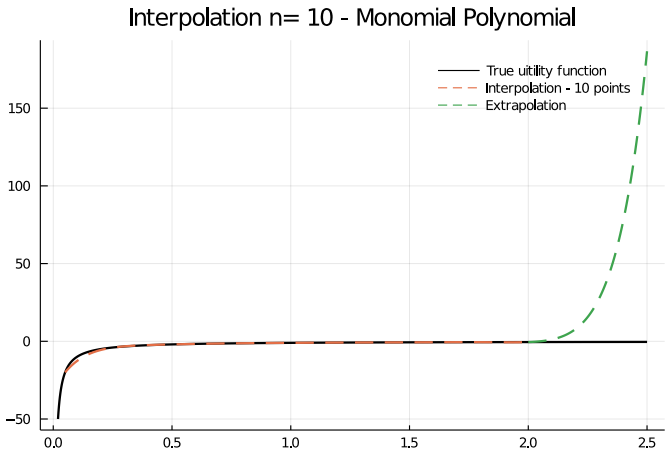

Figure 24: Extrapolation - CRRA utility with $\sigma = 2$

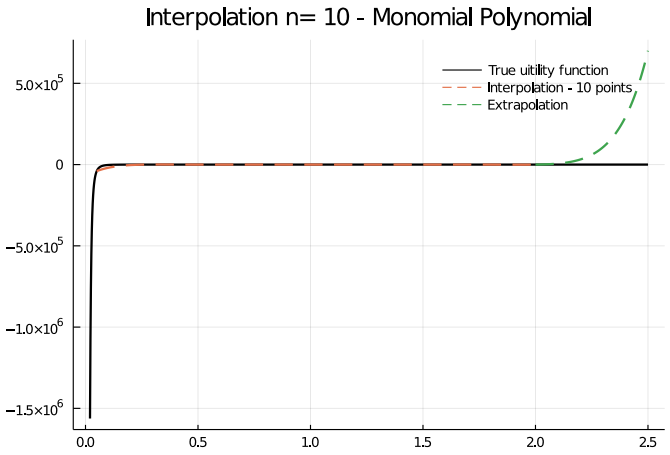

Figure 25: Extrapolation - CRRA utility with $\sigma = 5$

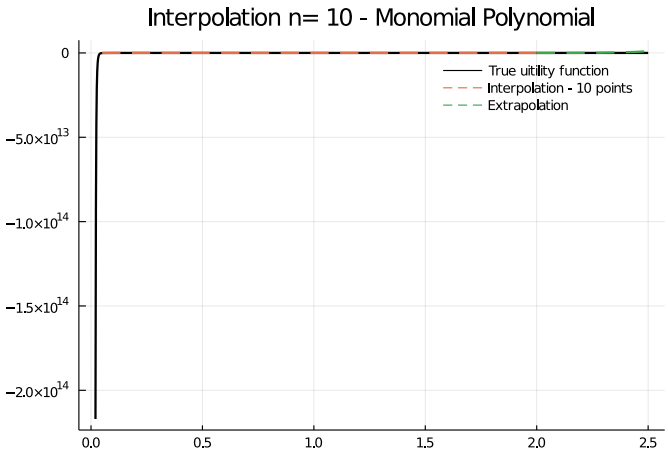

Figure 26: Extrapolation - CRRA utility with $\sigma = 10$