Emma Redfoot
Problem Set 5

Due Date: 3/10/17

Solve the following problems. The last date of submission is March 10, 2017.

1- Estimate the prompt neutron life time in a large thermal reactor fueled with ^{235}U and moderated with: (a) H_2O, (b) graphite, and (c) D_2O.

Assumptions

- ignore Delayed Neutrons
- large reactor, thus no leakage
- Assume

Average lifetime of n in reactor

$$l = [v \Sigma_a (1+L^2 B_g^2)]^{-1}$$

v = thermal velocity

L = Diffusion length = $\sqrt{D/\Sigma_a}$

B_g = Geometric Buckling

Σ_a = absorption cross section = .022 H_2O
(in water & 235?) , .0924 pg. 210

$$l = \left[v \Sigma_a + v \frac{\cancel{\Sigma_a} D}{\cancel{\Sigma_a}} B_g^2\right]^{-1}$$

$$l = [v \Sigma_a + \cancel{v D B_g^2}]^{-1} \quad 0 \text{ large ractor}$$

$$l = \frac{1}{v \Sigma_a}$$

a) $l_{H_2O} = \dfrac{1}{3\times10^5 \text{ cm/s} (.022 \text{ cm}^{-1} + .0924 \text{ cm}^{-1})} = 2.9\times10^{-5}$ s

b) $l_c = \dfrac{1}{3\times10^5 \text{ cm/s} (.0924 \text{ cm}^{-1} + 32 \text{ cm}^{-1})} = 1.039\times10^{-7}$ s

c) $l_{D_2O} = \dfrac{1}{3\times10^5 \text{ cm/s} (.0924 + 3.3)} = 9.826\times10^{-7}$ s

2- Calculate the effective neutron life time <l> for a thermal reactor fueled with either ^{233}U or ^{235}U and a fast reactor fueled with ^{239}Pu.

ume

de delayed
rons

$$\langle l \rangle = (1-\beta)\lambda + \sum_{i=1}^{6} \beta_i \left[\frac{1}{\lambda_i} + l\right]$$

$$\langle l \rangle_{U235} = .0845$$

$$\langle l \rangle_{Pu\,239} = .03126$$

See attached Spreadsheet / python script

3- Estimate the reactor period induced by a positive reactivity insertion of 1 \$ in an infinite ^{235}U-fueled thermal reactor moderated with light water and a fast ^{239}Pu-fueled reactor.

Assumptions

infinite reactor $\therefore$ leakage $= 0$

$$k = \eta f \epsilon p \cancel{P_{NL}} \cancel{P_{NL}}$$

$$\frac{\$}{\rho = \beta = \frac{k-1}{k}}$$

$$T = \frac{\cancel{\langle l \rangle}}{\cancel{k-1}} = \frac{(1-\beta)l + \sum_{i=1}^{6} \beta_i (1/\lambda_i + l)}{k-1}$$

$$T = \frac{1}{\lambda_1}$$

$$T = \frac{\langle l \rangle}{\rho_0} = \frac{(1-\beta)l + \sum_{i=1}^{6} \beta_i (1/\lambda_i + l)}{\beta}$$

$$T_{U235} = 13.2 \text{ sec}$$

$$T_{Pu239} = 14.88 \text{ sec}$$

See python script

4- Initially a reactor is operating at a steady-state power level of P_0. Using the point reactor kinetics equation with one equivalent group of delayed neutrons, determine the stable reactor period for positive step reactivity of insertion of 1 \$. Use the factor that $\lambda\Lambda/\beta \ll 1$ to simplify your answer. Calculate a numerical value for T in the case in which $\lambda = 0.1\ sec^{-1}$ and $\Lambda = 0.001\ sec$.

One Effective Delayed Group

Point Reactor Kinetics Eqn.

steady state

$$0 = \frac{dP}{dt} = \left[\frac{\rho_0 - \beta}{\Lambda}\right] P_0 + \lambda$$

$$0 = \frac{dC}{dt} = \frac{\beta}{\Lambda} P_0 - \lambda C_0$$

$$C_0 = \frac{\beta}{\lambda\Lambda} P_0$$

$$T = \frac{1}{S_1} = \frac{1}{-\frac{\lambda}{2} - \frac{\beta}{\Lambda}\sqrt{\frac{\Lambda\lambda}{\beta}}}$$

$$\boxed{\begin{array}{l} T = 1.322 \text{ Sec} \\ \text{For a thermal reactor} \\ \text{Fuel} = U\text{-}235 \\ \text{moderator} = H_2O \end{array}}$$

$\rho = \beta$

$$sP = \left(\frac{\cancel{\rho_0 - \beta}^{\,0}}{\Lambda}\right) P + \lambda C$$

$$sP = \lambda C$$

$$sC = \frac{\beta}{\Lambda} P - \lambda C$$

$\rho = \beta$

$$S_{1,2} = \frac{1}{2\Lambda}\left[-(\cancel{\beta - \rho_0} + \lambda\Lambda) \pm \sqrt{(\cancel{\beta - \rho_0}^{\,0} + \lambda\Lambda)^2 + 4\Lambda\lambda\rho_0}\right]$$

$$S_{1,2} = \frac{1}{2\Lambda}\left[-\lambda\Lambda \pm \sqrt{(\lambda\Lambda)^2 + 4\Lambda\lambda\rho_0}\right]$$

$$\left[S_{1,2} = \frac{-\lambda}{2} \pm \frac{1}{2\Lambda}\sqrt{\lambda^2\Lambda^2 + 4\Lambda\lambda\rho_0}\right]\frac{1}{\beta}$$

$$\left(\lambda^2\Lambda^2 + 4\Lambda\lambda\rho_0\right)^{1/2}\left(\frac{1}{\beta^2}\right)^{1/2}$$

$$\frac{S_{1,2}}{\beta} = \frac{-\lambda}{2\beta} \pm \frac{1}{\Lambda}\sqrt{\frac{\Lambda\lambda}{\beta}}$$

$$\sqrt{\frac{\cancel{\lambda^2\Lambda^2}^{\,0}}{\beta^2} + \frac{4\Lambda\lambda\rho_0}{\beta^2}}$$

$$S_{1,2} = \frac{-\lambda}{2} \pm \frac{\beta}{\Lambda}\sqrt{\frac{\Lambda\lambda}{\beta^2}}$$

$$= \sqrt{\frac{4\Lambda\lambda}{\beta}} = 2\sqrt{\frac{\Lambda\lambda}{\beta}}$$

$$S_1 = \frac{-\lambda}{2} + \frac{\beta}{\Lambda}\sqrt{\frac{\Lambda\lambda}{\beta^2}} = \frac{-.1}{2} + \frac{.0065}{.001}\sqrt{\frac{.001 \cdot .1}{.0065}}$$

$$= .75$$

$$S_2 = \frac{-\lambda}{2} - \frac{\beta}{\Lambda}\sqrt{\frac{\Lambda\lambda}{\beta^2}}$$

5- After one minute following the transient described in problem 4, all the control rods with a worth of 0.02 are inserted in the core. Calculate the power 15 minutes after the transient.

Prompt Drop U-235

$T = \frac{\rho}{k-1}$ $\rho = .0845$ from prob. 2

$P = P_0 e^{St}$

From Prob 4 $1.322 = \frac{.0845}{k-1}$

$P_1 = P_0 e^{st}$

$P_1 = P_0 e^{.75(60\, sec)}$

$P_1 = 3.493 \times 10^{19} P_0$

after 60 sec

$1.322k = .0845 + 1.322$

$k = 1.06392$

$\rho(60\, sec) = \frac{1.06392 - 1}{1.06392}$

$\rho_1 = .06008$

$\frac{P_2}{P_1} = \frac{\beta - \rho_1}{\beta - \rho_2}$

$P_2 = 3.493 \times 10^{19} P_0 \left(\frac{.064 - .06008}{.064 - .02 \cdot .064} \right)$

$\boxed{P_2 = 2.183 \times 10^{18} P_0 \text{ watts}}$