Sequences and Series - Lesson 1(Introduction to Sequences)

Grade 12 Mathematics

Introduction to Sequences

A sequence is a list of numbers arranged in a specific order. Each number in the sequence is called a term.

Example

- The sequence of natural numbers: $1, 2, 3, 4, 5, \ldots$
- The sequence of even numbers: $2, 4, 6, 8, 10, \ldots$
- The Fibonacci sequence: $0, 1, 1, 2, 3, 5, 8, 13, \ldots$
- The sequence defined by $a_n = \frac{1}{n}$: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

A sequence can be finite or infinite. An infinite sequence continues indefinitely.

Example

- Finite sequence: $3, 5, 7, 9$ (4 terms)
- Infinite sequence: $1, 2, 3, 4, \ldots$ (infinitely many terms)

In a sequence, each term can be identified by its position, called the index. The first term is T_1, the second term is T_2, and so on. The general term of a sequence can often be expressed as a function of its index n.

Example

- For the sequence $2, 4, 6, 8, \ldots$, the general term is $T_n = 2n$.
- For the sequence $1, 3, 5, 7, \ldots$, the general term is $T_n = 2n - 1$.

1 Types of Sequences

Sequences can be classified into different types based on their properties:

- Arithmetic sequences: Each term is obtained by adding a constant difference to the previous term.
- Geometric sequences: Each term is obtained by multiplying the previous term by a constant ratio.
- Harmonic sequences: The reciprocals of the terms form an arithmetic sequence.
- quadratic sequences: The difference between consecutive terms changes by a constant amount.

Example of Arithmetic Sequence

Example

Consider the arithmetic sequence $5, 8, 11, 14, \ldots$

- First term $a = 5$
- Common difference $d = 3$
- General term $T_n = a + (n-1)d = 5 + (n-1) \cdot 3 = 3n + 2$

Example of Geometric Sequence

Example

Consider the geometric sequence $3, 6, 12, 24, \ldots$

- First term $a = 3$
- Common ratio $r = 2$
- General term $T_n = a \cdot r^{n-1} = 3 \cdot 2^{n-1}$

Example of Harmonic Sequence

Example

Example of Harmonic Sequence Consider the harmonic sequence $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

- The reciprocals of the terms form an arithmetic sequence: $1, 2, 3, 4, \ldots$
- General term $T_n = \frac{1}{n}$

Example of Quadratic Sequence

Example

Consider the quadratic sequence $1, 4, 9, 16, \ldots$

- The general term is $T_n = n^2$
- The first differences are: $3, 5, 7, \ldots$
- The second differences are constant: $2, 2, 2, \ldots$

Summary

In this lesson, we introduced the concept of sequences, defined different types of sequences, and provided examples for each type. Understanding sequences is fundamental in mathematics, as they form the basis for series and many other mathematical concepts.