Asymmetric Environmental Regulation, Interfuel Substitution and Carbon Leakage*

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Abstract

This paper studies how firms reorganize their production when faced with asymmetric carbon pricing. While firms may compete with each other across geographical regions, some regions may have carbon pricing policies while others do not. This can lead to carbon leakage, where emissions shift from regulated to unregulated regions. I build a model of imperfect competition with multiple fuels as energy inputs, which allows for region-specific carbon taxes. I estimate the structural model with publicly available Canadian plant-level data on a wide range of pollutants emitted in the air to quantify the effect of the British-Columbia (B.C.) and Quebec carbon taxes that were implemented in 2008 and 2007, respectively. I find strong evidence of carbon leakage in other Canadian provinces, which mitigated 45% of emissions reduction efforts in BC and Quebec. I find that Canadian plants do not find it profitable to switch between fuels. As a result, regulated firms become less competitive, and much output is reallocated towards unregulated firms. A uniform carbon tax in all provinces, which was introduced in 2018, fully mitigates carbon leakage within Canada and reduces aggregate emissions by 21% relative to a 2.15% reduction with the asymmetric carbon tax.

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1 Introduction

Many countries are implementing regulations to combat pollution and its associated environmental challenges. As a result, there is an increasing interest in assessing the effectiveness of these policies in reducing pollution. The combustion of fossil fuels, a major contributor to pollution, creates a negative externality through the emission of greenhouse gases that contribute to global warming. Under Pigouvian theory, the optimal policy for addressing this externality is a carbon tax equal to the marginal social damages of GHG emissions. Although many countries and regions have implemented carbon taxes, the limited jurisdictional scope of regulation is such that even the most ambitious pollution reduction programs, such as the Kyoto Protocol and the Paris Agreement, are voluntary, and their effectiveness depends on the goodwill of governments. In this context, there is a risk of "carbon leakage", where emissions shift to unregulated regions as a result of asymmetric regulation.

This risk of carbon leakage is particularly relevant in the context of manufacturing activity because firms are known to compete across regions (Smith and Ocampo, 2020). Moreover, manufacturing activity contributes to 37% of global greenhouse gas emission (Worrell, Bernstein, Roy, Price and Harnisch, 2009) and is one of the main targets of carbon taxes.

Concerns about carbon leakage are accentuated in the case of sub-national and even sub-union regulation because production is more likely to shift across borders within a country/union due to limited trade barriers. Moreover, while carbon leakage is by itself an interesting phenomenon, its presence biases empirical studies that estimate the direct effect of carbon taxes. Indeed, the competitive nature of firms across regions makes it more difficult to exploit the jurisdictional boundaries of policies as natural experiments to form counterfactuals because it implies that the policy also treats unregulated firms. To use standard policy evaluation tools such as Difference-in-Difference to recover the effect of a policy, one must then make strong independence assumptions that rule out carbon leakage.

In this paper, I study carbon leakage in the context of the British Columbia (B.C.) and Quebec carbon taxes implemented in 2007 and 2008, respectively. I build a model of monopolistic competition with heterogeneous firms and multiple regions similar to Shapiro and Walker (2018) and Aichele and Felbermayr (2015). In the model, a carbon tax increases regulated firms' marginal costs, making them less competitive than unregulated firms. The increase in marginal costs depends on firms' capacity to substitute cleaner fuels for dirty fuels, and the extent of carbon leakage depends on how consumers are willing to substitute across firms. The model allows me to study both the direct effects of the carbon tax in regulated regions and the leakage effect in unregulated regions. Most importantly, I provide restrictions to quantify this model—and, by extension, carbon leakage–using publicly available emissions data/remote sensing data only.

Using publicly available Canadian data on a wide range of pollutants emitted in the air by manufacturing establishments, I estimate the model's parameters to quantify the impact of the British Columbia (B.C.) and Quebec carbon taxes on greenhouse gas (GHG) emissions across Canada prior to the uniform Canadian carbon tax implemented in 2018. Both carbon taxes were implemented in 2007 and 2008, respectively. I find strong evidence of carbon leakage to other Canadian provinces. Indeed, I find that the carbon taxes caused a decrease in aggregate emissions from Quebec and B.C. firms by 150 megatons of carbon dioxide equivalent (CO_{2e}) from 2007 to 2015. However, these carbon taxes caused an increase in emissions from firms in unregulated provinces by 67 megatons of CO_{2e}, mitigating 45% of emissions reduction efforts as output reallocated towards unregulated firms.

I find very little evidence of fuel substitution in response to the tax. Most firms use a dominant fuel and do not find substitution profitable. Moreover, the dominant fuel tends to be natural gas, whereas alternative fuels such as oil and coal are even more emission-intensive. As a result, B.C. firms pass on the cost increase to consumers and reduce output. This cost pass-through makes firms in unregulated provinces relatively more competitive, increasing their market share.[1]

The counterfactual introduction of a uniform carbon tax across all provinces dramatically reduces aggregate emissions[2]. Extending the carbon tax to all provinces reduces aggregate emissions by 21% relative to a 2.15% reduction in aggregate emissions from the B.C. and Quebec carbon taxes.

The model features monopolistic competition in the spirit of Melitz (2003a) and multiple regions. In a nontrivial extension of Copeland and Taylor (2004), I allow multiple fossil fuels to be used in production and specify pollution as a byproduct of fossil fuel combustion. Fossil fuel combustion generates energy for production but also releases greenhouse gases (GHG) into the atmosphere. This production function directly maps to emissions and carbon tax data. Indeed, I calculate quantities of fossil fuels from emissions data based on underlying chemical reactions. Carbon taxes are then

[1] However, I do not observe electricity consumption by firms, which could mitigate this leakage effect if regulated firms can substitute electricity for fossil fuels, especially in provinces such as Quebec, where electricity is "clean" as it comes mostly from hydro plants.

[2] Such as tax was implemented in 2018 by the Federal Government.

levied with a different per-unit tax rate on different fuels, reflecting the emission intensity of each fuel, which affects firms' cost of using different fuels[3].

This extension allows me to identify and estimate the model's parameters and provide novel empirical evidence on carbon leakage using only publicly available pollution release data, fuel price data, and industry aggregates, all commonly available. The key to my method is that, under a standard set of assumptions about firms' production process and profit maximization, pollution data contains information about which fuels firms use, how much of each fuel they use, and how productive they are relative to other firms in the same industry.

My method applies to contexts with abundant remote sensing data but where plant-level data is difficult to access or hardly even exists. It is also one of the first papers to empirically study carbon leakage at a granular level in a literature that has a longstanding theoretical foundation (Garella and Trentinaglia, 2019; Holland, 2012; Hoel, 1991) and computational general equilibrium (CGE) applications (Böhringer, Carbone and Rutherford, 2016; Felder and Rutherford, 1993). Moreover, since carbon tax rates are heterogeneous by fuel types, I allow firms to substitute across different fuels. Similar to the approaches of Atkinson and Luo (2023), Ganapati, Shapiro and Walker (2020) and Carlson, Burtraw, Cropper and Palmer (2000), fuel substitution provides an endogenous margin of pollution intensity adjustment. Interestingly, I find very little evidence of fuel substitution induced by the B.C. and Quebec carbon taxes.

I exploit carbon taxes and plausibly exogenous variation in world fuel prices to identify the model's parameters. I assume that the production technology features constant elasticity of substitution (CES), and I separate identification into two parts. The first part comes from firms' cost minimization problem of choosing relative fuel shares to form a composite fuel index. To this end, I show closed-form identification of reduced-form parameters that directly map into structural technology parameters: fuel-specific efficiency parameters that vary across industries and the elasticity of substitution across fuels. The second part comes from firms' profit maximization problem, in which they choose output quantity subject to a given level of the composite fuel index. I now observe an estimate of firms' fuel composite input, which contains information about their productivity. Under parametric assumptions about the productivity distribution, I use maximum likelihood to estimate the elasticity of substitution across firms, directly leveraging variation in the B.C. and Quebec carbon tax.

The data I use is the National Pollutant Release Inventory (NPRI) combined with industry-level

[3] This is how many carbon taxes are implemented in practices, including the B.C. carbon tax.

aggregates. The former contains all pollutants released by manufacturing plants in Canada. I specifically use pollutants released in the air that are known to come from the combustion of specific fossil fuels. I invert each fuel's chemical reaction to recover an estimate of fuel quantities from a range of pollutants. This procedure is key because the model relies on fuel quantities to identify the effect of carbon taxes.

Section 2 presents the intuition underlying carbon leakage. Section 3 presents the model in all its details. Section 4 presents the data. Section 5 presents identification and details on estimation. Section 6 presents the results on estimation. Section 7 presents counterfactual of interests and decomposes the effect of carbon taxes.

2 Carbon leakage mechanisms

In this section, I explain intuitively the different mechanisms through which an asymmetric carbon tax can lead to carbon leakage. Theoretically, there are two mechanisms at the intensive margins (within-firm adjustments) and one at the extensive margins (through entry and exit). First, the carbon tax increases the marginal cost of regulated firms, which leads to an increase in their output price and a reallocation of production across regions due to demand shifting from regulated to unregulated firms. Second, the increase in marginal costs can increase the productivity threshold required for a firm to be profitable, forcing some regulated firms to exit the market and vice versa in unregulated regions with firms entering the market. Third, the carbon tax also increases the relative price of most polluting fuels, and regulated firms will increase their demand for cleaner fuels. In general equilibrium, such variation in fuel demand will increase the relative gross price of cleaner fuels. Unregulated firms now face lower relative prices of polluting fuels, inducing substitution towards these polluting fuels.

I highlight the last two channels in red in Figure 1, indicating that these channels are not a priori relevant for Canadian firms in response to the B.C. and Quebec carbon taxes. Canadian firms are too small relative to the rest of the world to significantly impact aggregate fuel demand and induce change in world fuel prices. Moreover, I abstract from the entry/exit margin of adjustment as preliminary evidence suggests against it. Indeed, difference-in-difference (DiD) estimation on the number of firms operating in each province due to the B.C. and Quebec carbon taxes suggests an increase in the number of firms operating in both regulated provinces, which is at odds with theoretical predictions. See the Appendix for details. This result is not innocuous, especially for

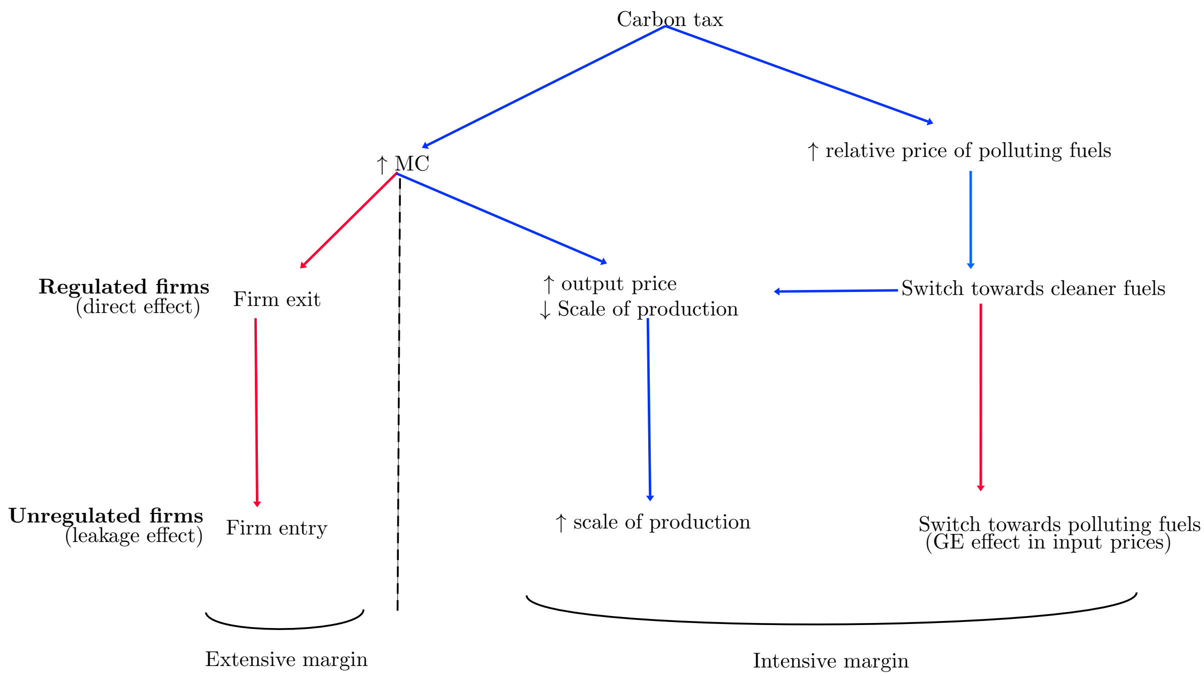


Figure 1: Channels for carbon leakage

British Columbia. It is consistent with previous evidence that the B.C. carbon tax fostered an small increase in aggregate employment due to its revenue neutrality (Yamazaki, 2017, 2022). For these reasons, I only look at the channel highlighted in blue.

3 Model

3.1 Structure of the Economy

The economy is characterized by firms who engage in monopolistic competition across multiple regions and industries and share similarities with Shapiro and Walker (2018) and Melitz and Redding (2014). I augment this framework with firm-specific production functions that take different fuels as inputs and allow for inter-fuel substitution. To study carbon leakage, I introduce an asymmetric carbon tax that only affects firms in specific regions. I present the main framework for a single region and will give details on multiple regions when I introduce the carbon tax. A Cobb Douglas production function takes the output of J industries and aggregates them into a final good:

$$Y = \prod_{j=1}^{J} Y_j^{\beta_j} \text{ with } \sum_j \beta_j = 1$$

Where β_j is the share of industry j's production allocated to final consumption. The final good producer chooses how much of each industry's aggregate is needed to maximize some fixed amount of final consumption subject to a standard budget constraint: $\sum_j P_j Y_j = C$ where C denotes aggregate income. Throughout the paper, I assume that aggregate income is in dollar units and that firms take it as given. The solution to this problem is standard and yields the share of total consumption produced by each industry, Y_j:

$$Y_j = \frac{\beta_j}{P_j} C \tag{1}$$

Where P_j is the industry price index. Next, within each industry, firms sell differentiated goods, $Y_j(A)$, indexed by productivity A, which can be substituted at rate $\rho > 1$ to form the CES composite industry output Y_j. ρ will be a crucial parameter to study carbon leakage as it defines the degree of competitivity across firms and how easily consumers can switch between firms.

$$Y_j \equiv \left(\int_{\Omega_j} Y_j(A)^{(\rho-1)/\rho} dA \right)^{\rho/(\rho-1)}$$

The difference in integrating regions Ω_j represents the idea that different industries have different masses of operating firms, indexed by N_j. From this, I can solve for the share of each firm's output $Y_j(A)$ as a fraction of the industry aggregate Y_j, which is easily found by choosing the cost-minimizing bundle of $\{Y_j(A)\}$ that produces a given amount of Y_j taking firm-level prices as given:

$$Y_j(A) = \left(\frac{P_j(A)}{P_j} \right)^{-\rho} Y_j \tag{2}$$

Where $P_j \equiv \left(\int_{\Omega_j} P_j(A)^{(1-\rho)} dA \right)^{1/(1-\rho)}$ is the industry-specific CES price index. Moving forward, I assume that firms compete in monopolistic competition.

3.2 Technology and Emissions

To the standard framework above, I add a CES technology that takes multiple fuels $\{q^\ell\}_{\ell=1}^{L}$ indexed by ℓ as inputs which form a fuel composite index F_j. This composite fuel represents the total quantity of energy service received from fuel combustion, a process that also emits greenhouse gases in the air. This technology is similar to the aggregate production function in Hassler, Olovsson and Reiter (2019), who study multiple energy sources in an integrated assessment model of climate change. Fuels have varying degrees of emission intensity, and firms can substitute between them at a rate $\sigma > 1$. Each industry j has baseline fuel-specific efficiency terms, $\lambda_{\ell j}$, where $\sum_\ell \lambda_{\ell j} = 1$. Each firm has a productivity level A drawn from a distribution $g_j(A)$. Productivity should be thought of as energy-augmenting rather than total factor productivity because it contains unobserved factors of production like labor and capital.

$$Y_j(A) = A \underbrace{\left(\sum_\ell \lambda_{\ell j} (q_j^\ell)^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}}_{F_j} \tag{3}$$

Let γ_ℓ be the coefficient that maps one unit of fuel ℓ to tons of carbon dioxide equivalent (CO_{2e}) equivalent, the measure for GHG emissions commonly used in the literature. Then, GHG emissions of firm i in industry j is the sum of all fuels used times their emission factor, which I decompose into the product of emission intensity and output quantity:

$$\begin{aligned} GHG_j(A) = \sum_\ell \gamma_\ell q_j^\ell(A) \equiv E_j(A) &= \frac{E_j(A)}{Y_j(A)} \times Y_j(A) \\ &= \underbrace{e_j(A)}_{\text{emission intensity (process factor)}} \times \underbrace{Y_j(A)}_{\text{output quantity (scale factor)}} \end{aligned}$$

It is useful to compare my technology to the canonical technological framework for pollution in the literature (Levinson and Taylor, 2008; Shapiro and Walker, 2018). I define pollution through a composite fuel index F_j, which is the CES aggregate of multiple fuels whereas they define an implicit pollution function:

$$\begin{aligned} \text{My model: } & Y_j(A) = AF_j \\ \text{Canonical model: } & Y_j(A) = (Al_j)^{1-\alpha_j}(z_j)^{\alpha_j} \end{aligned}$$

In the canonical model, l_j is labor, and z_j is total pollution such that one unit of z_j always pollute the same amount. However, firms can have varying emission intensity through investments in pollution abatement technologies, making pollution less intensive relative to labor (lower α_j). By contrast, I define pollution abatement endogenously through fuel substitution. One unit of F_j can emit different levels of GHG emissions depending on the underlying bundle of fuels that compose it, and that will create variation in emission intensity. A specification that maps variation in energy intensity to fuel substitution is desirable in the context of greenhouse gas emissions, where most emissions reductions come from substituting cleaner fuels for dirty fuels rather than the end-of-pipe solutions. Moreover, I can exploit the mapping between the inputs (fossil fuels) and emissions to study emissions reduction efforts.

This technology also has another important implication. By introducing fuel-specific efficiency terms $\lambda_{\ell j}$, firms in different industries face different realized fuel prices in terms of energy service per dollar. I call these prices "effective prices", even though underlying observed fuel prices (in units of fuel quantity per dollar) may be the same. This heterogeneity in effective prices implies that a firm with higher relative efficiency in a specific fuel perceives it as cheaper than other fuels because it can use it better. This is motivated by the empirical fact that firms in different industries purchase very different relative fuel quantities even though they often face the same fuel prices.

$$\textbf{Observed}: \begin{bmatrix} p_\ell \\ p \end{bmatrix} = \begin{bmatrix} p_{\ell s} \\ \left(\sum_\ell p_\ell^{1-\sigma}\right)^{1/(1-\sigma)} \end{bmatrix} \longrightarrow \textbf{Effective:} \begin{bmatrix} \tilde{p}_{\ell j} \\ \tilde{p}_j \end{bmatrix} = \begin{bmatrix} \frac{p_\ell}{\lambda_{\ell j}} \\ \left(\sum_\ell \lambda_{\ell j}^{\sigma} p_\ell^{1-\sigma}\right)^{1/(1-\sigma)} \end{bmatrix}$$

Moreover, due to the constant returns assumption, individual effective fuel prices can be aggregated into an effective fuel price index $\tilde{p}_j$ that buys quantities of the composite fuel index F_{ij}, which is akin to labor as a single input in Melitz (2003a)[4]. This makes the analysis of firms' scale decisions separate from the analysis of relative input choices. These features have important theoretical implications and will greatly simplify the model estimation.

Asymmetric carbon tax

[4] I assume constant returns to scale in the production of the fuel composite. However, this does not require constant returns in other inputs such as labor, capital, and intermediate materials. Here, other inputs get absorbed by productivity A because they are unobserved. If these inputs were observed, the constant returns fuel production function could be nested into another production function that takes the fuel composite along with labor, capital, and intermediate materials as inputs. This is the approach typically taken in the fuel substitution literature (Atkinson and Luo, 2023; Hyland and Haller, 2018; Wang and Lin, 2017; Ma et al., 2008; Cho et al., 2004; Pindyck, 1979).

I now introduce the main object of analysis to this framework: a carbon tax that affects a fraction $\tilde{N}_j^r$ of firms within each industry. Regulated firms are indexed by $s = r$ and face an additional per-unit tax rate $\{\tau_\ell\}_{\ell=1}^L$ which is added to gross fuel prices $p_{\ell r} = p_\ell + \tau_\ell \; \forall \ell$, while the remaining fraction $\tilde{N}_j^u$ unregulated firms are indexed by status $s = u$ and face same gross fuel price as before. Since this is a carbon tax, the fuel-specific tax rate is weakly increasing in fuel pollution intensity:

$$\gamma_\ell \geq \gamma_k \rightarrow \tau_\ell \geq \tau_k \; \forall k \neq \ell$$

The rationale behind this framework is that implementing a uniform tax rate on GHG emissions is typically achieved with a different tax rate across fuels due to varying fuel emission intensity γ_ℓ. This is exactly how B.C. introduced its carbon tax. For example, coal combustion emits, on average, twice as much CO_{2e} in the air as natural gas, and its tax rate is twice as high.

3.3 Firms' Optimal Decisions

Output Quantity and Price

By the constant returns to scale assumption, marginal costs are constant, and I can solve firms' problems in two parts which correspond to the scale factor and the process factor of GHG emissions. This is important because it allows me to quantify the model using pollution release data only. First, I solve the profit-maximizing amount of output quantity $Y_{js}(A)$ that a firm with productivity A in industry j and regulation regime s wants to produce from purchasing the composite fuel good $F_{js}(A)$, taking as given the composite effective fuel price index $\tilde{p}_{js}$. From this, I can know the equilibrium output of each firm $Y_{js}(A)$, the share of output allocated to fuels $F_{js}(A)$, and the output price $P_{js}(A)$, which will all be reflected in the scale factor of GHG emission. In the second part, I can solve the cost-minimizing bundle of fuels that will form $F_{js}(A)$. As such, I map the relative share of each fuel bought to the emission intensity of each firm, which forms the process factor of GHG emission.

I assume that firms compete over quantity and face an inverse demand derived from Equations 1 and 2. To avoid notational clutter, I will do the exposition for a single industry and remove the j subscript. Then, taking as given industry aggregates P and Y, a firm with productivity A in regulation status s solves:

$$\max_{Y_s(A)} \left\{ P_s(Y_s(A))Y_s(A) - \tilde{p}_s F_s(A) \right\}$$
$$\text{s.t. } P_s(Y_s(A)) = \left(\frac{Y_s(A)}{\beta C} \right)^{-1/\rho} P^{(\rho-1)/\rho}$$

Since $F_s(A) = \frac{Y_s}{A}$, each firm's marginal cost is the ratio of input price index to productivity: $c_s(A) = \frac{\tilde{p}_s}{A}$. This marginal cost leads to a standard monopolistic competition equilibrium pricing equation of a constant markup over marginal costs:

$$P(\tilde{p}_s, A) = \frac{\rho}{\rho - 1} \frac{\tilde{p}_s}{A} \tag{4}$$

Aggregate Productivity

This individual pricing equation can be aggregated into an industry price index as in Melitz (2003a). To do so, I assume that productivity comes from a known distribution with density $g(A)$. In principle, this productivity distribution could be used to derive the productivity distribution of active firms in each region, $\mu_s(A)$, which would depend on a productivity threshold that defines which firms operate in the market. This threshold would depend on regulation status since the carbon tax raises the relative marginal cost of regulated firms, thus increasing the productivity required for the marginal regulated firm to operate and decreasing the productivity required for the marginal unregulated firm. However, in the Appendix, I show that entry/exit does not appear to be an empirically relevant margin of adjustment in the context of the B.C./Quebec carbon taxes, which is why I shall assume for the remainder of this exposition that the productivity threshold is the same across regulation status $\mu_s(A) = \mu(A)$ and I assume an interior solution for all firms such that the distribution of productivity for active firms is the same as the distribution of productivity for all firms, $\mu(A) = g(A)$. Then,

$$\tilde{A} = \left(\int_0^\infty A^{\rho-1} g(A) dA \right)^{1/(\rho-1)} \tag{5}$$

With a share $\tilde{N}^r = N^r/N$ of regulated firms and $\tilde{N}^u = N^u/N$ of unregulated firms, the equilibrium industry price index will be a CES aggregate of regulated and unregulated individual firm prices.

Moreover, when $g_s(A) = g(A)$, this price index has the same structure as firm-level output prices, where aggregate marginal cost is a combination of regulated and unregulated input prices weighted by the respective mass of firms in each status over aggregate productivity:

$$
\begin{aligned}
P &= \left(\sum_s \tilde{N}^s \int_0^\infty P_s(A)^{1-\rho} g(A) dA \right)^{1/(1-\rho)} \\
&= \frac{\rho}{\rho - 1} \left(\tilde{p}_r^{1-\rho} \tilde{N}^r + \tilde{p}_u^{1-\rho} \tilde{N}^u \right)^{1/(1-\rho)} \left(\int_0^\infty A^{\rho-1} g(A) dA \right)^{1/(1-\rho)} \\
&= \frac{\rho}{\rho - 1} \frac{\left(\tilde{p}_r^{1-\rho} \tilde{N}^r + \tilde{p}_u^{1-\rho} \tilde{N}^u \right)^{1/(1-\rho)}}{\tilde{A}}
\end{aligned}
\tag{6}
$$

Note that absent of the carbon tax, this price index collapse to the price index in Melitz (2003a) where labor is replaced by the composite fuel index: $P = \frac{\rho}{\rho-1} \frac{\tilde{p}}{\tilde{A}}$. Putting everything together, I can now find the quantity of the composite fuel purchased by a firm with productivity A and regulation status s:

$$
F_s(A) = \frac{\rho - 1}{\rho} \frac{\tilde{p}_s^{-\rho}}{(\tilde{p}_r^{1-\rho} N^r + \tilde{p}_u^{1-\rho} N^u)} \beta C \left(\frac{A}{\tilde{A}} \right)^{\rho-1} \tag{7}
$$

The amount of composite fuel a firm wants decreases in the fuel price index and increases in relative productivity due to a competitive effect that allows more productive firms to produce more. Since I assume that aggregate consumption is Cobb-Douglas across industries, there is no substitution across industries (net of the income effect), and only within industry relative productivity matters for firms' decisions.

relative fuel share and input price index

To chose the cost-minimizing share of each fuel that composes $F_s(A)$, firms face the technology defined in equation 3, and their relative fuel choice will only be a function of the interior parameters of the technology, namely interfuel substitutability σ and fuel efficiencies λ_ℓ. Productivity will not matter for relative fuel quantities because it augments the composite fuel index rather than specific fuels. Moreover, I assume that firms take input prices as given and cannot affect such prices with their decisions because they are too small relative to the population of firms that make up global fuel demand. This is motivated by the idea that the set of Canadian manufacturing firms is small relative to the set of all firms and consumers that buy fuels around the world. Additionally, supply-side shocks such as new technologies (e.g., fracking) and geopolitical events that shift fuel supply in specific regions often drive fuel prices. Therefore, taking input prices and F as given, a firm in regulation status s solves the following:

$$\min_{\{q_s^\ell\}_{\ell=1}^L} \left\{ \sum_\ell p_{\ell s} q_s^\ell \right\} \text{ s.t. } F = \left(\sum_\ell \lambda_\ell (q_s^\ell)^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$$

The solution to this problem gives rise to the perceived input price index that varies across regulation status and fuel-specific perceived prices, whose implications were discussed earlier: $\tilde{p}_s = \frac{p_{\ell s}}{\lambda_\ell}$:

$$\tilde{p}_s = \left(\sum_\ell \lambda_\ell^\sigma p_{\ell s}^{1-\sigma} \right)^{1/(1-\sigma)}$$

This perceived price index is higher for regulated firms because $p_{\ell r} = p_\ell + \tau_\ell$ while $p_{\ell ur} = p_\ell$ and $\sigma > 1$. With this I can define the share of fuel ℓ that makes up the composite fuel F, which is a simple function of relative perceived input prices. Indeed, as the perceived price of a fuel increases, the relative quantity of that fuel decreases at rate σ due to substitution towards other fuels.

$$q_s^\ell(F) = \left(\frac{\tilde{p}_{\ell s}}{\tilde{p}_s} \right)^{-\sigma} F \tag{8}$$

Emission intensity

Emission intensity is the amount of GHG emissions as a fraction of output that a firm produces and can be endogenously determined by the conditional input demand of each fuel times its emission factor $\gamma_{\ell;}$ over the firm's output quantity:

$$\begin{aligned} e_s(A) &= \frac{\sum_\ell \gamma_\ell q_s^\ell(F_s)}{Y_s(A)} \\ &= \frac{1}{A} \sum_\ell \gamma_\ell \left(\frac{\tilde{p}_{\ell s}}{\tilde{p}_s} \right)^{-\sigma} \end{aligned}$$

As in Shapiro and Walker (2018), emission intensity is locally decreasing in productivity. Moreover, while there is no capital and labor in the technology, if a firm is highly intensive in capital and/or labor relative to fuel, this model will capture this as a higher productivity (less fuel required to produce a unit of output), hence a lower emission intensity, which is precisely what would happen if capital and labor were in the model[5].

When a firm gets regulated, it faces higher prices to purchase fuels that pollute more relative to other fuels. Due to possible substitution, it will change its optimal combination of fuels that make up F, and the distribution will shift towards fuels with lower emission factors, and emission intensity will decrease. For a set of fixed input prices, no carbon leakage is induced by emission intensity variation because unregulated firms still face the same gross price as before and will choose the same optimal bundle. Formally, it can be shown that for each fuel k, there is a threshold emission intensity, γ_k^* such that for each fuel more intensive than γ_k^*, increasing its prices will lead to a decrease in emission intensity in that industry ($\frac{\partial e_s(A)}{\partial p_k}\Big|_{\gamma_k > \gamma_k^*} < 0$) while the opposite is true for each fuel bellow γ_k^*, where

$$\gamma_k^* = p_k \frac{\sum_{\ell \neq k} \gamma_\ell (\lambda_\ell / p_\ell)^{-\sigma}}{\sum_{\ell \neq k} p_\ell^{\sigma-1} \lambda_\ell^\sigma} \tag{9}$$

For example, an increase in the relative price of fuels that pollute above γ_k^* will lead substitution towards less polluting fuels and a decrease in emission intensity, which is exactly what happens when firms face a carbon tax.

GHG emission and carbon leakage

Now that I have characterized the production structure in this economy, I can define equilibrium GHG emissions for a firm with productivity A, which is, by definition, the product of output quantity and emission intensity:

$$\begin{aligned}
GHG_s(A) &= Y_s(A)/A && \times\, e_s(A)A \\
GHG_s(A, \text{No tax}) &= \left(\frac{\rho-1}{\rho}\right)\frac{1}{\tilde{p}}\left(\frac{A}{\tilde{A}}\right)^{\rho-1}\beta C && \times\, \frac{\sum_\ell \gamma_\ell \tilde{p}_\ell^{-\sigma}}{\tilde{p}^{-\sigma}} \\
GHG_s(A, \text{Asymmetric tax}) &= \underbrace{\left(\frac{\rho-1}{\rho}\right)\frac{\tilde{p}_s^{-\rho}}{\left(\tilde{p}_r^{1-\rho}N^r + \tilde{p}_u^{1-\rho}N^u\right)}\left(\frac{A}{\tilde{A}}\right)^{\rho-1}\beta C}_{\text{scale factor}} && \times\, \underbrace{\frac{\sum_\ell \gamma_\ell \tilde{p}_{\ell s}^{-\sigma}}{\tilde{p}_s^{-\sigma}}}_{\text{process factor}}
\end{aligned}$$

[5]For this to be valid, however, I assume that fuel taxes/prices do not affect the price of unobserved inputs like capital and labor.

First, the scale factor is defined by macro parameters and relative perceived input price indices between regions. In contrast, the process factor is defined by relative input prices between different fuels that compose the input price index. As such, firms that are regulated face a higher relative perceived input price index and will produce less than before (scale effect). Moreover, regulated firms see a greater increase in the input price of the most polluting fuels, and they will substitute towards lesser polluting fuels, which will decrease emission intensity (process effect). From the perspective of unregulated firms, the increase in the perceived price index of regulated firms will increase their residual demand, and they will increase output. However, unregulated firms do not see any variation in relative fuel prices, and their emission intensity remains the same. The combination of higher output but unchanged emission intensity creates carbon leakage.

4 Data

The empirical application of this model will be a study of the BC carbon tax. I am initially considering three main fuel types: Natural Gas, Oil, and Coal. Since this is a closed-economy model, I am only looking at competition between Canadian manufacturing firms.

4.1 Pollution and fuel data

The primary dataset used for plant-level pollution is the National Pollutant Inventory Release (NPRI), which records each pollutant emitted by most Canadian plants since 2000. This dataset records 280 pollutants, but I will only look at five of the most relevant pollutants to differentiate different fuels (see the identification section for more details): Carbon Monoxide (CO), Mercury (H_g), Sulphur Dioxide (SO_2), Nitrogen Oxides (NO_x) and Particulate Matters (PM). Keeping only manufacturing plants, I get between 700 and 900 plants annually between 2002 and 2015.[6] To recover fuel quantities in mmBtu at the plant level, I invert each fuel's chemical reaction under standard stationary combustion practices[7]. Thus, I get five equations (pollutants) with three unknowns (fuels) that I solve by least squares minimization subject to a non-negativity constraint, and I do this procedure for each plant each year.

[6] I chose this time frame because most firms did not report to the NPRI before 2002 and because many other Canadian provinces introduced carbon taxes and other environmental regulations in 2016.

[7] There can be heterogeneity in the quantity of each pollutant released by each fuel primarily due to combustion efficiency, which will contribute to measurement error. However, heterogeneity in pollutants across fuels tends to be much larger.

$$CO_{it} = (Coal_{it} \times CO_c) + (Natgas_{it} \times CO_c) + (Oil_{it} \times CO_o)$$

$$H_{g,it} = (Coal_{it} \times H_{g,c}) + (Natgas_{it} \times H_{g,ng}) + (Oil_{it} \times H_{g,o})$$

$$SO_{2,it} = (Coal_{it} \times SO_{2,c}) + (Natgas_{it} \times SO_{2,ng}) + (Oil_{it} \times SO_{2,o})$$

$$NO_{x,it} = (Coal_{it} \times NO_{x,c}) + (Natgas_{it} \times NO_{x,ng}) + (Oil_{it} \times NO_{x,o})$$

$$PM_{it} = (Coal_{it} \times PM_c) + (Natgas_{it} \times PM_c) + (Oil_{it} \times PM_o)$$

Below is a table created from information provided by the U.S. Environmental Protection Agency (EPA) of the mapping between one mmBtu of each fuel and quantities (in pounds) of each pollutant released in the atmosphere:

Pollutant (mmBtu/lb)	Natual Gas	Oil	Coal
Carbon monoxide (CO)	0.04	0.033	0.208
Mercury (H_g)	0.001	1.122	2.591
Sulfur dioxide (SO_2)	0092	0.448	0.457
Nitrogen oxides (NO_x)	0.007	0.084	2.744
Particulate Matters (PM)	0	0.000007	0.000016

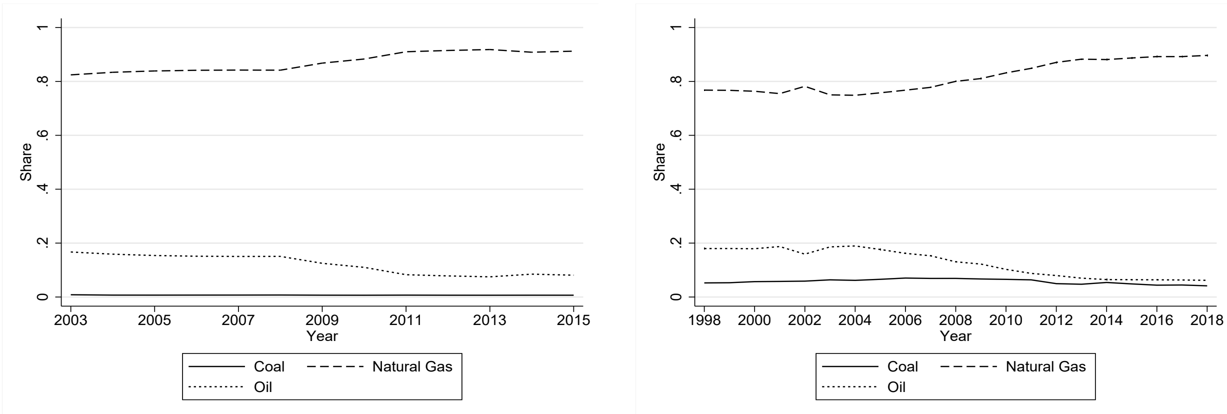
Table 1: Comparison of emission factors from natural gas, oil, and coal combustion

Source: EPA — AP-42 (2024). Notes: This table is constructed by averaging different measures of stationary emissions from the EPA's Compilation of Stationary Emission Factors (AP-42). Natural gas is a more homogeneous fuel than coal and oil. The emissions factors for coal are created by averaging emissions for different coal grades (anthracite, bituminous, and subbituminous). Similarly, the emissions factors for oil are the average emission factors for different types of fuel oils used in manufacturing, referring to different types of distillate and residual oils.

To validate this procedure, I compare an aggregate of estimated fuel shares from each plant to aggregate fuel shares available in the following Statistics Canada database: Manufacturing industries, total annual energy fuel consumption in gigajoules (Table: 25-10-0025-01). I also cross-checked this procedure with another dataset of U.S. firms that contained both plant-level fuel quantities and pollutants.

My estimation procedures recover oil shares quite well, but coal shares are underestimated relative to natural gas shares. This is partly because only a few firms use coal in Canada, with aggregate shares nearing 1%, potentially leading to unstable estimates.

Greenhouse gases



(a) Aggregates from plant level estimates

(b) Aggregtes from statscan

Figure 2: Aggregate fuel shares

Lastly, these pollutants are not greenhouse gases contributing to climate change (e.g., carbon dioxide CO_2, methane CH_4, and nitrous oxide N_2O) because the NPRI does not report greenhouse gases. Once I back out fuel quantities, I then use another conversion table from the EPA to convert fuel quantities to greenhouse gases. Using the GHG emission factors from Table 1, I then create a single measure of carbon dioxide equivalent CO_{2e} using the global warming potential method (EPA — GHG Emission Factors, 2024). This is the final measure I use in the paper to construct emissions.

	Carbon dioxide kg CO_2/mmBtu	Methane g CH_4/mmBtu	Nitrous Oxide g N_2O/mmBtu	Carbon dioxide equivalent kg CO_{2e}/mmBtu (100-Year GWP)
Coal	98.2	11	1.6	98.9
Oil	70.5	3	0.6	70.7
Natural Gas	56.1	1	0.1	56.2

Table 2: GHG Emission Factors

Source: EPA — GHG Emission Factors (2024). Notes: The last column is the final measure of GHG emissions.

While I use this procedure for only three fuels and five pollutants, it can be extended to include many more fuels and pollutants to improve accuracy. It can also be extended to study fuel usage by geolocated plants from remote sensing data, which may be interesting in regions that lack the regulatory body necessary to compile a dataset such as the NPRI.

4.2 Other Data

Fuel price data

Although there may be regional differences in fuel prices depending on the plant's location, I only look at global fuel prices to ensure the validity of the homogeneity assumption. Global fuel prices come from the World Bank database and are denominated in different units. To get around this issue, researchers usually convert fuel quantities into million British thermal units (mmBtu), which will be the unit of measure going forward. I use an average of crude oil prices across different sources for oil. For natural gas, I use the average price in the U.S., and for coal, I use the price of Australian coal as it is the most commonly used. Since prices are in USD, I convert them to nominal CAD prices.

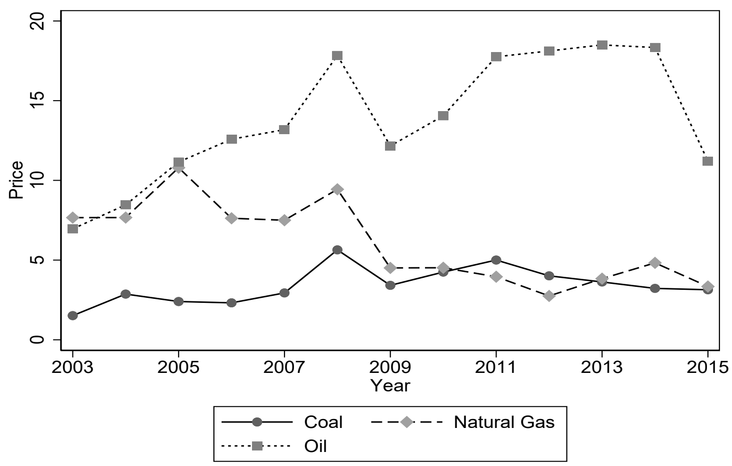


Figure 3: Fuel prices in unregulated provinces ($CAD/mmBtu)

Carbon tax

To construct the fuel-specific tax rates, I map one mmBtu of each fuel in tons of CO_{2e} equivalent using γ_ℓ which is calculated in Table 2, and I multiply this by the level of the carbon tax, which started at $10\$/ton$ in 2008 and increased by $5\$/ton$ every year until reaching $35\$/ton$ in 2013. Additionally, a much smaller carbon tax was introduced in 2008 in Quebec at $3.5\$/ton$:

	British Columbia						Quebec
	2008	2009	2010	2011	2012	2013-2015	2008-2015
Natural Gas ($/mmBtu)	0.53	0.8	1.06	1.33	1.59	1.86	0.19
Oil ($/mmBtu)	0.72	1.08	1.43	1.79	2.15	2.51	0.25
Coal ($/mmBtu)	0.99	1.48	1.98	2.47	2.97	3.44	0.35
CO_{2e} ($/ton)	10	15	20	25	30	35	3.5

Table 3: Carbon tax rates

Aggregate data

To get aggregate consumption over time, C_t, I use the share of Canadian nominal GDP from manufacturing. Then, I can get β_{jt}, the shares of Canadian manufacturing GDP that come from each 3-digit NAICS manufacturing industry. Since GDP is in dollar amount, this is going to be the share of manufacturing consumption associated with each industry: $\beta_{jt} = \frac{Y_{jt}P_{jt}}{C_j}$. Through period-by-period variation in C_t and β_{jt}, I allow for industry-specific demand shocks. To see this, if there is a one-time positive demand shock to industry j, this will increase aggregate demand C_t while simultaneously increasing β_{jt} and decreasing β_{kt} $\forall k \neq j$ such that the amount of consumption going to all other industries remains unchanged and $\sum_j \beta_{jt} = 1$ is still satisfied. Additionally, since there is only one big market in the model, a region-specific demand shock would effectively affect all firms regardless of their location. It would be captured by variation in C_t without changing relative industry shares.

5 Identification

Since the empirical application of this model is the B.C. and Quebec carbon taxes, I am only considering the universe of Canadian manufacturing firms under the plausible assumption that those firms take input prices as given and cannot affect global input prices with their individual decisions. As such, there will be no variation in aggregate fuel demand induced by carbon regulation that would affect equilibrium fuel prices, commonly referred to as the fossil-fuel channel of carbon leakage in the literature (Fowlie and Reguant, 2018). This mean that I can exploit observed fuel price variation to identify parameters in the model. In the data, such fuel price variation is due to global shocks such as the fracking boom that massively decreased natural gas prices during the sample period and variation in the supply of oil-producing countries. Added to these global shocks is the carbon tax that creates spatial variation in fuel prices and will be critical to identifying the elasticity of substitution across firms.

There are two sets of parameters in the model: parameters related to the production technology, such as baseline fuel shares, and interfuel substitutability: $\left\{\{\lambda_{\ell j}\}_{=1}^{L}\right\}_{j=1}^{L}, \sigma\}$ and parameters about firm-level output decisions (macro parameters), such as the elasticity of demand and the distribution of productivity: $\{\rho, \sigma_A^2\}$ and parameters of the productivity distribution. Ideally, if I had data on firm-level output and emission intensity, I could separately identify both sets of parameters like Shapiro and Walker (2018), Ganapati et al. (2020), and Aichele and Felbermayr (2015) since technology parameters only appear in emission intensity and other parameters only appear in firm-level output.

However, I only observe plant-level pollutants released along with aggregate industry statistics. An important contribution of this paper is to show identification of the direct and leakage effects of a counterfactual carbon policy on emissions under such flexible data requirements. To see this, I show in the following section how I can separate the estimation in two stages. The first stage relies *relative* fuel quantities within plants to identify technology and interfuel substitution parameters in closed-form using a linear seemingly unrelated regression model (SUR). In the second stage, I use the fuel quantities in *level* net of a fuel-specific component to measure the common scale component across fuels. This common scale component contains information about firm productivity and industry aggregates, which I use to identify macro parameters, such as the distribution of productivity and the elasticity of demand.

Before showing identification results, it is also to show some key equilibrium equations when I reintroduce the industry subscript j and year subscript t into the model:

$$\text{Composite fuel (scale): } F_{ijst} = \frac{\rho-1}{\rho}\frac{\tilde{p}_{jst}^{-\rho}}{(\tilde{p}_{jrt}^{1-\rho}N_{jt}^{r} + \tilde{p}_{jut}^{1-\rho}N_{jt}^{u})}\beta_{jt}C_t\left(\frac{A_{jst}}{\tilde{A}_{jt}}\right)^{\rho-1}$$

$$\text{Individual fuel quantities: } q_{ijst}^{\ell} = \left(\frac{\tilde{p}_{\ell jst}}{\tilde{p}_{jst}}\right)^{-\sigma}F_{ijst}$$

5.1 Stage 1: Identification of technology parameters

With estimated fuel quantities, I can use optimality conditions from the model to estimate interfuel substitution and fuel efficiency parameters in a seemingly unrelated regression model. Indeed, each fuel contains a fuel-specific factor driven by its “perceived price" and a common factor to all fuels that is proportional to the output of a given firm:

$$
\begin{aligned}
q^{\ell}_{ijst} &= \left(\frac{\tilde{p}_{\ell jst}}{\tilde{p}_{jst}}\right)^{-\sigma} F_{ijst} \\
&= \tilde{p}^{-\sigma}_{\ell jst} X_{ijst} \\
&= \underbrace{\left(\frac{p_{\ell st}}{\lambda_{\ell j}}\right)^{-\sigma}}_{\text{fuel-specific}} \underbrace{X_{ijst}}_{\text{common scale}}
\end{aligned}
$$

Where F_{ijst} is the factor common to all fuels. Since there are more than three fuels in reality, and since the pollutant factor to fuel conversion approximates chemical reactions under average conditions, it is likely that there is some measurement error in implied fuel quantities, $\exp(u^{\ell}_{ijst})$. Measurement error comes from unobservables such as heat at which combustion happens, the type of coal or oil product used, and the technology used for combustion, which I assume to be iid across time and firms:

$$
\hat{q}^{\ell}_{ijst} = \left(\frac{p_{\ell st}}{\lambda_{\ell ij}}\right)^{-\sigma} X_{ijst} \exp\left(u^{\ell}_{ijst}\right)
$$

Taking logs,

$$
\ln q^{\ell}_{ijst} = -\sigma \ln p_{\ell st} + \sigma \ln \lambda_{\ell j} + \ln X_{ijst} + u^{\ell}_{ijst}
$$

Since all fuels contain the common factor, I can eliminate it by subtracting each fuel $\ell = 2, 3$ by the first fuel, which is what allows me to identify technology parameters separately form the rest.

$$
\begin{aligned}
\ln q^{2}_{ijst} - \ln q^{1}_{ijst} &= \sigma(\ln p_{1st} - \ln p_{2st}) + \sigma(\ln \lambda_{j2} - \ln \lambda_{j1}) + (u^{2}_{ijst} - u^{1}_{ijst}) \\
\ln q^{3}_{ijst} - \ln q^{1}_{ijst} &= \sigma(\ln p_{1st} - \ln p_{3st}) + \sigma(\ln \lambda_{j3} - \ln \lambda_{j1}) + (u^{3}_{ijst} - u^{1}_{ijst})
\end{aligned}
$$

Intuitively, if the price of the first fuel increases, firms will substitute towards fuels 2 and 3, which will depend on the elasticity of substitution σ. I can rewrite this system of equations in matrix form:

$$\begin{bmatrix} \ln q^2_{ijst} - \ln q^1_{ijst} \\ \ln q^3_{ijst} - \ln q^1_{ijst} \end{bmatrix} = \sigma \begin{bmatrix} \ln p_{1st} - \ln p_{2st} \\ \ln p_{1st} - \ln p_{3st} \end{bmatrix} + \begin{pmatrix} \Gamma_{2j} \\ \Gamma_{3j} \end{pmatrix} + \begin{bmatrix} \epsilon^2_{ijst} \\ \epsilon^3_{ijst} \end{bmatrix}$$
$$\boldsymbol{\Delta} \ln \mathbf{q_{ijst}} = \sigma \boldsymbol{\Delta} \ln \mathbf{p_{st}} + \boldsymbol{\Gamma_j} + \boldsymbol{\epsilon_{ijst}}$$

Where $\Gamma_{\ell j} = \sigma(\ln \lambda_{j\ell} - \ln \lambda_{j1})$ and $\epsilon^{\ell}_{ijst} = u^{\ell}_{ijst} - u^1_{ijst}$. this is a SUR model because ϵ^2_{ijst} is correlated with ϵ^3_{ijst} but uncorrelated across firms and across time. Identification of the elasticity of substitution comes from variations in aggregate fuel prices over time, along with the carbon taxes that were implemented in B.C. and Quebec. Estimating $\{\sigma, \Gamma_{2j}, \Gamma_{3j}\}$ is a reduced form that will recover all technology parameters as follows:

$$\exp(\Gamma_{2j}/\sigma) = \frac{\lambda_{2j}}{\lambda_{1j}}$$
$$\exp(\Gamma_{3j}/\sigma) = \frac{\lambda_{3j}}{\lambda_{1j}}$$
$$\lambda_{1j} = 1 - \lambda_{2j} - \lambda_{3j]}$$

$\lambda_{1j}, \lambda_{2j}, \lambda_{3j}$ will then be the solution to the above system of three equations. Since I am only keeping oil and natural gas in the model, the fuel-specific efficiency terms can be recovered in closed form:

$$\hat{\lambda}_{ng,j} = \frac{\exp(\hat{\Gamma}_j/\hat{\sigma})}{\exp(\hat{\Gamma}_j/\hat{\sigma}) + 1}$$
$$\hat{\lambda}_{oil,j} = \frac{1}{\exp(\hat{\Gamma}_j/\hat{\sigma}) + 1}$$

5.2 Stage 2: Identification of macro parameters

Upon estimating technology parameters, I can use the levels of log fuel quantities substracted from a substitution term to recover the common scale component, F_{ijst} plus fuel-specific measurement error:[8]

[8] Note that F_{ijst} differs from the “true" scale component X_{ijst} because I also substract input price index in equation 10, which is common to all fuels.

$$\ln q_{ijst}^{\ell} + \hat{\sigma}(\ln \tilde{p}_{\ell jst} - \ln \tilde{p}_{jst}) = \ln F_{ijst} + u_{ijst}^{\ell} \quad \forall \ell \in \{oil, gas, coal\} \tag{10}$$

I can open up this expression in equilibrium because F_{ijst} is just an industry and time augmented version of equation 7. It is a function of the elasticity of demand, ρ, unobserved aggregate productivity $\tilde{A}_{jt}$, unobserved firm-specific productivity A_{ijst}, unobserved fuels-specific measurement error u_{ijst}^{ℓ}, and many quantities that are now observed (input price indices $\tilde{p}_{jst}$, share of firms in different regions $\tilde{N}_{jt}^{s}$, industry output share β_{jt}, and aggregate output C_t). Hence, in matrix form, we can write this as a system of estimating equations:

$$\begin{aligned}\underbrace{\ln \mathbf{q}_{\mathbf{ijst}}^{\ell} + \boldsymbol{\hat{\sigma}}(\ln \mathbf{\tilde{p}}_{\ell \mathbf{jst}} - \ln \tilde{p}_{jst})}_{y_{ijst}} = \underbrace{\ln\left(\frac{\rho-1}{\rho}\right) + \ln \beta_{jt} + \ln C_t - \rho \ln \tilde{p}_{jst} - \ln(\tilde{p}_{jrt}^{1-\rho}\tilde{N}_{jt}^{r} + \tilde{p}_{jut}^{1-\rho}\tilde{N}_{jt}^{u})}_{X_{ijst}(\rho)} \\ + \underbrace{(\rho-1)(\ln A_{ijst} - \ln \tilde{A}_{jt}) + \mathbf{u}_{\mathbf{ijst}}^{\ell}}_{\boldsymbol{\epsilon_{ijst}}}\end{aligned} \tag{11}$$

Where aggregate productivity in each industry was defined as:

$$\tilde{A}_{jt} = \left(\int_0^{\infty} A^{\rho-1} g_{jt}(A)\,dA\right)^{1/(\rho-1)}$$

I now assume a distribution for productivity and fuel-specific measurement error, allowing me to estimate this system with maximum likelihood. I assume that productivity is distributed log-normal with mean and variance common to all firms in all years $\tilde{A}_{jt} = \tilde{A}$, and that measurement error is normally distributed with mean zero and fuel-specific variance. I also allow measurement error to be correlated across fuels. The marginal distributions are then:

$$\begin{aligned}&A_{ijst} \sim LN(\mu, \sigma_A^2) \iff \ln A_{ijst} \sim N(\mu, \sigma_A^2) \\ &u_{ijst}^{\ell} \sim N(0, \sigma_{\ell}^2)\end{aligned}$$

Using the moment generating function for a log-normal distribution, I can get a closed-form solution for $\tilde{A}_{jt} = \tilde{A}$ (see appendix for details):

$$\tilde{A} = \left(\exp \left(\mu(\rho - 1) + (\rho - 1)^2 \sigma_A^2 / 2 \right) \right)^{1/(\rho - 1)} \tag{12}$$

With that in mind, the relative log productivity term will be normally distributed, such that the vector of fuel-specific error terms in estimating equation 11, $\boldsymbol{\epsilon_{ijst}}$, will be jointly normal (see appendix for details):

$$(\rho - 1)(\ln A_{ijst} - \ln \tilde{A}) \sim N\Big(-(\rho - 1)^2 \sigma_A^2 \big/ 2, (\rho - 1)^2 \sigma_A^2 \Big)$$

$$\begin{pmatrix} \epsilon_{ijst}^{ng} \\ \epsilon_{ijst}^{o} \end{pmatrix} \sim N\left(\begin{pmatrix} -(\rho - 1)^2 \sigma_A^2 \big/ 2 \\ -(\rho - 1)^2 \sigma_A^2 \big/ 2 \end{pmatrix}, \begin{pmatrix} (\rho - 1)^2 \sigma_A^2 + \sigma_{ng}^2 & (\rho - 1)^2 \sigma_A^2 + \sigma_{ng,o} \\ (\rho - 1)^2 \sigma_A^2 + \sigma_{ng,o} & (\rho - 1)^2 \sigma_A^2 + \sigma_o^2 \end{pmatrix} \right)$$

From this, I can construct the likelihood as the product of n bivariate normal density, take the natural logarithm, and estimate all the parameters: $\Big(\rho, \sigma_{ng}^2, \sigma_o^2, \sigma_A^2, \sigma_{ng,o}\Big)$.

To get a bit of intuition behind estimating the within-industry elasticity of substitution across firms ρ, note that input price variation only identifies ρ when it is spatially differentiated, in this case through the asymmetric carbon tax. To see this, if input prices are the same everywhere in a given period ($\tilde{p}_{jrt} = \tilde{p}_{jurt} = \tilde{p}_{jt}$), then firms' choice of the composite fuel index collapses to:

$$\ln F_{ijst} = \ln \left(\frac{\rho - 1}{\rho} \right) + \ln \beta_{jt} + \ln C_t - \ln \tilde{p}_{jt}$$

Intuitively, this happens because variation in input prices is the same for all firms in the industry, hence there is no incentive for consumers to substitute across firms. With the introduction of a tax, consumers will switch towards unregulated firms at rate $\rho > 1$ because the tax raises the marginal costs of regulated firms, which raises their output prices due to the monopolistic competition pricing rule.

6 Results

Technology parameters

Below are estimates of technology parameters. Since quantities of estimated coal are very low and only represent 1% of fuel consumption, I only keep natural gas and oil in the analysis. The standard error for the interfuel substitution parameter $\hat{\sigma}$ is an OLS standard error, and the standard errors for the fuel shares parameters are derived using the Delta Method (see appendix for details). I also construct 95% bootstrap confidence intervals.

Many firms often consume a dominant fuel; the model captures these quasi-corner solutions by $\hat{\lambda}_{j,\ell}$ close to either one or zero. Since natural gas is by far the most prominent fuel, $\hat{\lambda}_{j,oil}$ is often close to zero, which lead to a perceived price distribution that is much larger for oil than natural gas such that firms in most industries would never want to purchase oil:

	N	Mean	Std. Deviation	Min	Max
$\tilde{p}_{oil}$	10,532	1.62e+08	2.86e+08	6.957819	9.32e+08
$\tilde{p}_{natgas}$	10,532	645.9813	17690.24	2.752033	797652.4

Table 4: Summary statistics, perceived prices $\left(\tilde{p}_{\ell jt} = \frac{p_{\ell t}}{\lambda_{j\ell}}\right)$

Macro parameters

To get standard errors, I bootstrapped the entire dataset by blocks of individual firms, each sample being every year a firm operates. This allows for within-firm correlation across years, which is to be expected.

	Within-industry elasticity of substitution across firms	Variance of productivity	Variance of measurement error, natgas	Variance of measurement error, oil	Covariance between measurement error
	$\hat{\rho}$	$\hat{\sigma}_A^2$	$\hat{\sigma}_{ng}^2$	$\hat{\sigma}_o^2$	$\hat{\sigma}_{ng,o}$
Estimate	2.242	1.623	71.5	188.184	-39.86
S.E.	(0.7837)	(1.909)	(2.77)	(4.68)	(3.28)
95 % C.I.	[1.155, 3.57]	[0.46,6.27]	[67.72,77.79]	[182.11,196.56]	[-44.16,33.98]
Observations	10,532	10,532	10,532	10,532	10,532

Table 5: Estimates of macro parameters

				Baseline Natural Gas share: $\hat{\lambda}_{ng,j}$					
Industry	311	312	314	315	321	322	323	324	325
Estimate	0.984	0.871	0.007	0.000	1.000	1.000	1.000	0.983	1.000
S.E.	(0.023)	(0.176)	(0.104)	(0.001)	(0.000)	(0.002)	(0.000)	(0.022)	(0.001)
95 % CI lb	0.931	0.444	0.000	0.000	1.000	0.995	1.000	0.936	0.996
95 % CI ub	0.999	0.993	0.224	0.000	1.000	1.000	1.000	0.999	1.000
$\sum_{t=2003}^{2015} N_{jt}$	1,005	145	40	16	2,669	1,305	24	1,036	1,378
				Baseline oil share $\hat{\lambda}_{oil,j}$					
Industry	311	312	314	315	321	322	323	324	325
Estimate	0.016	0.129	0.993	1.000	0.000	0.000	0.000	0.017	0.000
S.E.	(0.023)	(0.176)	(0.104)	(0.001)	(0.000)	(0.002)	(0.000)	(0.022)	(0.001)
95 % CI lb	0.001	0.007	0.776	1.000	0.000	0.000	0.000	0.001	0.000
95 % CI ub	0.069	0.556	1.000	1.000	0.000	0.005	0.000	0.064	0.004
$\sum_{t=2003}^{2015} N_{jt}$	1,005	145	40	16	2,669	1,305	24	1,036	1,378
				Baseline Natural Gas share: $\hat{\lambda}_{ng,j}$					
Industry	326	327	331	332	333	335	336	337	339
Estimate	0.960	0.994	0.997	1.000	1.000	0.821	1.000	1.000	0.519
S.E.	(0.068)	(0.011)	(0.008)	(0.001)	(0.000)	(0.314)	(0.000)	(0.002)	(0.301)
95 % CI lb	0.792	0.967	0.979	0.998	1.000	0.069	1.000	0.995	0.050
95 % CI ub	0.998	1.000	1.000	1.000	1.000	0.998	1.000	1.000	0.961
$\sum_{t=2003}^{2015} N_{jt}$	195	903	980	189	57	42	328	99	121
				Baseline oil share $\hat{\lambda}_{oil,j}$					
Industry	326	327	331	332	333	335	336	337	339
Estimate	0.040	0.006	0.003	0.000	0.000	0.179	0.000	0.000	0.481
S.E.	(0.068)	(0.011)	(0.008)	(0.001)	(0.000)	(0.314)	(0.000)	(0.002)	(0.301)
95 % CI lb	0.002	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.039
95 % CI ub	0.208	0.033	0.021	0.002	0.000	0.931	0.000	0.005	0.950
$\sum_{t=2003}^{2015} N_{jt}$	195	903	980	189	57	42	328	99	121
				Interfuel elasticity of substitution: $\hat{\sigma}$					
Estimate					1.856				
S.E.					0.416				
95 % CI lb					1.040				
95 % CI ub					2.671				
N_{total}					10,532				

Table 6: Estimates of technology parameters

7 Counterfactuals

7.1 Causal effects of the B.C. and Quebec carbon taxes

I now have everything needed to recover counterfactual GHG emissions levels had the carbon tax not been introduced in B.C. and Quebec. In this counterfactual, regulated firms are always subject to unregulated input prices, and firms in other provinces do not get competitive gains from the input taxes raising input prices in B.C. and Quebec. Doing this exercise for regulated and unregulated firms separately will recover the tax's direct effect and the tax's carbon leakage effect separately. Let T_τ be the year the tax was implemented. GHG emissions then refer to the sum of GHG across firms for a given industry and year post-treatment (inner sum), the sum over all industries in a given year (middle sum), and the sum over all years post-treatment (outer sum). I do this for regulated and unregulated firms separately. I compare total GHG emissions generated by the model when prices include the tax (in red) subtracted to GHG emissions when prices exclude the tax (in blue) to get the effect of the carbon taxes.

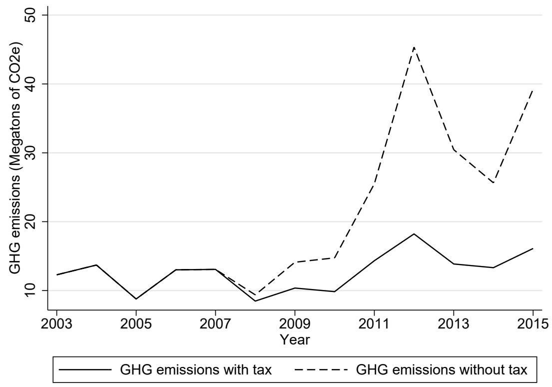
$$\text{Direct effect (BC firms): } \left(\sum_{t\geq T_\tau}\sum_{j}\sum_{i:r}\widehat{GHG}_{ijt}(\tilde{p}_{jrt},\tilde{p}_{jut})\right) - \left(\sum_{t\geq T_\tau}\sum_{j}\sum_{i:r}\widehat{GHG}_{ijt}(\tilde{p}_{jrt}=\tilde{p}_{jut},\tilde{p}_{jut})\right) =$$

$$= \sum_{t\geq T_\tau}\sum_{j}\sum_{i:r}\frac{\hat{\rho}-1}{\hat{\rho}}\beta_{jt}C_t\left(\frac{A_{ijrt}}{\tilde{A}}\right)^{\hat{\rho}-1}\left(\underbrace{\frac{\tilde{p}_{jrt}^{-\hat{\rho}}}{(\tilde{p}_{jrt}^{1-\hat{\rho}}+\tilde{p}_{jut}^{1-\hat{\rho}})}}_{\text{scale channel}}\underbrace{\frac{\sum_\ell \gamma_\ell \tilde{p}_{\ell jrt}^{-\hat{\sigma}}}{\tilde{p}_{jrt}^{-\hat{\sigma}}}}_{\text{process channel}} - \underbrace{\frac{1}{\tilde{p}_{jut}}}_{\text{scale channel}}\underbrace{\frac{\sum_\ell \gamma_\ell \tilde{p}_{\ell jut}^{-\hat{\sigma}}}{\tilde{p}_{jut}^{-\hat{\sigma}}}}_{\text{process channel}}\right)$$

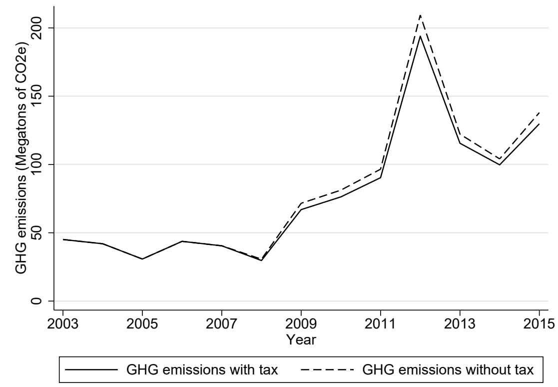
$$\text{Carbon leakage (Other firms): } \left(\sum_{t\geq T_\tau}\sum_{j}\sum_{i:u}\widehat{GHG}_{ijt}(\tilde{p}_{jrt},\tilde{p}_{jut})\right) - \left(\sum_{t\geq T_\tau}\sum_{j}\sum_{i:u}\widehat{GHG}_{ijt}(\tilde{p}_{jrt}=\tilde{p}_{jut},\tilde{p}_{jut})\right) =$$

$$= \sum_{t\geq T_\tau}\sum_{j}\sum_{i:u}\frac{\hat{\rho}-1}{\hat{\rho}}\beta_{jt}C_t\left(\frac{A_{ijut}}{\tilde{A}}\right)^{\hat{\rho}-1}\frac{\sum_\ell \gamma_\ell \tilde{p}_{\ell jut}^{-\hat{\sigma}}}{\tilde{p}_{jut}^{-\hat{\sigma}}}\underbrace{\left(\frac{\tilde{p}_{jrt}^{-\hat{\rho}}}{(\tilde{p}_{jrt}^{1-\hat{\rho}}+\tilde{p}_{jut}^{1-\hat{\rho}})} - \frac{1}{\tilde{p}_{jurt}}\right)}_{\text{scale channel}}$$

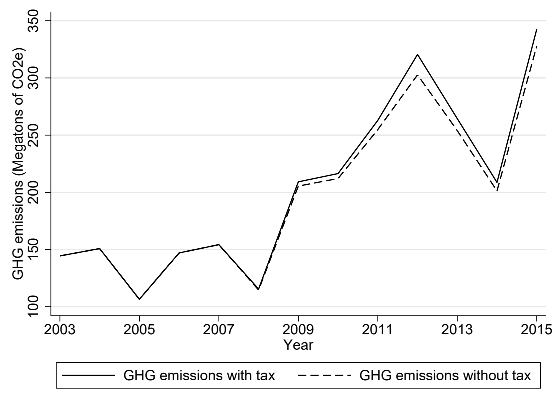
Moreover, I can decompose both the direct and carbon leakage effects into a scale channel (change in aggregate output) and a process/technique channel (change in aggregate emission intensity due to interfuel substitution). There is no process effect for unregulated firms because they do not face the carbon tax, and relative fuel prices stay constant. I also did this exercise for Quebec firms that faced a smaller carbon tax around the same period. Hence, the effect of the carbon tax on unregulated firms will be calculated by setting both the Quebec and B.C. carbon taxes to zero. I estimate an overall decrease in GHG emissions of 37.6 % in BC and 4.87 % in Quebec and an increase in GHG emissions of 2.6 % in unregulated provinces, which can be attributed to carbon leakage.



(a) British Columbia



(b) Quebec



(c) Unregulated Provinces

Figure 4: Estimated and counterfactual GHG emissions (kilotons $CO2_e$)

While aggregate percentage changes in emissions are suggestive that emissions decreased much more in British Columbia than they increased in unregulated provinces, this is misleading because the level of emissions in unregulated provinces is much larger at the baseline. There are many more firms in unregulated provinces, and those firms tend to concentrate on more polluting industries. As a result, significant leakage mitigates 45% of emissions reduction efforts.

In the next section, I document which effect drives most of this variation in GHG emissions. I also show what happens under a uniform carbon tax across all provinces, which the federal government introduced in 2018 with the Greenhouse Gas Pollution Pricing Act.

	Aggregate Emissions (kilotons CO_{2e}) (2003-2015) **No Tax**	Aggregate Emissions (kilotons CO_{2e}) (2003-2015) **Carbon Tax**	Level Difference (Carbon Tax - No Tax)	Percentage Difference (Carbon Tax - No Tax)
British Columbia	265	165	-100	-37.6%
Quebec	1,055	1,004	-51.4	-4.87%
Unregulated Provinces	2,575	2,643	67.4	2.6%

Table 7: Effect of Carbon Taxes on Emissions of Regulated and Unregulated Firms

Role of Fuel Substitution

The introduction of multiple fuels allows firms to substitute across fuels, which determines the GHG emissions process channel. Indeed, absent fuel substitution, firms only choose how much of the composite fuel they want to purchase, which will form their output (scale) decisions. By including the tax only in the perceived price of composite fuel $\tilde{p}_{sjt}$ and keeping the fuel specific perceived prices without the tax $\tilde{p}_{\ell jt}$ (and vice versa), I am able to decompose the overall effect of the carbon tax into the scaling factor and the fuel substitution factor.

My results suggest that almost all changes in GHG emissions (98.51% for B.C., 96.1% in Quebec, and 100% in unregulated provinces) are due to firms adjusting output, and only a tiny fraction is due to fuel substitution. In unregulated provinces, this is expected because unregulated firms did not face the tax directly, so there is no role fuel substitution. However, this is the case in B.C. and Quebec because most firms are consuming a dominant fuel, with the perceived price of other fuels being extremely high. Most dominant fuels tend to be natural gas, and the carbon tax raised the price of oil more than it raised the price of natural gas because the former releases more $CO_{2e}/mmBtu$. Thus, there is a limited role for substituting towards the newly relatively cheaper fuel (natural gas) because most firms already use large quantities of natural gas. The lack of a cleaner alternative to natural gas partly explains why carbon leakage is so large.

	Effect of carbon tax on GHG emissions (%)		
	British Columbia	Quebec	Unregulated Provinces
Scale	-37.1	-4.68	2.62
Fuel substitution	-1.2	-0.2	0
Scale and fuel substitution	-37.66	-4.87	2.62

Table 8: Difference in GHG emissions with counterfactual no policy, percentage

7.2 Uniform policy

An interesting aspect of this model is that I can impose a uniform carbon tax on all firms in all provinces such that there is no risk of carbon leakage, and I can see how it compares with the actual policy. I find that the reduction in GHG emissions would be between 19% and 22% depending on the province, which is lower than the 37.66% decrease of the actual policy in British Columbia

	Change in GHG emissions (%)
British Columbia	-23.77
Quebec	-22.82
Unregulated Provinces	-19.90

Table 9: Effect on GHG emissions of introducing a uniform carbon tax across Canada with a tax rate equivalent to the BC carbon tax (percentage)

Interestingly, this happens because as all firms face the same tax rate, there is no competitive hedge from being in one province versus another and all the production that would have shifted provinces due to the asymmetry of the carbon tax stays in the province of origin. The aggregate emission decrease is simply due to a uniform increase in marginal costs. Overall though, a uniform carbon tax is much better at reducing GHG emissions, estimated at 21% relative to a total decrease of 2.15% in the case of the asymmetric tax.

	Total change in GHG emissions (%)
BC carbon tax	-2.15
Uniform carbon tax	-21.15

Table 10: Total effect of implementing a uniform carbon tax across Canada relative to total effect of B.C. carbon tax (percentage)

8 Conclusion

In this paper, I build a production model where manufacturing firms can substitute between different fuels and compete across different regions. Using a panel of publicly available emissions data from Canadian plants, I can recover counterfactual emissions of regulated and unregulated plants following British and Quebec carbon taxes implemented in 2007 and 2008, respectively. I find that carbon leakage increased GHG emissions by 2.62 % in unregulated provinces relative to a decrease in emissions by 37.6% in British Columbia and 4.87% in Quebec and that most of the decrease in emissions by regulated plants was due to variation in output rather than substitution towards cleaner fuels.

However, these percentage differences hide a much larger effect of carbon leakage when looking at the level of emissions. Indeed, both carbon taxes accounted for a decrease in emissions by 150 kilotons of CO_{2e} by regulated firms relative to an increase in emissions of 67 kilotons of CO_{2e} by unregulated firms. Carbon leakage thus mitigated 45% of emissions reduction efforts in BC and Quebec. Introducing a uniform carbon tax across Canada as was legislated in the Greenhouse Gas Pollution Pricing Act of 2018 reduces total Canadian greenhouse gas emissions by 21 %, relative to a net decrease in emissions by 2.15% if the carbon tax was only implemented in BC and Quebec, significantly mitigating leakage concerns.

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Appendix

.1 Preliminary evidence for entry/exit

To investigate entry/exit as a result of the carbon tax, I run a simple difference-in-difference regression with the number of firms as the dependent variable. In both BC and Quebec, the control group comprises of firms in all other provinces and the treated period is 2008 onwards for both provinces. As an asymmetric carbon tax raises the marginal cost of regulated firms relative to the marginal cost of unregulated firms, standard Melitz theory suggests that the minimum productivity required to operate in regulated provinces would increases, decreasing the number of operating firms, and vice versa for unregulated provinces. Hence, the DiD coefficients should be negative. Here, it is positive, which is why I assume that the distribution of productivity remains the same after the tax. As discussed in the main text, this result is consistent with multiple findings in the literature.

	(1)	(2)	(3)
	___000001	___000001	___000001
time	-100.0***	-100.0***	-100.0***
	(2.57)	(2.57)	(2.43)
treated	-97.24***	-97.24***	-21.74***
	(4.19)	(4.19)	(3.38)
_diff	79.40***	79.40***	71.66***
	(5.55)	(5.55)	(4.47)
_cons	242.7***	242.7***	242.7***
	(1.89)	(1.89)	(1.79)
N	7888	7888	8813

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 11: Difference-in-Difference on Firm Entry/Exit

.2 Derivation perceived prices and relative fuel quantities

Starting from firm's cost minimization problem:

$$
\min_{\{q_s^\ell\}_{\ell=1}^L} \left\{ \sum_\ell p_{\ell s} q_s^\ell \right\} \text{ s.t. } F = \left(\sum_\ell \lambda_\ell (q_s^\ell)^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}
$$
$$
\mathcal{L} = \sum_{\ell=1} p_{\ell s} q_s^\ell + \mu \left(F - \left(\sum_\ell \lambda_\ell (q_s^\ell)^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \right)
$$

FOC:

$$
p_{\ell s} = \mu \left(\frac{\lambda_\ell}{q_s^{\ell 1/\sigma}} \left(\sum_\ell \lambda_\ell q_s^{\ell(\sigma-1)/\sigma} \right)^{1/(\sigma-1)} \right) \ \forall\ \ell
$$

I can divide fuel ℓ's FOC by fuel k's FOC:

$$
\frac{p_{\ell s}}{p_{ks}} = \left(\frac{q_s^k}{q_s^\ell} \right)^{1/\sigma} \frac{\lambda_\ell}{\lambda_k}
$$

Then,

$$
q_s^\ell = \left(\frac{p_{ks}}{p_{\ell s}} \frac{\lambda_\ell}{\lambda_k} \right)^\sigma q_s^k
$$

I can plug $q_s^\ell(q_s^k)$ into the technology:

$$
F = \left(\sum_\ell \left[\left(\frac{p_{ks}}{p_{\ell s}} \frac{\lambda_\ell}{\lambda_k} \right)^\sigma q_s^k \right]^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}
$$
$$
= q_s^k \left(\frac{p_{ks}}{\lambda_k} \right)^\sigma \left(\sum_\ell \lambda_\ell^\sigma p_{\ell s}^{1-\sigma} \right)^{\sigma/(\sigma-1)}
$$

Now I can define perceived prices:

$$
\begin{aligned}
\tilde{p}_{ks} &= \frac{p_{ks}}{\lambda_k} \\
\tilde{p}_s &= \Big(\sum_{\ell} \lambda_\ell^{\sigma} p_{\ell s}^{1-\sigma}\Big)^{1/(1-\sigma)}
\end{aligned}
$$

Then, I get equation (8) in the paper:

$$
\begin{aligned}
F &= q_s^k \left(\frac{\tilde{p}_{ks}}{\tilde{p}_s}\right)^{\sigma} \\
q_s^k &= \left(\frac{\tilde{p}_{ks}}{\tilde{p}_s}\right)^{-\sigma} F
\end{aligned}
$$

.3 Derivation of the log-likelihood for estimation of macro parameters

Recall the main estimating equation (eq. 10 in the paper):

$$
\begin{aligned}
\underbrace{\ln q_{ijst} + \boldsymbol{\hat{\sigma}}(\ln \mathbf{\tilde{p}_{\ell jst}} - \ln \mathbf{\tilde{p}_{jst}})}_{\mathbf{y_{ijst}}} = \underbrace{\ln\left(\frac{\rho-1}{\rho}\right) + \ln \beta_{jt} + \ln C_t - \rho \ln \tilde{p}_{jst} - \ln(\tilde{p}_{jrt}^{1-\rho} N_{jt}^{r} + \tilde{p}_{jurt}^{1-\rho} N_{jt}^{ur})}_{X_{ijst}(\rho)} \\
+ \underbrace{(\rho-1)(\ln A_{ijst} - \ln \tilde{A}_{jt}) + \mathbf{u_{ijst}}}_{\boldsymbol{\epsilon_{ijst}}}
\end{aligned}
$$

Assuming that $g_{jt}(A) = g(A)$, then $\tilde{A}_{jt} = \tilde{A}$. I now assume a normal distribution for the distribution of log-productivity and both measurement errors:

$$
\begin{aligned}
A_{ijst} &\sim LN(\mu, \sigma_A^2) \\
\iff \ln A_{ijst} &\sim N(\mu, \sigma_A^2) \\
u_{ijst}^{\ell} &\sim N(0, \sigma_\ell^2)
\end{aligned}
$$

Recall the definition of aggregate productivity:

$$\tilde{A} = \left(\int_0^\infty A^{\rho-1} g(A) dA \right)^{1/(\rho-1)}$$

Using the moment generating function for a log-normal distribution, I know that

$$\mathrm{E}[A^{\rho-1}] = \exp\left((\rho-1)\mu + (\rho-1)^2\sigma_A^2/2 \right)$$

I also know that $\tilde{A} = \mathrm{E}[A^{\rho-1}]^{1/(\rho-1)}$. Then,

$$\tilde{A} = \left(\exp\left(\mu(\rho-1) + (\rho-1)^2\sigma_A^2/2 \right) \right)^{1/(\rho-1)}$$

I now have everything to derive the distribution of the relative log-productivity term where $\ln \tilde{A}$ is just a constant that gets added to the mean but does not change the variance:

$$(\rho-1)(\ln A_{ijst} - \ln \tilde{A}) \sim N\left(-(\rho-1)^2\sigma_A^2 \Big/ 2, (\rho-1)^2\sigma_A^2 \right)$$

I can now write the joint distribution for the composite error term $\boldsymbol{\epsilon_{ijst}}$, which will be used to form the log-likelihood:

$$\begin{pmatrix} \epsilon_{ijst}^{ng} \\ \epsilon_{ijst}^{o} \end{pmatrix} \sim N\left(\begin{pmatrix} -(\rho-1)^2\sigma_A^2 \Big/ 2 \\ -(\rho-1)^2\sigma_A^2 \Big/ 2 \end{pmatrix}, \begin{pmatrix} (\rho-1)^2\sigma_A^2 + \sigma_{ng}^2 & (\rho-1)^2\sigma_A^2 + \sigma_{ng,o} \\ (\rho-1)^2\sigma_A^2 + \sigma_{ng,o} & (\rho-1)^2\sigma_A^2 + \sigma_o^2 \end{pmatrix} \right)$$

.4 Derivation of variance for fuel efficiency parameters

Recalling the relative fuel equation to estimate technology parameters when I only have two fuels:

$$\ln q_{ijst}^{ng} - \ln q_{ijst}^{o} = \sigma(\ln p_{o,st} - \ln p_{ng,st}) + \Gamma_j + \epsilon_{ijst}$$

Where

$$\Gamma_j = \sigma(\ln \lambda_{j,ng} - \ln \lambda_{j,o})$$

To estimate these reuced-form industry fixed-effects, I redefine them as a constant plus an industry-specific term: $\Gamma_j = \gamma + \delta_j$. To get the variance of $\lambda_{j,o}, \lambda_{j,ng}$, I first need to derive the variance of $\Gamma_j = \gamma + \delta_j$ where γ and δ_j are the estimated parameters:

$$\sqrt{n}(\hat{\gamma} + \hat{\delta}_j - (\gamma + \delta_j)) \sim \mathcal{N}(0, \sigma_\gamma^2 + \sigma_{\delta_j}^2)\ \forall j$$
$$\hat{\sigma}_{\Gamma_j}^2 = \hat{\sigma}_\gamma^2 + \hat{\sigma}_{\delta_j}^2$$

Next, I can used the delta method to recover the variance of structural parameters, $\lambda_{ng,j}, \lambda_{oil,j}\ \forall j$, where I showed in the paper that those are functions of the interfuel elasticity of subsitution, σ and reduced-form parameters Γ_j.

$$\hat{\lambda}_{ng,j} = \frac{\exp(\hat{\Gamma}_j/\hat{\sigma})}{\exp(\hat{\Gamma}_j/\hat{\sigma}) + 1}$$
$$\hat{\lambda}_{oil,j} = \frac{1}{\exp(\hat{\Gamma}_j/\hat{\sigma}) + 1}$$

By the delta method,

$$\sqrt{n}\begin{bmatrix}(\hat{\lambda}_{ng,j} - \lambda_{ng,j})\\(\hat{\lambda}_{oil,j} - \lambda_{oil,j})\end{bmatrix} \sim \begin{bmatrix}\frac{\partial \lambda_{ng,j}}{\partial \Gamma} & \frac{\partial \lambda_{ng,j}}{\partial \sigma}\\ \frac{\partial \lambda_{oil,j}}{\partial \Gamma} & \frac{\partial \lambda_{oil,j}}{\partial \sigma}\end{bmatrix}\begin{bmatrix}\mathcal{N}(0, \sigma_{\Gamma_j}^2)\\ \mathcal{N}(0, \sigma_\sigma^2)\end{bmatrix}$$

Where, the gradients are as follows:

$$\begin{bmatrix} \frac{\partial \lambda_{ng,j}}{\partial \Gamma} & \frac{\partial \lambda_{ng,j}}{\partial \sigma} \\ \frac{\partial \lambda_{oil,j}}{\partial \Gamma} & \frac{\partial \lambda_{oil,j}}{\partial \sigma} \end{bmatrix} = \begin{bmatrix} \frac{\exp(\Gamma_j/\sigma)}{\sigma(\exp(\Gamma_j/\sigma)+1)^2} & \frac{-\Gamma_j \exp(\Gamma_j/\sigma)}{\sigma^2(\exp(\Gamma_j/\sigma)+1)^2} \\ \frac{-\exp(\Gamma_j/\sigma)}{\sigma(\exp(\Gamma_j/\sigma)+1)^2} & \frac{\Gamma_j \exp(\Gamma_j/\sigma)}{\sigma^2(\exp(\Gamma_j/\sigma)+1)^2} \end{bmatrix}$$

Hence, the asymptotic variance of structural fuel shares by industry is as follow:

$$\begin{aligned} \sigma^2_{\lambda_{ng,j}} &= \left(\frac{\exp(\Gamma_j/\sigma)}{\sigma(\exp(\Gamma_j/\sigma)+1)^2}\right)^2 \sigma^2_{\Gamma_j} + \left(\frac{\Gamma_j \exp(\Gamma_j/\sigma)}{\sigma^2(\exp(\Gamma_j/\sigma)+1)^2}\right)^2 \sigma^2_{\sigma} \\ &= \sigma^2_{\lambda_{oil,j}} \end{aligned}$$

And I can use sample estimates of all those quantities to get the sample variance of fuel shares.