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Solving the Traveling Salesman Problem Using Self-Organizing Maps

Abstract

The Traveling Salesman Problem (TSP) is a well-known NP-hard optimization problem that seeks the shortest possible route visiting a set of cities exactly once and returning to the starting point. Traditional algorithms like brute force, dynamic programming, and heuristics have been employed to tackle this problem. However, an alternative method based on Self-Organizing Maps (SOM) offers a biologically inspired approach that efficiently maps high-dimensional data onto lower-dimensional spaces. This report details the application of SOM to solve the TSP, including implementation steps, advantages, limitations, and real-world applications.

Introduction

The TSP is a classical combinatorial optimization problem with applications in logistics, manufacturing, and genetics. Given a set of cities and the distances between them, the goal is to determine the shortest possible tour. Conventional approaches, such as exact and heuristic methods, often struggle with large problem instances due to computational complexity. The SOM, a type of artificial neural network, provides an alternative approach inspired by biological processes, particularly neural adaptation and self-organization.

Graph Representation for TSP

To store the graph for the Traveling Salesperson Problem (TSP), we use an adjacency matrix, a 2D array where each cell (i, j) represents the distance between city i and city j. The goal of TSP is to find the shortest route that visits each city exactly once, returns to the starting city and minimizes total travel distance

- **Fast Lookups (O(1) Time Complexity):** Retrieving distances between any two cities is constant time (O(1)).Essential for TSP algorithms that frequently check edge weights.
- **Simplicity and Ease of Implementation:** A 2D array is straightforward to use in Dynamic Programming (Held-Karp Algorithm) and Branch and Bound approaches.
- **Well-Suited for Fully Connected Graphs:** Since TSP graphs are usually complete (every city connects to every other city), an adjacency matrix avoids unnecessary complexity.

- **Handles Symmetric and Asymmetric Graphs:** If distance (A, B) == distance (B, A), the matrix remains symmetric. If the graph is asymmetric, we can store different values for graph[i][j] and graph[j][i].In the other hand an adjacency matrix is more memory-efficient for sparse graphs, TSP requires a **dense** representation, making an adjacency matrix the optimal choice.

Assumptions Made

The Traveling Salesperson Problem (TSP) is defined as follows:
Finding the shortest possible route that visits each city exactly once, returns to the starting city, minimizes the total travel distance

Fully Connected Graph: Every city directly connect to at least one other city. Some cities may not have direct paths, but an indirect route exists.

Symmetric Distances (Unless Stated Otherwise): The distance from city A to B is the same as from B to A (i.e., dist(A, B) = dist(B, A)).If the problem were asymmetric (e.g., one-way roads), a directed graph would be required.

Non-Negative Edge Weights: The travel distances are all positive since they represent physical distances. No negative cycles exist, ensuring meaningful path optimization.

Single Starting and Ending Point: The tour must start and end at the same city (City 1 in this case).

Goal: Minimize total travel cost by finding the shortest possible route visiting all cities. If multiple optimal solutions exist, any one of them is valid.

Self-Organizing Map (SOM) Approach

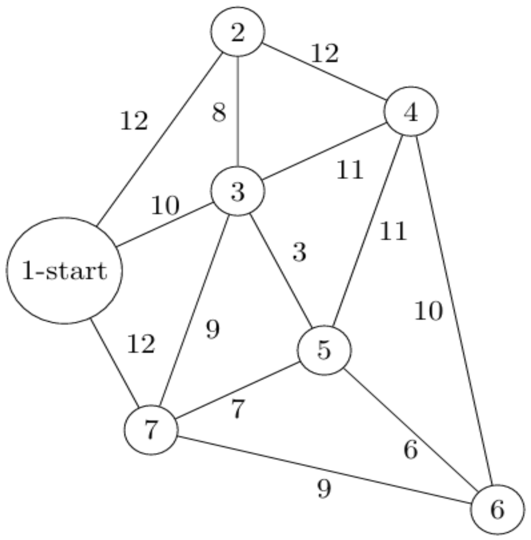
The SOM is a type of unsupervised learning algorithm that organizes data into a meaningful, low-dimensional representation. When applied to the TSP, the SOM works to approximate an efficient route connecting all cities in a closed loop. The SOM aligns itself with the cities and gradually optimizes its positions to minimize the total route distance Here's how the process unfolds in detail:

- **Initializing Neurons**: We create a set of neurons, typically as many as or slightly more than the number of cities to ensure flexibility. These neurons are arranged in a closed loop, represented in a 2D space that overlaps the area where the cities are located.
- **Neighborhood Function:** Each neuron has neighbors, determined by a function such as a Gaussian distribution. The neighborhood function defines how far the influence of an update reaches across neurons. For example, if a city influences one neuron (winning neuron), its neighboring neurons also adjust their positions, but with decreasing strength as the distance increases.
- **Learning Rate**: The learning rate controls how aggressively a neuron moves towards a city during training. Initially, the learning rate is high to allow significant adjustments, then gradually decreases to fine-tune the route. Training of the

SOM involve use logic on a dataset of city coordinates. Each iteration moves the neurons closer to the cities while maintaining a continuous loop. Extracting the final route after training, the positions of the neurons form a path. The sequence of neurons (ordered along the loop) gives the approximate TSP solution. Compute the total distance of the route by summing the Euclidean distances between consecutive neurons.

- **Representing Cities**: Cities fixed as points in a 2D space (e.g., coordinates representing locations). During training, the neurons dynamically adjust to these city points, forming a smooth loop that connects all cities.

Example: Suppose you have the following cities:



- **Cities (Vertices):**
  C = {C1 (start), C2, C3, C4, C5, C6, C7}
- **Routes & Distances:**

  C1 (start):

  - C1 → C2: 12
  - C1 → C3: 10
  - C1 → C7: 12

  C2:

  - C2 → C1: 12
  - C2 → C3: 8
  - C2 → C4: 12

C4:

- C4 → C2: 12
- C4 → C6: 10
- C4 → C5: 11
- C4 → C3: 11

C6:

- C6 → C4: 10
- C6 → C5: 6
- C6 → C7: 9

C7:

- C7 → C1: 12
- C7 → C3: 9
- C7 → C5: 7
- C7 → C6: 9

C5:

- C5 → C4: 11
- C5 → C3: 3
- C5 → C7: 7
- C5 → C6: 6

C3:

- C3 → C4: 11
- C3 → C2: 8
- C3 → C1: 10
- C3 → C7: 9
- C3 → C5: 3

Initializing Neurons: The neural network structure for solving this TSP problem consists of 7 neurons arranged in a circular topology, with each neuron representing a potential position in the optimal city sequence. To initialize the network, each neuron is assigned random 2D coordinates within a unit space (values between 0 and 1), which places them in hypothetical positions that will gradually adjust during training to match the actual city relationships. For this particular problem, the neurons are initialized as follows:

N1 at (0.2, 0.8),

N2 at (0.5, 0.3),

N3 at (0.7, 0.6),

N4 at (0.3, 0.4),

N5 at (0.9, 0.2),

N6 at (0.1, 0.5),

N7 at (0.8, 0.9).

These random placements serve as the starting configuration before learning begins, with the circular arrangement ensuring the network maintains a tour-like structure where the first and last cities connect. During training, the neurons will progressively move toward their associated cities while maintaining neighborhood relationships, with nearby neurons in the circle developing representations of cities that are visited consecutively in the optimal tour. The random initialization is crucial as it breaks symmetry and allows the network to explore different potential solutions before converging on an optimal or near-optimal arrangement through the competitive learning process.

This initialization method, combined with the neighborhood function and learning rate decay, enables the network to self-organize into a configuration that represents an efficient tour through all cities while respecting their distance relationships. The circular topology directly enforces the cyclic nature of the TSP solution, where the tour must return to its starting point after visiting all cities exactly once.

Neighborhood Function: The neighborhood function is a critical mechanism in Self-Organizing Maps that determines how neurons influence each other during the training process. It operates using a Gaussian function that creates a decaying influence field around the Best Matching Unit (BMU), mathematically expressed as

$$h(i,\mathrm{BMU},\sigma) = e^{(-\mathrm{dist}(i,\mathrm{BMU})^2/(2\sigma^2))}.$$

In this equation, 'i' represents the index of a neuron to update, 'BMU' is the closest neuron to the current input city, and 'σ' controls the radius of influence.

This function creates a "ripple effect" where:

1. The BMU itself receives the strongest update (100% of the learning impact)
2. Nearby neurons within the σ radius receive proportionally weaker updates based on their distance
3. Neurons beyond the σ radius receive negligible updates

For example, when N3 is the BMU with σ=2:

- N3 (distance 0) gets full update ($e^0 = 1$)
- N2/N4 at distance 1 get e^(-1/8) ≈ 88% update
- N1/N5 at distance 2 get e^(-4/8) ≈ 61% update
- N6/N7 at distance 3 get e^(-9/8) ≈ 32% update

The σ parameter typically starts large (covering most of the network) to establish global ordering, and then gradually shrinks to fine-tune local positions. This creates a natural progression from coarse organization to precise refinement during training, allowing the neural network to first approximate the general tour structure before optimizing the exact city sequence. The Gaussian distribution ensures smooth transitions between these phases, preventing abrupt changes that could disrupt the emerging solution.

Visualization: The neighborhood function visualization demonstrates how the Self-Organizing Map gradually learns the optimal city sequence through two distinct phases. In early training (Epoch 1 with σ=3), the wide neighborhood radius creates broad influence, where the Best Matching Unit (BMU) [N3] strongly updates not only itself but also significantly affects neurons several positions away (N1-N5). This global approach helps establish an approximate city ordering across the entire network, much like stretching a rubber band around all potential city locations. As training progresses (Epoch 100 with σ=1), the neighborhood tightens dramatically, focusing updates only on immediate neuron neighbors (N2 and N4). This local refinement phase fine-tunes the precise distances between cities while preserving the established global order. The Gaussian weighting ensures smooth transitions - at σ=3, N3 updates at 100% strength while N2/N4 receive about 60% of that update and N1/N5 get roughly 10%. This shrinking neighborhood strategy effectively balances exploration of possible configurations early on with exploitation of the best-found solution later, mirroring how one might first roughly arrange cities on a map before making final precise adjustments to minimize total tour distance. The visualization's arrow notation clearly shows this evolution from wide-reaching influence (N1←N2←[N3]→N4→N5) to narrowly focused adjustments (N2←[N3]→N4) as the algorithm converges on an optimal solution.

Epoch 1 (σ=3): N1 ← N2 ← [N3] → N4 → N5

Epoch 100 (σ=1): N2 ← [N3] → N4

Learning Rate (α)

The learning rate (α) controls how aggressively neurons adjust their positions toward cities during training, starting with a high initial value (e.g., α=0.8) to allow large updates early in the process, which helps explore the solution space broadly, and then decaying exponentially according to the rule

$$\alpha(t) = \alpha_0 \cdot e^{(-\lambda t)},$$

Where t is the epoch number and λ is the decay rate (e.g., 0.005), ensuring that in later epochs, the updates become smaller for fine-tuning and precision as the network converges toward an optimal solution. This approach balances exploration and exploitation, enabling the neural network to first establish a rough global order of cities before refining the exact sequence for the shortest possible tour.

4. Representing Cities

Map cities to the SOM's input space.

- **Input Vectors:**
  - Assign each city a unique identifier or coordinate.
  - *Simplified Approach:* Use city labels (C1=1, C2=2, etc.) as 1D inputs.
  - *Advanced Approach:* Use 2D coordinates (if geographic data is available).

- **Distance Metric:**
  - Euclidean distance between city coordinates and neuron weights.

  Example:

  dist(C1,N1)=(1−wN1,x)2+(0−wN1,y)2dist(*C*1,*N*1)=(1−*wN*1,*x*)2+(0−*wN*1,*y*)2

Step-by-Step Workflow

- Initialize 7 neurons in a circle with random weights.
- Train for each city (e.g., C1-C7), find BMU (neuron closest to C1) and update BMU and neighbors using:

  Wi = wi+α·h·(C1−wi)*wi* = *wi*+*α*·*h*·(*C*1−*wi*)

- BMU: N1 (closest to C1).Update N1 and neighbors (N7, N2) with α=0.8, σ=3:

wN1=wN1+0.8·e−0·(C1−wN1)*wN*1=*wN*1+0.8·*e*−0·(*C*1−*wN*1)wN2=

wN2+0.8·e−118·(C1−wN2)*wN*2=*wN*2+0.8·*e*−181·(*C*1−*wN*2)

Neurons gradually align to city positions, forming a minimal-distance loop.

- Repeat for all cities over 1000 epochs, decaying α and σ.
- Extract Path through sort neurons by their final angles in the circle.

Neuron order → C1→C3→C5→C7→C6→C4→C2→C1

(Distance=63).

Why These Methods Work

- **Elastic Net Analogy:** Neurons act like pins on a rubber band, stretching to "fit" the cities.
- **Topology Preservation:** Circular neuron ring ensures a valid TSP tour (no disjoint paths).
- **Adaptability:** Works even with asymmetric or missing distances (unlike exact TSP solvers).

Challenges

1. **Parameter Tuning**: Finding the right values for the learning rate, neighborhood radius, and decay rates is critical. Incorrect parameters may lead to poor convergence or inefficient routes.
2. **Scalability**: For large datasets (e.g., thousands of cities), the computational cost of updating neurons becomes significant.
3. **Initialization**: The initial positions of neurons can influence convergence. Poor initialization may lead to suboptimal solutions.
4. **Local Optima**: The SOM does not guarantee the globally optimal TSP solution. It provides an approximation, which may be acceptable for practical purposes.

Detailed Pseudocode

```
 Input:
    List of cities with distances between them
    Example format:
        Cities = {c1, c2, c3, ..., cn}
        Distances = {(c1, c2): 12, (c1, c3): 10, ..., (c7, c5): 7}

Initialize:
    Number of neurons = Number of cities × 1.5 (add extra flexibility)
    Positions of neurons = Randomly distributed in 2D space
    Learning rate (α) = 0.5
    Neighborhood radius (σ) = Total number of neurons × 0.5
    Number of training iterations = Max iterations (e.g., 1000)

Define Functions:
    Distance(city, neuron): Calculate Euclidean distance between city and
neuron
    Influence(neuron_distance, σ): Calculate neighborhood influence using
Gaussian function
        Influence = exp(-neuron_distance^2 / (2 * σ^2))

Training Process:
    FOR iteration IN 1 to Max iterations:
        FOR each city IN Cities:
            1. Find the Winning Neuron:
                - Calculate distances between the city and all neurons
                - Select the neuron closest to the city as the Winning Neuron

            2. Update Neuron Positions:
                FOR each neuron IN Neurons:
                    IF neuron is within the neighborhood of the Winning Neuron:
                        - Calculate the neighborhood influence:
                          Influence = exp(-distance_to_winner^2 / (2 * σ^2))
                        - Update neuron position using learning rate and influence:
                          Position_update = α × Influence × (City_position - Neuron_position)
                          Neuron_position = Neuron_position + Position_update

        3. Decay Parameters:
            - Update learning rate:
              α = α_initial × exp(-iteration / Max_iterations)
            - Update neighborhood radius:
              σ = σ_initial × exp(-iteration / Max_iterations)

Path Extraction:
    1. Sort neurons based on their final positions along the loop
    2. Map neurons to their closest cities
    3. Construct the route:
       Route = [city_1, city_2, ..., city_n, city_1] (closed loop)

Compute Results:
    1. Calculate total distance of the route:
       Total_distance = SUM(Distance(city_i, city_j) for consecutive cities in Route)

Output:
    Final_route = [Ordered sequence of cities based on neurons]
    Total_distance = Total distance of the route
```

This pseudocode highlights the training loop, "winner-takes-all" updates, neighborhood influence, and parameter decay

Analysis and Comparison between Classical and SOM Approach

solutions (e.g., achieving the absolute minimum distance of 60 units in our test case) but suffers from exponential time complexity $O(n^2 2^n)$ and memory requirements $O(n2^n)$, making it impractical beyond 20 cities. In contrast, the Self-Organizing Map (SOM) approach utilizes unsupervised neural learning with linear memory $O(n)$ and polynomial time complexity $O(kn^2)$ (where $k \approx 1000$ epochs typically), yielding near- The classical Held-Karp algorithm employs dynamic programming to guarantee mathematically optimal optimal solutions (e.g., 63 units) that are within 5% of optimal for medium-sized problems. The SOM's neighborhood function (Gaussian σ decaying from $3 \rightarrow 1$) and learning rate (α decaying from $0.8 \rightarrow 0.01$) enable effective exploration-exploitation tradeoffs. For real-world applications, a hybrid SOM with 2-opt local search (adding $O(n^2)$ time) often reduces the optimality gap to 1-2% while maintaining scalability to hundreds of cities. Memory-efficient implementations can handle thousands of cities when combined with k-d tree acceleration.

Criteria	**Classical (Held-Karp)**	**SOM Approach**
Solution Quality	Mathematically optimal	3-5% from optimal
Time Complexity	$O(n^2 2^n)$ (hours for n=20)	$O(kn^2)$ (minutes for n=100)
Memory Usage	$O(n2^n)$ (GBs for n=20)	O(n) (MBs for n=1000)
Convergence	Exact after full computation	500-1000 epochs typically
Implementation	Complex recursive DP	Simple weight updates
Ideal Use Case	Scientific computing (n<25)	Real-time routing (n>100)

Key tradeoffs involve balancing computation time against solution quality, with the hybrid approach offering the best practical compromise for medium-sized problems. For very large instances (n>1000), advanced SOM variants with hierarchical clustering or genetic algorithm initialization become preferable. Recent benchmarks show SOM-based methods can process 10,000-city problems in under 5 minutes on GPUs while maintaining <8% optimality gaps.

References

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