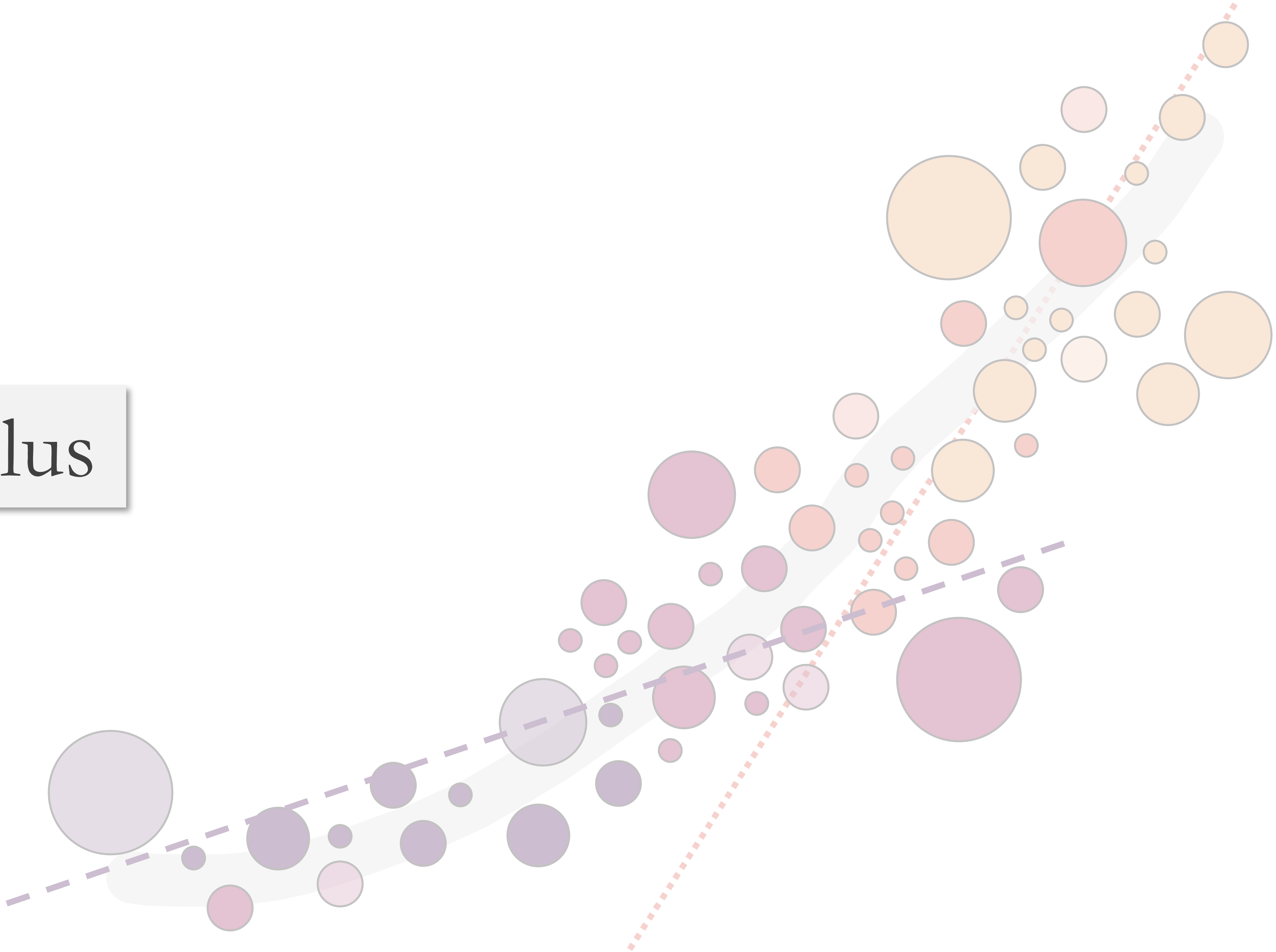
Scalar calculus



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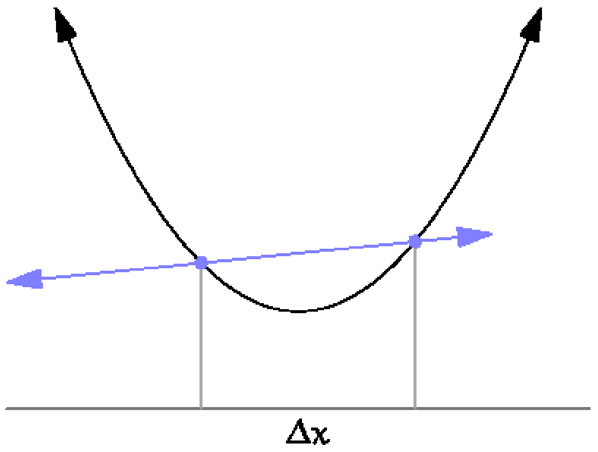
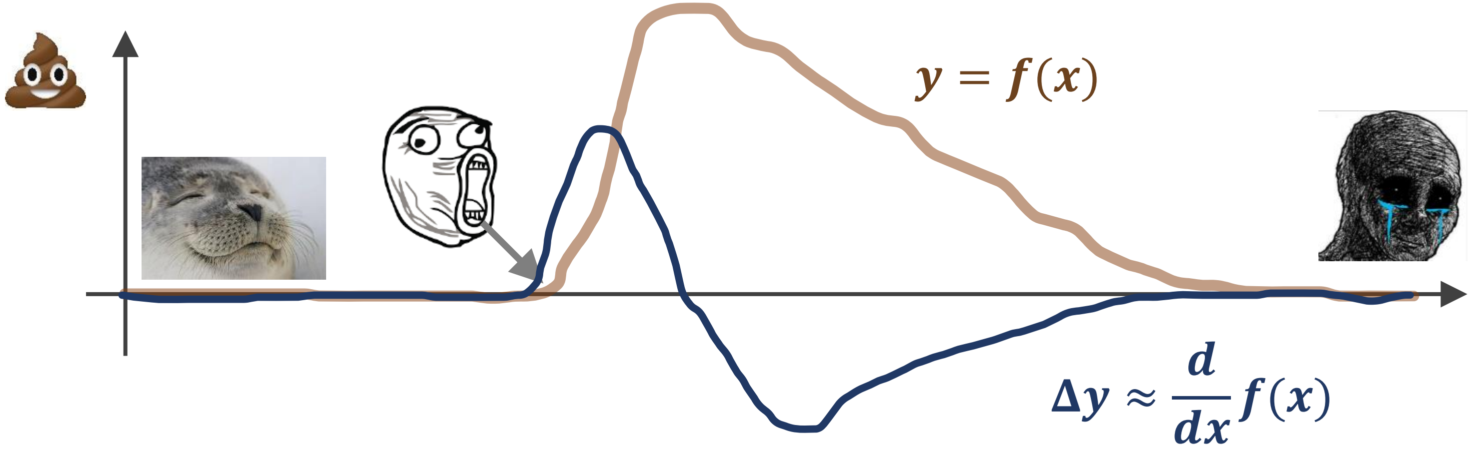
Limits and derivatives



Differentials (Δ) measure the change in the dependent variable while **derivatives** ($\frac{d}{dx}f(x)$) measure the rate of the change of the dependent variable with respect to the independent variable.



$$f'(x) = \frac{d}{dx}f(x) = \frac{df(x)}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Newton's notation

Leibniz's notation

Numerical differentiation

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

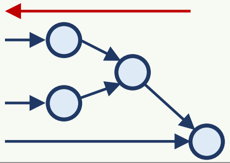
When x is known, h is very small and f unknown.

Symbolic differentiation

$$f(x) = x^2 \rightarrow \frac{d}{dx}f(x) = 2x$$

Automatic differentiation

Convert a function into a differentiable computational graph



julia

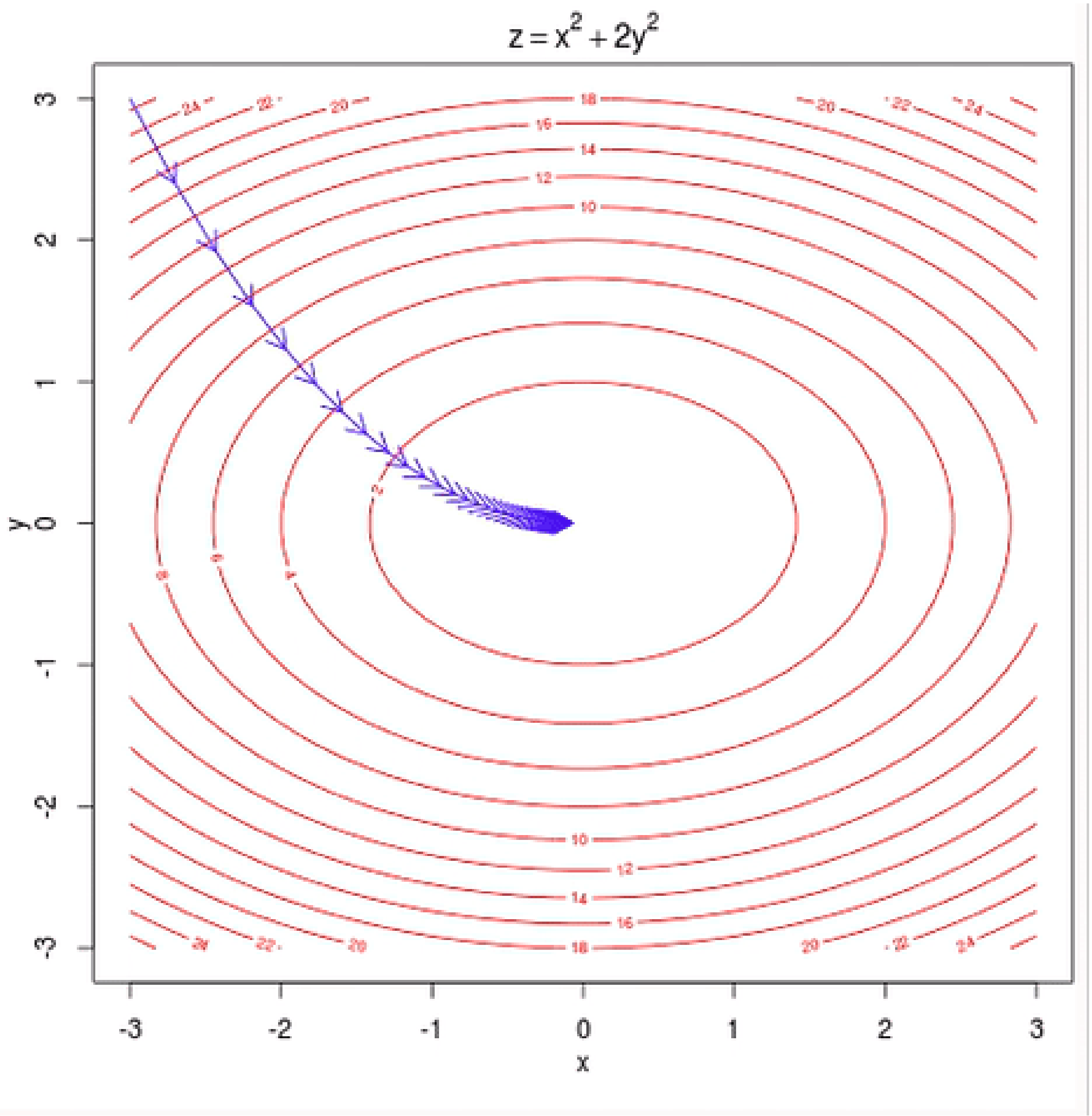
PyTorch

PYTENSOR

Gradient Descent

If the multivariable function $F(x)$ is defined and differentiable in a neighborhood of a point a, then $F(x)$ decrease fastest if one goes from a in the direction of the negative gradient of F at **a**.

$$\boldsymbol{a}_{n+1} = \boldsymbol{a}_n - \gamma \Delta F(\boldsymbol{a}_n)$$



Derivative rules

Common Functions	Function	Derivative
Constant	c	0
Line	x	1
	ax	a
Square	x^2	$2x$
Square Root	$\sqrt{x}$	$(½)x^{-½}$
Exponential	e^x	e^x
	a^x	$\ln(a)\ a^x$
Logarithms	$\ln(x)$	$1/x$
	$\log_a(x)$	$1\ /\ (x \ln(a))$
Trigonometry (x is in radians)	$\sin(x)$	$\cos(x)$
	$\cos(x)$	$-\sin(x)$
	$\tan(x)$	$\sec^2(x)$
Inverse Trigonometry	$\sin^{-1}(x)$	$1/\sqrt{(1-x^2)}$
	$\cos^{-1}(x)$	$-1/\sqrt{(1-x^2)}$
	$\tan^{-1}(x)$	$1/(1+x^2)$

Rules	Function	Derivative
Multiplication by constant	cf	cf'
Power Rule	x^n	nx^{n-1}
Sum Rule	$f + g$	$f' + g'$
Difference Rule	$f - g$	$f' - g'$
Product Rule	fg	$f\,g' + f'\,g$
Quotient Rule	f/g	$f'\,g - g'\,f\mathbf{g^2}$
Reciprocal Rule	$1/f$	$-f'/f^2$
Chain Rule (as "Composition of Functions")	$f \circ g$	$(f' \circ g) \times g'$
Chain Rule (using ')	$f(g(x))$	$f'(g(x))g'(x)$
Chain Rule (using $\frac{d}{dx}$)	$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$	