Gerrymandering and Impossibility

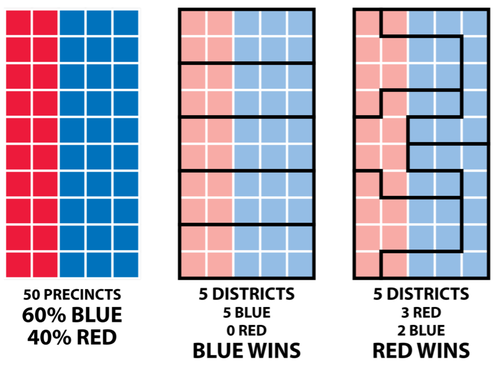
Emma Remy '18

Swarthmore College

What is Gerrymandering?

Gerrymandering is the act of deliberately setting or manipulating political district boundaries so as to favor (or disfavor) a particular group.

There are two methods of gerrymandering: *Packing* occurs when a gerrymander packs members of a particular group into only a few districts districts, so they will only win there and nowhere else. *Cracking* occurs when a gerrymander splits members across districts, so they can't acheive a majority anywhere.



Evaluation Metrics

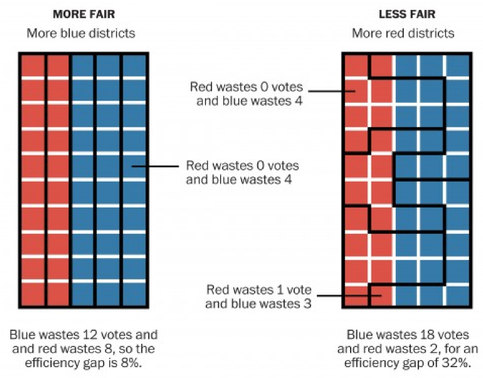
There are a number of existing metrics for evaluating whether or not a given district is excessively gerrymandered.

- *Equal population.* Having districts of roughly equal population throughout the state ensures that each individual's vote counts for approximately the same amount.
- *Continuity.* A district is continuous if it possible to travel from any point in the district to any other point in the district without leaving the district.
- *Existing boundaries.* Many states require that districts follow existing political boundaries, such as city limits.
- *Compactness.* Conventional wisdom suggests that oddly-shaped districts (with long, winding boundaries) are more likely to be created as the result of gerrymandering. Using the Polsy-Popper definition of compactness, a district is compact if $\exists\ C > 0$ such that for districts D_i with perimeter $|\partial D_i|$ and area $|D_i|\ \forall i \in \{1, \ldots, k\}$,

$$|\partial D_i|^2 \leq C|D_i|.$$

Efficiency Gap

The efficiency gap is currently being evaluated by the Supreme Court as a legal measure of partisan gerrymandering. Proposed by Stephanopoulos and McGhee in 2015, the efficiency gap measures the number of "wasted" votes in a district. Votes are considered "wasted" if they are for the losing party, or over the majority for the winning party.



Formally, with A, B disjoint sets of voters in two different parties and districts $D_i \forall i \in \{1, \ldots, k\}$, the efficiency gap is given by

$$\frac{1}{|A \cup B|} \sum_{i=1}^{k} (w_{A,i} - w_{B,i})$$

where $w_{A,i}$ is the wasted votes in $A \cap D_i$,

$$|A \cap D_i| - \lceil \frac{1}{2} |(A \cup B) \cap D_i| \rceil.$$

Theorem 1

As recently proven by Alexeev and Mixon, it's impossible to have a district that fulfills all three of one vote per person, Polsy-Popper compactness, and the efficiency gap.

Proof idea

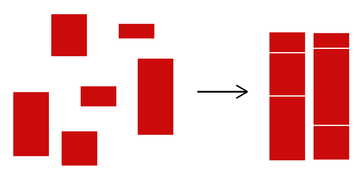
The proof for Theorem 1 is structurally very similar to the proof of Arrow's Impossibility Theorem. Take one vote per person (approximately equal population across districts) and Polsby-Popper compactness to be true, and show with algebra that the efficiency gap as defined here cannot exist.

Generating District Boundaries

With existing metrics for evaluating gerrymandering conflicting and sometimes seeming arbitrary, it may seem like we need to find a different solution to this problem. Many people have turned to computers to produce reasonable districts with complex programs, but using simulations to generate district boundaries comes with its own difficulties.

Theorem 2

As shown with polynomial-time reductions to NP-complete problems like 3-Partitioning (visualized below), generating optimal districts with reasonable metrics is at least an NP-complete problem.



Conclusions and Implications

While these results may seem discouraging toward actually preventing/eliminating partisan gerrymandering, things may not be as hopeless as they seem.

Theorem 1 does not imply that the efficiency gap is an ineffective metric for evaluating partisan gerrymandering. It simply means that districts that satisfy the efficiency gap may not fulfill our traditional, intuitive senses of what a non-gerrymandered district looks like.

Similarly, even though generating optimal districts is NP-complete, we can use heuristics to generate districts much better than many of those that currently exist. However, what "optimal" means in this context is contested.

References

[1] Boris Alexeev and Dustin G. Mixon. "An impossibility theorem for gerrymandering". In: (2017).

[2] Micah Altman. "The Computational Complexity of Automated Re-districting: Is Automation the Answer?" In: *Rutgers Computer and Law Technology Journal* (1997).