Gerrymandering and Impossibility

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1 Introduction

Gerrymandering, particularly partisan gerrymandering, has always been a popular topic in U.S. politics, as it is one of few issues that has bipartisan disapproval. However, in light of events like the 2016 election and in anticipation of the 2018 Congressional election, where Democrats hope to take back majority, research on how to both identify and confront gerrymandering has recently redoubled. The Supreme Court is even currently considering a new measure for evaluating gerrymandering.

Gerrymandering is the act of deliberately setting or manipulating political district boundaries so as to favor (or disfavor) a particular party or group. In some cases, this is done with good intent: a district might be formed to represent a particular group of people with shared interests that would not be represented otherwise [Lev17]. However, partisan gerrymandering, where districts are deliberately crafted to favor a particular party (or disfavor another party), is illegal when excessive.

There are two methods of gerrymandering: packing and cracking [Wan16]. Packing occurs when a gerrymanderer packs all of a particular voting bloc into one district, so that they will only win there and nowhere else. Cracking occurs when the bloc is broken apart across districts, so they cannot get a majority anywhere. See Figure 1 for a visual.

Identifying where gerrymandering is actually occurring is a nontrivial problem. What counts as a "good" district? What is "excessively" partisan? Another factor to consider is accidental gerrymandering: because Democrats and Republicans are not randomly distributed (e.g. people in urban areas tend to be Democrats), partisan gerrymandering can easily occur completely by accident. Many states have adopted standards based on metrics like contiguity and compactness to try to identify illegal gerrymandering [Wan16; Lev17].

However, despite these measures, excessive partisan gerrymandering is still occurring. As a result, many political scientists, mathematicians, and statisticians have proposed new methods of evaluating gerrymandering. One such method is the efficiency gap, which measures the number of wasted votes cast for a party divided by the total number of votes cast for that party [SM15]. Simulations are also being used to model possible districts in states suspected

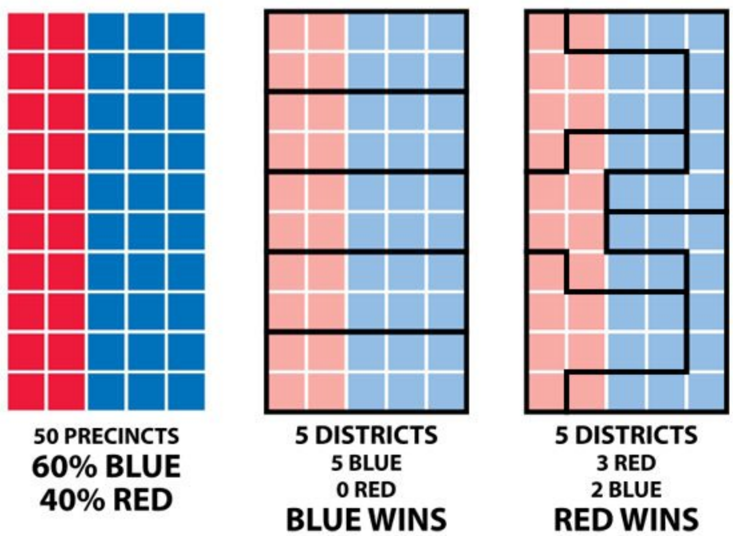

Figure 1: In the middle, the red voters are "cracked" across the districts, so that they have no representation. To the right, the blue voters are "packed" into a few districts, so red wins the majority of the state even though there are more blue voters overall.

of gerrymandering [CR15]. These simulations generate many possible district maps and compare the partisanship of these maps to the real map: if the simulated maps result on average in $\frac{6}{10}$ Democratic districts, but the current map has $\frac{9}{10}$ Democratic districts, this may be a sign of gerrymandering in favor of Democrats.

With so many mathematicians and policy-makers putting so much effort into this, it may be surprising that a concrete solution to fixing gerrymandering has not emerged. In this paper, we explain and prove exactly why this is such a difficult problem. Following some background on existing metrics for evaluating districts, we present the recent finding that using currently well-established metrics of evaluating gerrymandering is impossible with the addition of the efficiency gap [AM17]. This is particularly notable because the Supreme Court is currently considering it as a viable standard for evaluating gerrymandering [CB17]. Finally, we present a proof that legal, non-partisan district optimization is an NP-hard problem [LCW16], so generating non-gerrymandered districts with computer simulations is not possible without approximation or heuristics.

2 Existing metrics

The metrics used for evaluating gerrymandering vary across states, but many of agree that a winding, oddly-shaped district is more likely to be egregiously gerrymandered, as is intuitive [Lev17]. In this section, several salient existing metrics for evaluating districts are explained.

2.1 Equal population

It is federally mandated that all districts across a state have approximately equal population [Lev17]. This ensures that each person's vote counts for around the same amount. If districts did have highly variable population, then a person's vote from a district with a very high population would count for less than a person's vote from a district with a very low population. So, requiring equal population is effectively requiring that there be one vote per person. We can formalize this as such: there exists $\delta \in [0, 1)$ such that where A and B are sets of voter locations for two major political parties (where there are no third-party voters) and $\{D_i\}_{i=1}^{k}$ are proposed districts,[AM17]

$$(1-\delta)\lfloor\frac{|A \cup B|}{k}\rfloor \leq |(A \cup B) \cap D_i| \leq (1+\delta)\lceil\frac{|A \cup B|}{k}\rceil \ \forall i \in \{1 \ldots k\}.$$

Notice that $|A \cup B|$ is the population of the entire state, $|(A \cup B) \cap D_i|$ is the population of a district i, and δ allows for a small margin of error in enforcing equal population across districts. So, this is essentially just requiring that all districts have a population within a reasonable margin of the average number of people in a district.

2.2 Contiguity

Contiguity is fairly intuitive: a district is contiguous if you can travel from any point in a district to another without crossing the district boundary [Lev17]. This implies that the district is not broken apart into many pieces. This is the most common metric for standardizing districts, and is required in most states that have any such metrics.

2.3 Compactness

In general, compactness is a measure of how winding and oddly-shaped a district is. In Figure 1, the districts in the center state are compact, as they are all very regularly shaped. The districts in the rightmost state are long and wobbly, and are therefore less compact. See Figure 2 for a real example of a very non-compact district.

One particular measure of compactness we can use is Polsby-Popper compactness, that there exists $C > 0$ for districts $\{D_i\}_{i=1}^{k}$ such that

$$|\partial D_i|^2 \leq C|D_i| \ \forall i \in \{1 \ldots k\},$$

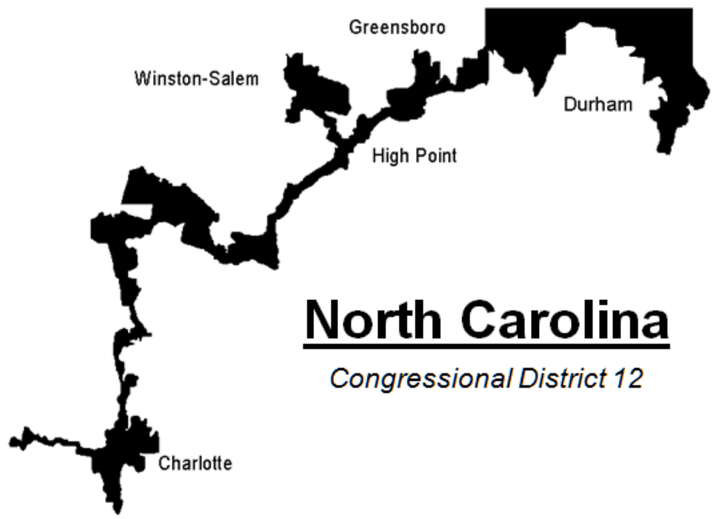

Figure 2: This is one iteration of North Carolina's 12th district, which has been redrawn many times in recent years.

where $|\partial D_i|$ is the perimeter of district D_i and $|D_i|$ is the area [AM17]. So, the perimeter and the area of a district must be reasonably proportionate.

2.4 Other common requirements

The majority-majority standard requires that, if the overall majority vote across a state or region goes to a particular party, then this should be reflected in the results by district [Lev17]. That is, if 90% of the total vote goes to Democrats, then roughly 90% districts should go to Democratic representatives.

Additionally, many states require that districts follow existing political boundaries, i.e. one city can't be split into multiple districts. This can prevent cracking of a community between districts, and also can help ensure that a community's interests are represented in their district.

3 The efficiency gap

A newer metric is the *efficiency gap*, proposed by Nicholas Stephanopoulos in 2015, is currently being considered by the Supreme Court in the case Gill v. Whitford.[SM15; Baz17] The plaintiffs use the efficiency gap as a quantifier to claim that Wisconsin's district boundaries reflect excessive partisan gerrymandering.

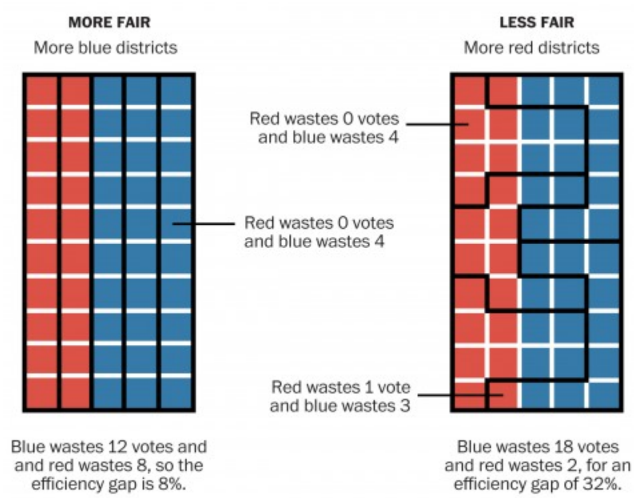

Figure 3: It's clear that the districting to the left is more fair than the districting to the right. Notice that the fair districting still requires long, skinny district boundaries that are not very compact.

Unlike most of the existing metrics to evaluate gerrymandering, the efficiency gap specifically considers partisanship. Generally, it is a measure of the difference between the number of wasted votes between the two major parties. See Figure 3 for a visual example.

A vote is *wasted* if it is for the minority party in a district, or if it is in the majority party but over 50%. For example, if a 80% of a district voted for a Democratic candidate then the 20% of Republican votes are wasted, as are the 30% of Democratic votes over majority. The efficiency gap here is 10%.

We can formalize this for sets of voters for two parties A and B and districts $\{D_i\}_{i=1}^k$ [AM17]. Wasted votes $w_{A,i}$ for a party A in a district D_i are

$$w_{A,i} := |A \cap D_i| - \lceil \frac{1}{2} |(A \cup B)| \cap D_i| \rceil$$

and can be defined similarly for party B. The efficiency gap is defined as

$$EG(D_1, \ldots, D_k; A, B) := \frac{1}{|A \cup B|} \sum_{i=1}^{k} (w_{A,i} - w_{B,i}).$$

Informally, this is the difference in number of wasted votes between two parties, divided by the total population of the state.

We say districts satisfy the efficiency gap (and are therefore not excessively partisan gerrymandered) if there exists $\alpha, \beta > 0$ such that the districts always satisfy

$$|EG(D_1, \ldots, D_k; A, B)| < \frac{1}{2} - \alpha, \text{ whenever } ||A| - |B|| < \beta |A \cup B|.$$

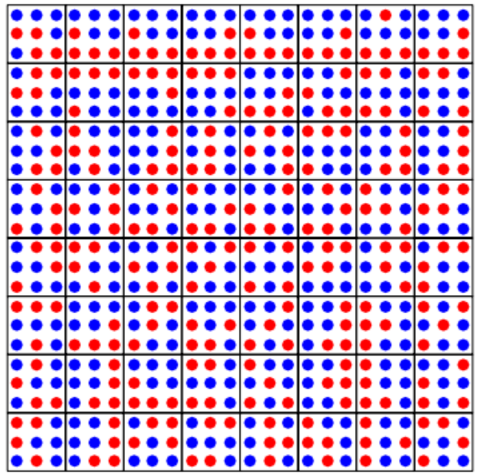

Figure 4: Here, $l = 3$, $n = 8$, $a = 5$ and $b = 4$. The lattice is the collection of all colored points (voters), where there are nine voters in each grid square.

This metric seems to accurately identify partisan gerrymandering, as it matches our sense of which states and districts have been most egregiously gerrymandered based on political context [CB17].

3.1 Proof of impossibility

According to a recent paper by Boris Alexeev and Dustin Mixon, it is impossible to have one vote per voter, Polsby-Popper compactness, and partisan efficiency at the same time [AM17]. We will prove this using the formal definitions of each, as given above.

Theorem 1. For every $\delta, C, \alpha, \beta, k$, there exist A and B such that any districting $\{D_i\}_i^k = 1$ violates at least one of one vote per voter, Polsby-Popper compactness, and the partisan efficiency gap. (Recall that δ is required to satisfy equal population across districts – which guarantees one vote per voter – α, β are similarly required for partisan efficiency, C for compactness, and k is the number of districts.)

Let us begin the proof by setting up a "state". Take the unit square, $[0,1]^2$. Now, partition the unit square into a n by n grid, where $\epsilon = \frac{1}{n}$. That is, ϵ is the length/width of one grid square. From here, we will refer to grid squares as $\epsilon-$squares.

Choose $a, b > 0$ such that $a + b = l^2$. a will be the number of voters in A in one grid square, and similarly for b. Take a lattice L, $\left(\frac{1}{nl}\left(\mathbb{Z} + \frac{1}{2}\right)\right)^2$. This is simply a collection of points arranged such that there are l^2 points inside each $\epsilon-$square of our grid on the unit square. Each point in the lattice represents one voter – this gives us a state with a homogeneous mixture of partisanship across voters. See Figure 4 for a visual of a potential state using this system.

Consider a partitioning of voters into k districts $\{D_i\}_{i=1}^k$ such that both approximately equal population per district and Polsby-Popper compactness are satisfied.

We want to find bounds for the number of $\epsilon-$squares that contain the boundary of a district ∂D_i. Consider an $\epsilon\sqrt{2}-$thickened version of ∂D_i. This just means that the boundary is $\epsilon\sqrt{2}$ thick, rather than having no thickness. We do this to account for any voters in the lattice that are along the border of the district. Considering the turns at the corners, the maximum total area of the thickened boundary is

$$\sqrt{2}|\partial D_i|\epsilon + 2\pi\epsilon^2,$$

so the maximum number of $\epsilon-$squares ∂D_i contains is

$$E := \sqrt{2}\frac{|\partial D_i|}{\epsilon} + 2\pi.$$

Thus, the number of $\epsilon-$squares contained in D_i is between $\frac{|D_i|}{\epsilon^2} - E$ and $\frac{|D_i|}{\epsilon^2}$.

Notice that the number of votes for party A in district D_i is greater than or equal to a times the minimum possible number of votes included in the district (including its boundary). That is,

$$|A \cap D_i| \geq a\left(\frac{|D_i|}{\epsilon} - E\right).$$

Similarly, the number of votes for party B in district D_i is less than or equal to b times the maximum possible number of votes included in the district. Or,

$$|B \cap D_i| \leq b\left(\frac{|D_i|}{\epsilon} + E\right).$$

We can conclude that party A will have the majority of votes (and therefore win a district-wide election) if the number of votes for party A if the minimum number of votes for party A is greater than the number of votes for party B. That is,

$$|A \cap D_i| \geq a\left(\frac{|D_i|}{\epsilon} - E\right) \geq b\left(\frac{|D_i|}{\epsilon} + E\right) \geq |B \cap D_i|.$$

Because $b \geq 0$ (and therefore all values involved are greater than zero), we can rearrange the center inequality of this to

$$\frac{b}{a} \leq \frac{|D_i| - \epsilon^2 E}{|D_i| + \epsilon^2 E}.$$

Consider Polsby-Popper compactness to get

$$\frac{b}{a} \leq \frac{|\partial D_i|^2 - C\epsilon^2 E}{|\partial D_i|^2 + C\epsilon^2 E}.$$

So, we have a modified form of the criterion for party A winning the district's election.

Now consider our requirement of approximately equal population per district. Recall that it requires that

$$|(A \cup B) \cap D_i| \geq (1-\delta)\lfloor \frac{|A \cup B|}{k} \rfloor.$$

Note that $n^2 l^2 = |A \cup B|$, as this is the number of voter across the entire "state". So, we can extend this to

$$|(A \cup B) \cap D_i| \geq (1-\delta)\lfloor \frac{|A \cup B|}{k} \rfloor \geq (1-\delta)\frac{|A \cup B|}{2k} = (1-\delta)\frac{n^2 l^2}{2k}.$$

Because of the finite nature of our lattice L of voters, we can tell from this result that there must exist two voters in district D_i that are at least $\sqrt{\frac{1-\delta}{2k}}$ in distance from each other. We will refer to this distance as F.

This bound on distance has implications for the size of the boundary of the district. That is,

$$|\partial D_i| \geq F = \sqrt{\frac{1-\delta}{2k}}.$$

With this bound, we can again consider our criterion for party A winning the election. Let us substitute $\sqrt{2}\frac{|\partial D_i|}{\epsilon} + 2\pi$ back in for E:

$$\frac{b}{a} \leq \frac{|\partial D_i|^2 - C|\partial D_i|\sqrt{2}\epsilon - 2C\pi\epsilon^2}{|\partial D_i|^2 + C|\partial D_i|\sqrt{2}\epsilon + 2C\pi\epsilon^2}.$$

We can achieve a very similar inequality using F as a bound on $|\partial D_i|$ and the value of E:

$$\frac{b}{a} \leq \frac{F^2 - CF\sqrt{2}\epsilon - 2C\pi\epsilon^2}{F^2 + CF\sqrt{2}\epsilon + 2C\pi\epsilon^2}.$$

We may select a, b so that $\gamma = 1 - \frac{b}{a} > 0$ is arbitrarily small and this inequality is true, and party A wins every district.

As such, all $bn^2 = (1-\gamma)an^2$ votes for party B are wasted, and at most $an^2 - bn^2 = \gamma n^2$ votes for party A are wasted. This gives us a partisan efficiency score of

$$EG(D_1, \ldots, D_k; A, B) \leq \frac{\gamma n^2 - bn^2}{an^2 + bn^2} = \frac{2\gamma - 1}{2-\gamma}.$$

This is the value that, in order to satisfy partisan efficiency, the absolute value of should be less than $\frac{1}{2} - \alpha$, in the case where $||A| - |B|| < \beta|A \cup B|$. In this case,

$$\frac{||A| - |B||}{|A \cup B|} = \frac{an^2 - bn^2}{an^2 + bn^2} = \frac{\gamma}{2-\gamma} < \beta.$$

Because of our choice of γ, this value is arbitrarily small and therefore the inequality holds. However, our value for $\frac{1}{2} - \alpha$ is arbitrarily close to $\frac{-1}{2}$. The absolute value of this is close to $\frac{1}{2}$, and therefore we must select α as arbitrarily close to zero, which violates the criteria for satisfying partisan efficiency.

Thus, every districting system that satisfies approximately equal population for every district and Polsby-Popper compactness necessarily violates criterion for partisan efficiency.

3.2 Implications

Approximately equal populations across districts in a state is federally mandated, so the implications of this result lead to a potential tradeoff between compactness and partisan efficiency. Particularly, if the Supreme Court rules that the efficiency gap is a viable metric for quantifying partisan gerrymandering, there may occasionally be contradictions that are extremely confusing for various legislative or judiciary officials.

For example, the efficiency gap may rule a district with a particularly winding and irregular shape is not gerrymandered, which contradicts our many existing metrics that limit the shape of a district. Similarly, many regularly-shaped districts may be ruled excessively gerrymandered by the efficiency gap.

So, we see that creating metrics to determine whether or not a district is gerrymandered is not a simple problem. Even with so many people working on this problem for so long, our metrics contradict each other. But, if these metrics aren't effective, why not use our improved computing ability from the past few decades to just create good districts? In the next section, we will discuss why this is difficult.

4 Hardness of generating legal districts

We have known for twenty years that the problem of creating optimal districts (for many different definitions of optimal) is not easily solvable. In fact, the problem of creating district boundaries is at least NP-hard [Alt97].

4.1 Characterizing difficulty

We can classify problems based on how difficult they are to solve. Difficult here typically means how long a problem takes to solve in the fastest case. [1]

The set of problems solvable in polynomial time is called P. We generally consider problems in P reasonable, (not very difficult to find solutions to).

NP is the set of problems that are *verifiable* in polynomial time. This means that, if we have a proposed solution the problem, we can verify that it is correct in a polynomial way. With our current knowledge, NP $\supseteq$ P. That is, all problems that are solvable in polynomial time are also verifiable in polynomial

[1] Readers who are familiar with computational complexity theory will notice that this section's explanations are a little simplistic (or perhaps a bit naive). The goal here is to simply prepare readers unfamiliar with the material for the following proof.

time. However, some of these problems may not be solvable in polynomial time [2].

NP-hardness is a characteristic of problems that are at least as hard as the hardest problems in NP. So, if some problem in NP may not be solvable in polynomial time, problems that are NP-hard are also not solvable in polynomial time [3].

A problem is NP-hard if it is *polynomial time reducible* from every problem in NP. So, we can show that a problem is NP-hard by showing that it is reducible in polynomial time from another problem known to be both NP-hard and in NP. This is known as *NP-complete*.

A polynomial time reduction takes the new problem as a "black box" inside of a known NP-hard problem, and shows that the inputs to the known problem can be transformed into the inputs to the black box with polynomial map, and similarly for the outputs from the black box to the outputs for the known problem.

4.2 Proof of NP-hardness

Consider a state as a set of census blocks, and the problem of creating district boundaries as a problem of partitioning the census blocks into sets. While there are many different sub-problems of this that are all NP-hard, we will focus on the problem where the districts are required to have equal population [Alt97]. We have chosen this as our focus because approximately equal population is a federal requirement for U.S. districts, and therefore the hardness of this problem is representative of the hardness of districting as a whole.

We define the problem of DISTRICTING here as taking inputs $X = \{x_i\}_{i=1}^n$ a set of n census blocks and k the number of districts to partition the census blocks into. This problem outputs $\{D_i\}_{i=1}^k$ a set of a k districts, each of which is a set of census blocks, such that every district has equal population.

Let us first consider the case where we are splitting our state into only two districts, or $k = 2$.

The known NP-complete problem SET PARTITION takes a finite set of integers $A = \{a_i\}_{i=1}^n$, and splits the integers into two subsets such that the sums of the two subsets are equal.

Let the problem of creating districts with equal population be our "black box". If we take A as the set of census blocks X, each with a population equal to the value of the integer a_i and $k = 2$, this is equivalent to DISTRICTING with only two districts. The outputs of each problem are two subsets of the original input, and the requirement that the population of each district be equal is equivalent to the requirement that the sum of each subset be equal. Thus, the inputs and outputs are effectively the same, and this is polynomial reduction for $k = 2$ districts, so drawing district boundaries with only two districts is an NP-hard problem.

[2] Whether P = NP is an ongoing point of discussion and research.

[3] With our current knowledge.

For $k \geq 3$, let us consider the known NP-complete problem of Three-Partitioning. Given an integer k and a set of n integers $S = \{s_1, s_2, \ldots, s_n\}$ where $n = 3k$, we partition S into k disjoint subsets, each with equal sum. The input integers have one additional qualification:

$$\forall i \in 1, \ldots, n \ \frac{B}{4} < s_i < \frac{B}{2}$$

for some positive integer B, and the sum over all S is kB. This condition ensures that all final partitions are triplets, or sets of three integers.

Again, Districting is our "black box". We take the set of n integers S as a set of n census blocks X, where each census block's population is equal to the value of the corresponding integer s_i. The value k from Three-Partitioning becomes the value k for Districting. The output from Districting will be k triplets of census blocks with equal population, due to the restrictions on the possible values of integers in S. This is equivalent to the desired output for Three-Partitioning, and thus we have a polynomial reduction for $k \geq 3$.

So, creating any number of districts with equal population is an NP-hard problem.

4.3 Implications

How does the NP-hardness of this problem actually impact our creating districts? It's probably okay if our districts are not absolutely, optimally perfect. So why does it matter if the optimality problem is NP-hard?

The NP-hardness of the various subproblems that make up creating district boundaries simply guarantees that some concessions and compromises must be made, and we can't simply say "Yes, this is absolutely the best solution." What concessions are made, and who decides what concessions are made, are contest. There is no perfect, objective solution. Thus, there is room for partisan bias and subjectivity in our districting methods.

Additionally, even with good approximations for optimality algorithms, what are we optimizing for? Aside from achieving approximately equal population across districts, do we optimize for compactness? the partisan efficiency gap? This brings us back to the same problems from the first section of this paper, where our metrics for evaluating gerrymandering are imperfect.

5 Conclusion

Throughout the existence of the United States (even before the term was coined in 1812), gerrymandering has been a concern. As discussed in this paper, it isn't an easy problem to solve. However, our discussion of gerrymandering here is even a simpler view of the issue than it truly is [4].

[4] I highly recommend FiveThirtyEight's Gerrymandering Project if you would like to learn more [Fiv17].

Beyond partisan gerrymandering, racial gerrymandering is an issue. The Voting Rights Act mandates that districts be drawn such that racial minority groups have a say in their elections. However, this can lead to racial groups being packed into just a few districts. Racial gerrymandering has been more acknowledged by official legislation than partisan gerrymandering, in part due to the existence of the Voting Rights Act.

Here we have mostly discussed deliberate gerrymandering, where a particular group carefully manipulates district boundaries for their own gain (as happened in Wisconsin, and prompted the Supreme Court case considering the efficiency gap). However, both partisan and racial gerrymandering can happen accidentally. Liberal people, often people belonging to racial minority groups, tend to live in the cities. Increasingly, conservative voters tend to live suburban and rural spaces. So, even without deliberately packing or cracking any particular group, district boundaries tend to give the advantage to conservative voters, who are more spread out. Many researchers are trying confront this by using simulations to generate random (but valid) districts, and measuring the expected tilt [CR15].

Mathematicians are also approaching this from different angles: a summer program led by Tuft's Dr. Moon Duchin considering compactness in non-Euclidean spaces [Cha17]. Hopefully, with work like this, we can minimize excessive gerrymandering, and have more fair elections, even as soon as 2018 or 2020.

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