Week 5 Video 1

Relationship Mining

Correlation Mining

Relationship Mining

- Discover relationships between variables in a data set with many variables

- Many types of relationship mining

Correlation Mining

- Perhaps the simplest form of relationship mining
- Finding substantial linear correlations between variables
  - Remember this from earlier in the class?
- In a large set of variables

Use Cases

- You have 100 variables, and you want to know how each one correlates to a variable of interest
  - Not quite the same as building a prediction model

- You have 100 variables, and you want to know how they correlate to each other

Many Uses…

- Studying relationships between questionnaires on traditional motivational constructs (goal orientation, grit, interest) and student reasons for taking a MOOC
- Correlating features of the design of mathematics problems to a range of outcome measures
- Correlating features of schools to a range of outcome measures

The Problem

- You run 100 correlations (or 10,000 correlations)
- 9 of them come up statistically significant
- Which ones can you "trust"?

If you…

- Set p=0.05
- Then, assuming just random noise
- 5% of your correlations will still turn up statistically significant

The Problem

- Comes from the paradigm of conducting a single statistical significance test

The Solution

- Adjust for the probability that your results are due to chance, using a *post-hoc control*

Two paradigms

- **FWER** – Familywise Error Rate
  - Control for the probability that any of your tests are falsely claimed to be significant (Type I Error)

- **FDR** – False Discovery Rate
  - Control for the overall rate of false discoveries

Bonferroni Correction

- The classic approach to FWER correction is the Bonferroni Correction

Bonferroni Correction

- Ironically, derived by Miller rather than Bonferroni

Bonferroni Correction

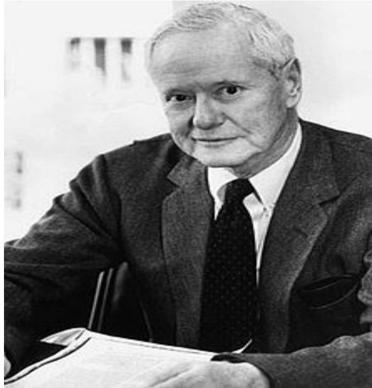
- Ironically, derived by Miller rather than Bonferroni
- Also ironically, there appear to be no pictures of Miller on the internet

Bonferroni Correction

- A classic example of Stigler's Law of Eponomy
  - "No scientific discovery is named after its original discoverer"

Bonferroni Correction

- A classic example of Stigler’s Law of Eponomy
  - “No scientific discovery is named after its original discoverer”
  - Stigler’s Law of Eponomy was proposed by Robert Merton



Bonferroni Correction

- If you are conducting *n* different statistical tests on the same data set

- Adjust your significance criterion α to be
  - α / n

- E.g. For 4 statistical tests, use statistical significance criterion of 0.0125 rather than 0.05

Bonferroni Correction: Example

- Five tests
  - p=0.04, p=0.12, p=0.18, p=0.33, p=0.55
- Five corrections
  - All p compared to $\alpha = 0.01$
  - None significant anymore
  - p=0.04 seen as being due to chance

Bonferroni Correction: Example

- Five tests
  - $p=0.04$, $p=0.12$, $p=0.18$, $p=0.33$, $p=0.55$

- Five corrections
  - All p compared to $\alpha = 0.01$
  - None significant anymore
  - $p=0.04$ seen as being due to chance
  - Does this seem right?

Bonferroni Correction: Example

- Five tests
  - $p=0.001$, $p=0.011$, $p=0.02$, $p=0.03$, $p=0.04$

- Five corrections
  - All p compared to $\alpha = 0.01$
  - Only $p=0.001$ still significant

Bonferroni Correction: Example

- Five tests
  - p=0.001, p=0.011, p=0.02, p=0.03, p=0.04

- Five corrections
  - All p compared to α= 0.01
  - Only p=0.001 still significant
  - Does this seem right?

Quiz

- If you run 100 tests, which of the following are statistically significant?

A) 0.05

B) 0.01

C) 0.005

D) 0.001

E) All of the Above

F) None of the Above

Bonferroni Correction

- Advantages
  - You can be "certain" that an effect is real if it makes it through this correction
  - Does not assume tests are independent
    - In our "100 correlations with the same variable" case, they aren't!
- Disadvantages
  - Massively over-conservative
  - Throws out everything if you run a lot of correlations

Often attacked these days

- Arguments for rejecting the sequential **Bonferroni** in ecological studies. MD Moran - Oikos, 2003 - JSTOR
- Beyond **Bonferroni**: less conservative analyses for conservation genetics. SR Narum - Conservation Genetics, 2006 – Springer
- What's wrong with **Bonferroni** adjustments. TV Perneger - Bmj, 1998 - bmj.com
- p Value fetishism and use of the **Bonferroni** adjustment. JF Morgan - Evidence Based Mental Health, 2007

There are FWER corrections that are a little less conservative…

- Holm Correction/Holm's Step-Down (Toothaker, 1991)
- Tukey's HSD (Honestly Significant Difference)
- Sidak Correction

- Still generally very conservative
- Lead to discarding results that probably should not be discarded

FDR Correction

- (Benjamini & Hochberg, 1991)

FDR Correction

- Different paradigm, arguably a better match to the original conception of statistical significance

Statistical significance

- $p<0.05$

- A test is treated as rejecting the null hypothesis if there is a probability of under 5% that the results could have occurred if there were only random events going on

- This paradigm accepts from the beginning that we will accept junk (e.g. Type I error) 5% of the time

FWER Correction

- $p<0.05$
- Each test is treated as rejecting the null hypothesis if there is a probability of under 5% divided by N that the results could have occurred if there were only random events going on
- This paradigm accepts junk far less than 5% of the time

FDR Correction

- $p<0.05$

- Across tests, we will attempt to accept junk exactly 5% of the time
  - Same degree of conservatism as the original conception of statistical significance

FDR Procedure (Benjamini & Hochberg, 1991)

- Order your n tests from most significant (lowest p) to least significant (highest p)
  - Test your first test according to significance criterion $\alpha*1 / n$
  - Test your second test according to significance criterion $\alpha*2 / n$
  - Test your third test according to significance criterion $\alpha*3 / n$
  - Quit as soon as a test is not significant

FDR Correction: Example

- Five tests
  - p=0.001, p=0.011, p=0.02, p=0.03, p=0.04

FDR Correction: Example

- Five tests
  - p=0.001, p=0.011, p=0.02, p=0.03, p=0.04
- First correction
  - p = 0.001 compared to α= 0.01
  - Still significant!

FDR Correction: Example

- Five tests
  - $p=0.001$, $p=0.011$, $p=0.02$, $p=0.03$, $p=0.04$
- Second correction
  - $p = 0.011$ compared to $\alpha = 0.02$
  - Still significant!

FDR Correction: Example

- Five tests
  - $p=0.001$, $p=0.011$, $p=0.02$, $p=0.03$, $p=0.04$
- Third correction
  - $p = 0.02$ compared to $\alpha = 0.03$
  - Still significant!

FDR Correction: Example

- Five tests
  - $p=0.001$, $p=0.011$, $p=0.02$, $p=0.03$, $p=0.04$

- Fourth correction
  - $p = 0.03$ compared to $\alpha = 0.04$
  - Still significant!

FDR Correction: Example

- Five tests
  - p=0.001, p=0.011, p=0.02, p=0.03, p=0.04
- Fifth correction
  - p = 0.04 compared to α= 0.05
  - Still significant!

FDR Correction: Example

- Five tests
  - p=0.04, p=0.12, p=0.18, p=0.33, p=0.55

FDR Correction: Example

- Five tests
  - p=0.04, p=0.12, p=0.18, p=0.33, p=0.55

- First correction
  - p = 0.04 compared to α= 0.01
  - Not significant; stop

Conservatism

- Much less conservative than Bonferroni Correction

- Much more conservative than just accepting $p<0.05$, no matter how many tests are run

q value extension in FDR (Storey, 2002)

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- p = probability that the results could have occurred if there were only random events going on

- q = probability that the current test is a false discovery, given the post-hoc adjustment

q value extension in FDR (Storey, 2002)

- q can actually be lower than p
- In the relatively unusual case where there are many statistically significant results

Closing thought

- Correlation mining can be a powerful way to see what factors are mathematically associated with each other
- Important to get the right level of conservatism

Next lecture

- Causal mining