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1983 Phys. Med. Biol. 28 535

(http://iopscience.iop.org/0031-9155/28/5/007)

Phys. Med. Biol., 1983, Vol. 28, No. 5, 535–545. Printed in Great Britain

Projected ranges and effective stopping powers of electrons with energy between 20 eV and 10 keV

H Iskef, J W Cunningham and D E Watt

Department of Medical Biophysics, The University, Dundee DD1 4HN, Scotland

Received 24 September 1981, in final form 8 October 1982

Abstract. Experimental projected ranges of electrons between 20 eV and 10 keV have been correlated by a simple scaling factor to yield 'best-fit' expressions applicable to all media. Extrapolated ranges, R_{ex} (in $\mu g\,cm^{-2}$) are given by

$$\ln((Z/A)R_{ex}) = -4.546\,7 + 0.311\,04 \ln E + 0.077\,73 (\ln E)^2$$

valid for $20\ eV < E < 10^4$ eV with a precision of 25% in $(Z/A)R_{ex}$ or

$$(Z/A)R_{ex} = \left(2.37\ {}^{+0.79}_{-0.59}\right) \times 10^{-3} E, \qquad 20\ eV \leq E \leq 370\ eV$$

and

$$(Z/A)R_{ex} = \left(1.294\ {}^{+0.291}_{-0.237}\right) \times 10^{-4} E^{1.492}, \qquad 370\ eV \leq E \leq 10^4\ eV$$

where Z/A is the ratio of charge to mass numbers for the absorbing medium. From an analysis of the transmission curves, the following relation between median or mean projected ranges and the extrapolated ranges is obtained

$$R_{50} = (0.3655 \pm 0.05) R_{ex}^{1.024}, \qquad 20\ eV \leq E \leq 10^4\ eV.$$

Differentiation of the extrapolated range-energy expression yields an effective stopping power along the projected track of the electron and is given by

$$dE/dR_{ex} = E/[R_{ex}(0.15546 \ln(E) + 0.31104)]\ eV\ cm^2\ \mu g^{-1},\ 20\ eV \leq E \leq 10^4\ eV$$

with a precision of 30%. Improved accuracy of the experimental data is required for track structure applications and to resolve phase effects.

1. Introduction

For use in the interpretation of track structure effects in radiobiology, published experimental information on penetration depths of electrons in the important δ-ray energy region between 20 eV and 10 keV has been correlated and compared, where possible, with theory.

Experimental ranges are usually determined from measurement of the fraction, η, transmitted through a known absorber thickness, x (figure A1, ICRU 1970). In practice, because of the influence of scattering and straggling effects on the shape of the transmission curve of number plotted against thickness, several definitions of range have evolved (ICRU 1970). The most important of these ranges are:

(i) The maximum range, R_m, is the thickness of absorber at which the recorded transmission reduces to an undetectable level. It is therefore a rather arbitrary quantity depending on the threshold sensitivity of the detector. Usually it will be larger than the extrapolated range.

(ii) The extrapolated range, R_{ex}, is determined by extrapolation of the linear portion of the transmission curve to the abscissa. At low electron energies subjective judgement is involved in determining this quantity because of the often short linear portion of the transmission curve (see section 5). For low-energy electrons, because of the occurrence of large energy transfers and angular deviations, R_{ex} is the quantity which, on the available experimental evidence, most closely corresponds to the theoretical R_{CSDA} range in the continuous slowing down approximation in materials with Z values ranging up to that of silver (Tung *et al* 1979). (For heavy particles the mean range, $\bar{R}$, corresponds most closely to the R_{CSDA}.)

(iii) The median range, R_{50}, is the thickness of absorber at which the transmitted fraction is reduced to 50%. In ICRU (1970) the median range is identified with the mean range to comply with general usage but, strictly, these two quantities can be different at low electron energies when the shape of the transmission curve is non-symmetrical about the 50% transmission point.

(iv) The mean range, $\bar{R}$, is the centre of gravity of the number relative to penetration depth transmission curve, $\eta(x)$: thus

$$\bar{R} = \int_0^{R_m} x \frac{d\eta}{dx} dx$$

to a good approximation determined by the reliability of R_m.

(v) Additionally, transmission ranges, R_q, are in use in which q is the residual transmitted percentage selected arbitrarily, e.g., 5%, 1%, etc.

Only recently have reasonably successful theoretical treatments of the ranges of low-energy electrons in absorbing media appeared in the literature (e.g., Tung *et al* 1979) but these are CSDA ranges and consequently it is not yet possible to relate these ranges accurately to the experimental quantities.

In the following sections empirical range–energy relations, which are applicable to all media, are derived from experimental data for electrons with energies between 20 eV and 10 keV. Ranges discussed are extrapolated ranges and mean or median ranges. From the extrapolated ranges effective stopping powers are derived.

2. Extrapolated range–energy relations

Weber (1964) has proposed a semi-empirical equation to calculate the extrapolated range of electrons in aluminium in the energy region 3 keV–3 MeV, which Kobetich and Katz (1968, 1969) later extended for application to low-energy electrons of 0.3 keV–20 MeV in water. Tabata *et al* (1972) generalised the Weber equation for absorbers of atomic number 6–92 for electrons in the energy region 0.3 keV–30 MeV (their calculated results are shown later in figure 3). A number of investigators have proposed empirical range–energy expressions to describe low-energy electron projected ranges in various absorbing media in solid and gaseous form. These are summarised in table 1. Recently Akkerman and Chernov (1980) have proposed a few empirical formulae for different solid materials.

The types of ranges measured have been identified on the basis of the foregoing definitions. Generally the assumption is made that the range–energy relationship has the form

$$R = kE^n. \tag{1}$$

Here n is a constant which some investigators find to be dependent on the absorber

Table 1. Summary of projected range–energy measurements for electrons <10 keV.

Investigator	Range–energy expression	Units of E and R	Valid for electron energy (keV)	Type of range measured	Absorbing medium
Lane and Zaffarano (1954)	$R = 5.463\times10^{-5}E^{1.67}$	E, eV R, μg cm^{-2}	1.5 → 40	Extrapolated range	Al, collodion, Formvar, plastic
Young (1956a, b)	$R = 142.763\times10^{-5}E^{1.3}$ $R = 102.494\times10^{-5}E^{1.35}$	E, eV R, μg cm^{-2}	0.3 → 10	Extrapolated range	Al Al_2O_3
Grün (1957)	$R = 2.570\times10^{-5}E^{1.75}$	E, eV R, μg cm^{-2}	5 → 54	Extrapolated range	Air
Feldman (1960)	$R = 250(A/\rho Z^{n/2})E^{n}$ $n = 1.2/(1-0.29\log Z)$	E, keV R, Å	1 → 10	Maximum range	Mg, Al, ZnS, Ni, Ag, Sn, Au, Pb, CaF_2, $MgSiO_3$, Zn_2SiO_4, $CaWO_4$
Cosslett and Thomas (1964)	$R = 29.09\times10^{-5}E^{1.5}$ $R = 27.23\times10^{-5}E^{1.5}$ $R = 27.46\times10^{-5}E^{1.5}$	E, eV R, μg cm^{-2}	5 → 15	Extrapolated range	Cu Ag Au
	$R = 86.62\times10^{-5}E^{1.37}$ $R = 100.56\times10^{-5}E^{1.34}$	E, eV R, μg cm^{-2}	2 → 5	Extrapolated range	Cu Al
Kanicheva and Barzdo (1964)	$R = 2.42\times10^{-5}E^{1.8}$	E, eV R, μg cm^{-2}	2 → 15	Maximum range	Cu
Cole (1969)	$R = 2.111\times10^{-5}$ $(E+367)^{1.77}-0.7$	E, eV R, μg cm^{-2}	0.02 → 100	Extrapolated range	Collodion, air
Thomas and Pattison (1970)	$R = 256.5\times10^{-5}E^{1.3}$	E, eV R, μg cm^{-2}	0.3 → 12	Maximum range	Al
Gledhill (1973)	$y = -5.100+1.358x+$ $0.215x^2-0.043x^3$ where $y=\log R$ $x=\log E$	E, keV R, g cm^{-2}	0.1 → 100	Extrapolated range	Air and other light material
Fitting (1974)	$R = 900\,\rho^{-0.8}E^{1.3}$	E, keV R, Å ρ, g cm^{-3}	0.1 → 10	Maximum range	Be, Al, Ge, Cu, Ag, Au
Wulff and Gledhill (1974)	$R = 2.513\times10^{-5}E^{1.74}$	E, eV R, μg cm^{-2}	2 → 20	Extrapolated range	Based on calculated data of Berger and Seltzer for air

medium (Young 1956a, b, Feldman 1960) or on the electron energy (Cosslett and Thomas 1964), whereas others find it to be non-parametric. However Sugiyama's (1974) semi-empirical expression, based on modified Bethe theory for electron energy below 10 keV, gives a good representation of stopping power for electrons at energies down to 200 eV (Iskef *et al* 1981) and it indicates that the parameter k is directly proportional to the ratio of mass to effective charge, A/Z^*, of the constituent atoms of the absorber. The ratio Z^*/A for Al to Au varies by a factor of 2.66 at 200 eV to 1.23 at 6 keV. This suggests that Z^*/A will be a better scaling factor than simply Z/A. However Z^* is obtained empirically and is given in a rather complex formula. Its divergence from Z becomes particularly important at electron energies below 1 keV for $Z < 50$. Because of the wide spread in the experimental data at low electron energies, and the limitations of Sugiyama's semi-empirical expression, it was thought more appropriate to scale the ranges by the purer Z/A ratio. This normalisation may not be optimum as there could be an additional Z dependence as predicted by theory and observed unambiguously only at electron energies higher than those considered here (see section 5).

Figure 1 shows the extrapolated ranges, scaled by the factor Z/A, as a function of electron energy. Consideration of the trend of the available stopping power information (mostly theoretical) with energy, leads to the conclusion that an expression of the form

$$\ln[(Z/A)R_{ex}] = a_0 + a_1 \ln E + a_2(\ln E)^2 \tag{2}$$

if fitted to the experimental extrapolated range–energy data (figure 1) should give a good representation which is self-consistent with the underlying theoretical trends in stopping power. The 'best-fit' coefficients for equation (2) are $a_0 = -4.5467$, $a_1 = 0.31104$, and $a_2 = 0.07773$, based on 261 datum points. R_{ex} is in $\mu g\,cm^{-2}$ and E in eV. The standard error of estimate is 25%.

Alternatively, the experimental data in figure 1 may be represented in the more convenient empirical form given by equation (1) which, upon least squares analysis, becomes

$$(Z/A)R_{ex} = \left(2.37^{+0.79}_{-0.59}\right) \times 10^{-3} E, \qquad 20\,eV \leq E \leq 370\,eV \tag{3}$$

and

$$(Z/A)R_{ex} = \left(1.294^{+0.291}_{-0.237}\right) \times 10^{-4} E^{1.492}, \qquad 370\,eV \leq E \leq 10^4\,eV \tag{4}$$

where E is in eV and R_{ex} in $\mu g\,cm^{-2}$. The errors are standard errors of estimate and reflect the wide spread in the experimental results.

Maximum range data, shown in figure 1 for comparison, are omitted from the least squares analysis. These are the data of Fitting (1974), who measured 1% residual transmission ranges which do, nevertheless, correspond well with the extrapolated ranges above 1 keV, and of Thomas and Patterson (1970). Also omitted are the extrapolated ranges of Adams and Hansma (1980), whose results are abnormally low for reasons which are not clear but possibly because of failure to correct for optical absorption in their frozen gas absorber layers, and the theoretical data of Berger and Seltzer (1970) as these are CSDA ranges.

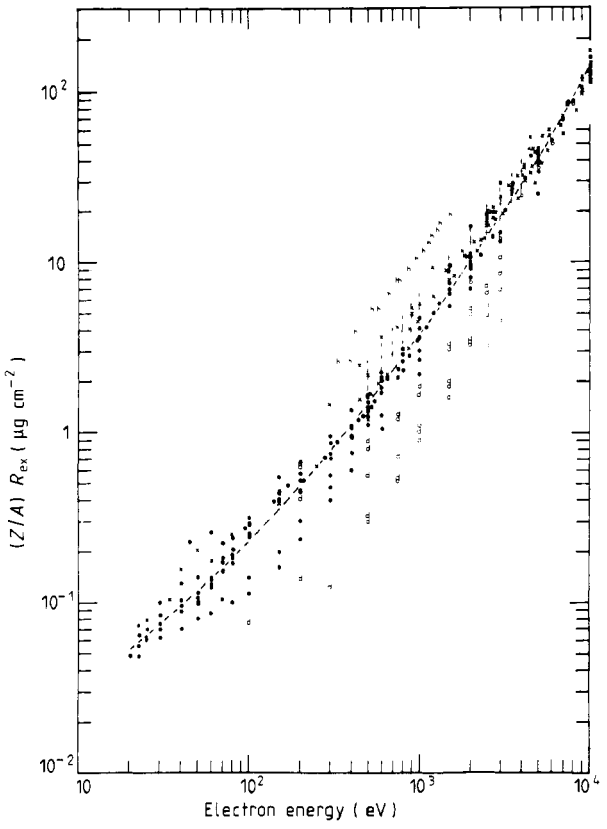

Figure 1. Reduced extrapolated ranges, $(Z/A)R_{ex}$ (μg cm^{-2}), deduced from the results of the investigators given below are shown for various media. Results indicated by filled circles (gases) and crosses (solids) were used to obtain equations (2), (3) and (4). The broken line was obtained using equation (2).

	Media	**Investigator**
Gases	TE gas, CH_4, CO_2, N_2, Ar	Smith and Booz (1978)
	N_2, CH_4	Waibel and Grosswendt (1981)
	N_2	Barrett and Hays (1976)
		Cohn and Caledonia (1970)
	Air	Cole (1969)
		Hartman (1968)
		Grün (1957)
Solids	H_2O vapour	ICRU (1970)
	Collodion	Cole (1969)
		Lane and Zaffarano (1954)
	Barium stearate	Mori *et al* (1980)
	Al, Au	Holliday and Sternglass (1959)
	Al	Klemperer and Thetford (1960)
	Al_2O_3	Hoffman (1955)
	Al, Cu, Au, Ag	Cosslett and Thomas (1964)
	Al, Au	Kantner and Sternglass (1962)
	Au	Kanicheva and Burtsev (1959)
	Al, Al_2O_3	Young (1956)

Results indicated by: f, h, d and b are excluded (see text).

Key:			
	f	Be, Al, Ge, Ag, Au	Fitting (1974)
	h	Al	Thomas and Pattinson (1970)
	d	Solid N_2, O_2, Ar, Kr, Xe	Adams and Hansma (1980)
	b	Air (calculated)	Berger and Seltzer (1970)

Several authors (Cole 1969, Gledhill 1973, Mori *et al* 1980, ICRU 1970) have compared their extrapolated ranges with Davis's (1954a, b, c, 1955) penetration depths deduced from enzyme inactivation studies and have indicated poor agreement. Such a comparison is invalid as Davis's technique yields a range approaching the mean range, $\bar{R}$, and not the extrapolated range (see for example the appendix in Watt and Sutcliffe (1975)). Consequently Davis's ranges have been included with the set of mean ranges where it will be seen that her results are in good agreement with others. Davis observed an apparently constant range at energies less than 100 eV. This is an artifact due to the finite size of the detector, i.e., the enzyme molecule. These constant range values are not included in the analyses.

The spread of results for the extrapolated ranges is such that no definite conclusions can be drawn about the existence of physical phase effects for electrons (Ha:nm *et al* 1981) although there does seem to be some indication of an excess range in solids compared with gases at energies below 200 eV (figure 1). A comparison with the R_{CSDA} ranges is discussed in section 4.

3. Mean and median ranges

With the exception of penetration depths determined by enzyme inactivation (Davis 1954a, b, c, 1955) there are no reported mean electron ranges. It is not possible to deduce mean ranges from the other ranges without knowledge of the transmission curve and this is available in very few instances. However, calculations based on the typical shapes of these observed transmission curves prove that the mean range and the median range are sometimes identical and, at worst, the mean range may exceed the median range by 15%. Consequently, in the following correlation, mean ranges and R_{50} ranges are treated as being identical quantities.

Experimental transmission curves representative of different media are obtainable from the work of Cole (1969, collodion), Smith and Booz (1978, gases), Mori *et al* (1980, barium stearate), and Cosslett and Thomas (1964, metals). A good empirical representation of the transmitted fraction is given by Vyatskin and Pilyankevich (1962), Vyatskin and Trunev (1967, 1970, 1972) and Fitting (1974) as

$$\eta = \exp(Cx^p) \tag{5}$$

where C is a function of atomic number and electron energy, and p is a function of the atomic number only. By combining this information it became possible to deduce realistic R_{50} ranges in those cases where only extrapolated ranges were reported. These data agree well with directly reported R_{50} values (Cole 1969, Davis 1954a, b, c, 1955).

A plot of the R_{50} ranges versus R_{ex} leads to the relationship

$$R_{50} = (0.3655 \pm 0.05) R_{ex}^{1.024} \tag{6}$$

valid for electron energies between 20 eV and 10 keV. The power dependence in equation (6) reflects the slightly increasing skewness of the transmission curve with decreasing electron energy and the error is the standard error of the plotted curve.

Figure 2 shows the R_{50} ranges scaled by Z/A for a wide range of materials as a function of electron energy. At electron energies above 500 eV Davis's results for mean penetration depths in invertase and doexyribonuclease are seen to be consistent with the other data.

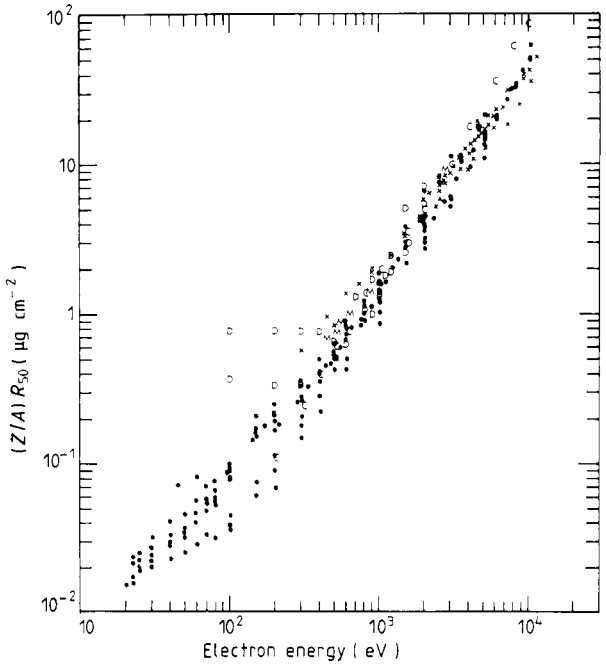

Figure 2. Reduced R_{50} ranges, $(Z/A)R_{50}$ ($\mu g\,cm^{-2}$), determined by combining the data from figure 1 with typical observed transmission curves, are shown as a function of R_{ex}. Symbols are as for figure 1 with additionally: C, collodion (Cole 1969); M, barium stearate (Mori *et al* 1980); and D, invertase, deoxyribonuclease (Davis 1954a). Data with letter symbols are direct measurements.

4. Stopping power and ranges

In principle stopping power can be deduced from the derivative of the R_{CSDA} versus electron energy curve. Since for low energy electrons, R_{ex} is the observed quantity which most closely approaches the R_{CSDA} it is interesting to compare the R_{ex} data (equation (2)) with the theoretically determined R_{CSDA} values (shown for aluminium in figure 3). Sources used are Holt (1970), Gryzinski model; Ashley *et al* (1975), statistical electron gas model; Sugiyama (1974, 1976), modified Bethe theory; and Terrissol and Patau (1980), Monte Carlo code. Holt, Ashley *et al* and Sugiyama's ranges are for inelastic collisions only and therefore overestimate the total range at low energies (<100 eV) where elastic energy losses become important (Ritchie *et al* 1978). Terrissol and Patau's ranges include all forms of energy loss. At energies above 100 eV, their R_{CSDA} ranges show an energy dependence similar to the trend of the experimental data which are, however, about 30% less in magnitude. The extrapolated ranges of Tabata *et al* (1972) correspond well with the R_{CSDA} at low energy. This is because their equation is based on only a few experimental data points for solids. Consequently stopping power deduced from the R_{ex} against E curve using equation (2) should give the correct trend with energy but the magnitude should be proportionately larger with respect to the theoretical values. This effective stopping power dE/dR_{ex} applicable to the projected extrapolated range, can be obtained for any media from the scaled relation,

$$dE/dR_{ex} = E/[R_{ex}(0.15546\ln(E)+0.31104)]\ \text{eV cm}^2\ \mu\text{g}^{-1} \tag{7}$$

which is the derivative of equation (2). Results for stopping power based on equation

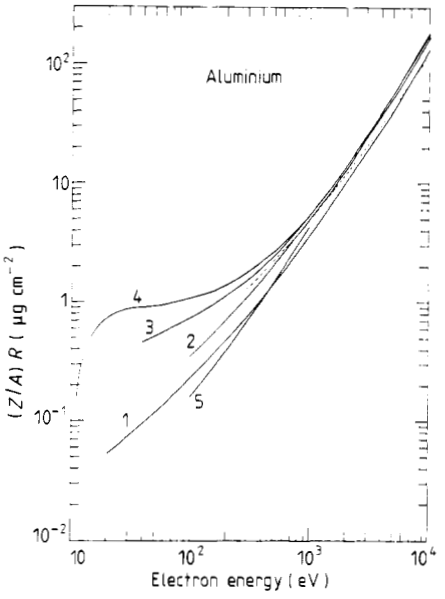

Figure 3. Theoretical R_{CSDA} ranges in aluminium are compared with the extrapolated ranges given by equation (2) in the text. Curves: (1) extrapolated ranges from the present work; (2) Terrissol and Patau (1980), Monte Carlo calculated; (3) Sugiyama (1974), modified Bethe theory; (4) Ashley *et al* (1975) electron gas model; (5) Holt (1970), Gryzinski model; broken line, extrapolated ranges from Tabata *et al* (1972) semi-empirical equation.

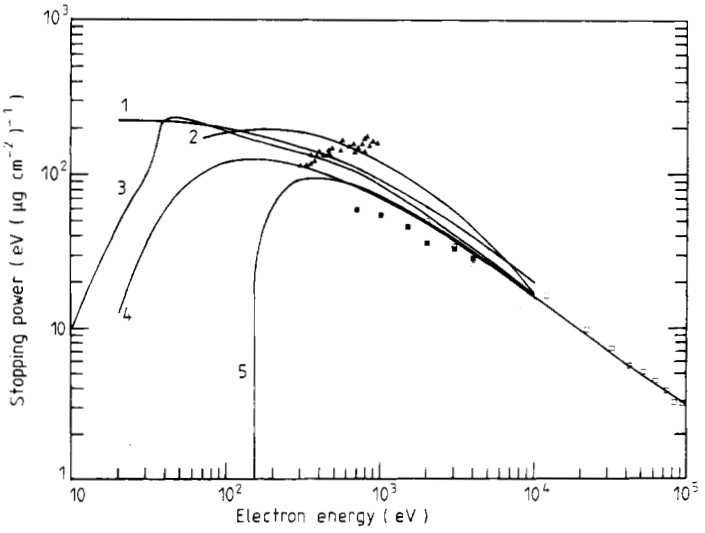

Figure 4. Theoretical and experimental stopping powers for electrons in aluminium. Curves: (1) present results for equation (7); (2) Terrissol and Patau (1980); (3) Ashley *et al* (1979); (4) Sugiyama (1974, 1976); (5) Berger and Seltzer's (1964) calculations extended below 10 keV. Experimental measurements: ▲, Garber *et al* (1971); ■, Fitting (1974); ○, Ishigura *et al* (1978); □, Kalil *et al* (1959).

(7) are compared in figure 4 with the theoretical data of Ashley *et al* (1979), Sugiyama (1974, 1976), Terrissol and Patau (1980) and Berger and Seltzer (1964) (extended below 10 keV by the present authors to show the low energy trend of fast electron theory). Ashley's predictions are seen to be the most consistent with the results of equation (7), which is based on range data and confirms work described elsewhere (Iskef *et al* 1981) in which a comparison is made with directly measured stopping powers.

5. Discussion and conclusions

Equation (2) or equations (3) and (4) give projected extrapolated ranges as a function of energy in all media to a precision of better than 30%. Medium projected ranges in all media are given by equation (6) to a precision of 14%. This error represents the fit to the experimental points in figure 2 and does not include the error in R_{ex}. If the latter error is included equation (6) becomes

$$R_{50} = (0.3655 \pm 0.106) R_{ex}^{1.024}.$$

Clearly these large errors represent an unsatisfactory situation. The main difficulties are associated with the lack of accurate knowledge about the structure, thickness and uniformity of thin solid films; multiple scattering corrections for the path travelled; the uncertainty in extrapolation of the number transmission curve to determine the extrapolated range particularly at very low electron energies where there may be no straight portion; and the choice of parameters for adequate normalisation to produce a universal range–energy relationship.

For electron energies below 2 keV the transmission curve as a function of absorber thickness, in gases, has no clear straight portions to permit reliable extrapolation ranges to be found (see, for example, figure 6 in Smith and Booz (1978)). This uncertainty probably accounts for some of the large spread in data from different authors. Usually these quoted extrapolated ranges are within the experimental error of the 1% transmission ranges which are more reliably defined. On the other hand, a plot of the transmitted fraction of electrons through a constant absorber thickness against different primary electron energies yields a curve with well defined straight portions down to 400 eV from which an extrapolated range can be found with good accuracy (see, for example, figure 3 in Young (1956a, b)). It is not obvious that both approaches to determining extrapolated ranges will give the same results and this may also account for some of the spread in the data.

The ranges in figure 1 are the various authors' quoted extrapolated ranges which have been scaled by the electron density for each medium to yield an approximate universal curve. All theoretical treatments of range indicate that there should be an additional Z dependence and many workers have confirmed this at higher energies. At lower energies (<10 keV) there is no clear experimental evidence for an additional Z dependence, possibly because of the large experimental errors involved. Examination of Fitting's (1974) data shows that any such Z dependence will be less than 25% with a possible maximum influence at the transition elements. Some authors find that the range–energy relationship is a function of ρ, the density of the medium to a power somewhat less than unity. One may imply that some degree of electronic screening is active particularly in higher Z elements. In the absence of accurate data we have made no allowance for a screening correction. Also we have considered it

to be inappropriate to apply any correction for the different mean excitation potentials as the Bethe theory becomes invalid in this energy region.

Effective stopping power over the extrapolated range is given to a precision of 30% by equation (7). These data are in closest harmony with the theoretical calculations of Ashley *et al* but better precision in experimental ranges and stopping power is clearly desirable for track structure application and to quantify phase effects.

Résumé

Due to circumstances beyond our control it has been impossible to obtain translations of the abstract into French. Should these become available later, they will be published separately.

Zusammenfassung

Projizierte Reichweite und effektives Bremsvermögen von Elektronen mit Energien zwischen 20 eV und 10 keV.

Experimentelle projizierte Reichweiten von Elektronen zwischen 20 eV und 10 keV wurden durch einen einfachen Skalenfaktor miteinander in Beziehung gesetzt, um 'best-angepaßte' Ausdrücke zur Anwendung auf alle Materialien zu erhalten. Extrapolierte Reichweiten R_{ex} in $\mu g\,cm^{-2}$ sind gegeben durch: gültig für $20\,eV \leqslant E \leqslant 10^4$ eV mit einer Genauigkeit von 25% bei $(Z/A)R_{ex}$ oder wobei (Z/A) das Verhältnis von Ordnungszahl zu Atomgewicht des Absorbers ist. Aus einer Analyse der Transmissionskurven erhielt man folgende Beziehung zwischen dem Mittelwert der mittleren projizierten Reichweiten und der extrapolierten Reichweite: Die Differentiation der extrapolierten Reichweite-Energie-Beziehung liefert das effektive Bremsvermögen entlang der projizierten Spur des Elektrons, das gegeben ist durch: für $20\,eV \leqslant E \leqslant 10^4$ mit einer Genauigkeit von 30%. Für Spurstruktur-Anwendungen und zur Auflösung von Phaseneffekten ist eine verbesserte Genauigkeit der experimentellen Werte erforderlich.

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