

Emergent Analogical Reasoning in Large Language Models

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Abstract

The recent advent of large language models — large neural networks trained on a simple predictive objective over a massive corpus of natural language — has reinvigorated debate over whether human cognitive capacities might emerge in such generic models given sufficient training data. Of particular interest is the ability of these models to reason about novel problems *zero-shot*, without any direct training on those problems. In human cognition, this capacity is closely tied to an ability to reason by analogy. Here, we performed a direct comparison between human reasoners and a large language model (GPT-3) on a range of analogical tasks, including a novel text-based matrix reasoning task closely modeled on Raven’s Progressive Matrices. We found that GPT-3 displayed a surprisingly strong capacity for abstract pattern induction, matching or even surpassing human capabilities in most settings. Our results indicate that large language models such as GPT-3 have acquired an emergent ability to find zero-shot solutions to a broad range of analogy problems.

1 Introduction

Analogical reasoning is at the heart of human intelligence and creativity. When confronted with an unfamiliar problem, human reasoners can often identify a reasonable solution through a process of structured comparison to a more familiar situation [1]. This process is an essential part of human reasoning in domains ranging from everyday problem-solving [2] to creative thought and scientific innovation [3]. Indeed, tests of analogical reasoning ability are uniquely effective as measures of fluid intelligence: the capacity to reason about novel problems [4, 5].

Recently, there has been considerable debate about whether and how a capacity for analogical thought might be captured in deep learning systems [6]. Much of this recent work has focused on training neural networks on very large datasets (containing millions of problems in some cases) of analogy problems [7, 8]. Though this is a challenging task that has spurred the development of some interesting approaches [9, 10], this approach does not address the issue of whether analogical reasoning can emerge *zero-shot* (i.e., without direct training), the capacity most central to human thought.

An alternative approach, also based on deep learning, involves large language models (LLMs) [11]. LLMs have recently sparked great interest (and great controversy) for their potential to perform few-shot, and even zero-shot, reasoning. These models employ relatively generic neural network architectures with up to billions of parameters, and are trained using a simple predictive objective (predicting the next token in a sequence of text) with massive web-based text corpora consisting of billions of tokens. Though there is significant debate about the capabilities of these models, particularly with respect to their commonsense knowledge [12], a potential advantage is their ability to solve problems with little direct training, sometimes requiring only a few examples, or even a simple task instruction (typically without any updating of model parameters). This feature raises the question of whether LLMs might be capable of human-like, zero-shot analogical reasoning.

To answer this question, we evaluated the language model GPT-3 [11] on a range of zero-shot analogy tasks, and performed a direct comparison with human behavior. These tasks included a novel text-based matrix reasoning task with comparable problem structure and complexity as Raven’s Progressive Matrices [13], a visual analogy problem set that is commonly viewed as one of the best measures of fluid intelligence [5]. Strikingly, we found that GPT-3 performed as well or better than humans in most conditions, despite receiving no direct training on this task. GPT-3

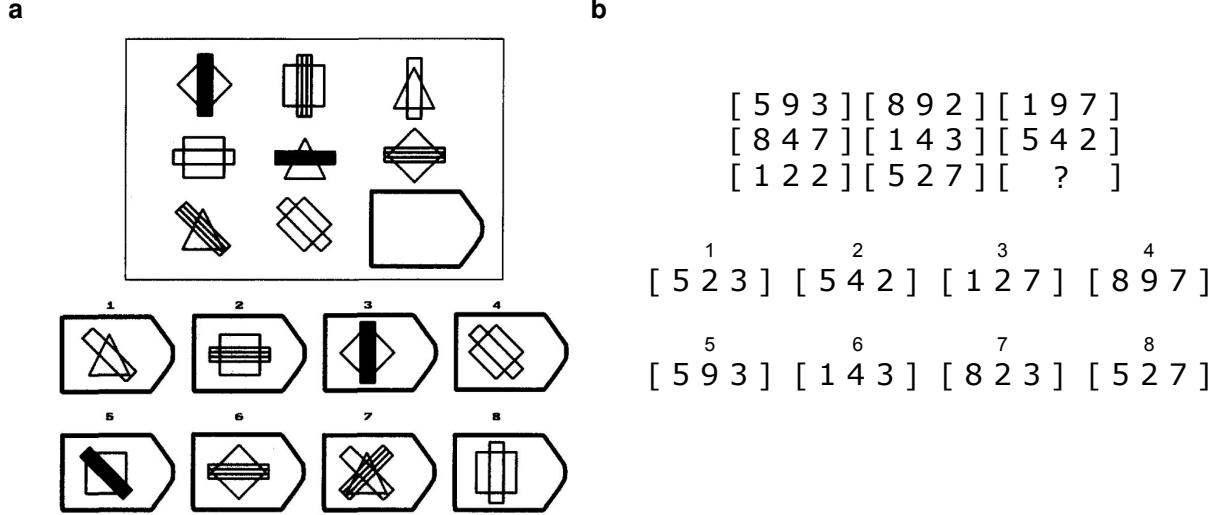


Figure 1: Matrix reasoning problems. (a) Example problem depicting structure of Raven’s Progressive Matrices [13] (reproduced from [19]). Problems consist of a 3×3 matrix populated with geometric forms, in which each row or column is governed by the same set of abstract rules. Problem solvers must induce these rules, and use them to infer the missing cell in the lower right, by selecting from the set of 8 choices below. (b) Example problem illustrating the novel Digit Matrices problem set. Problems consist of a 3×3 matrix, in which each cell is demarcated by brackets, and populated by digits. The problems are governed by the same rule structure as Raven’s Standard Progressive Matrices. The example problems in (a) and (b) are structurally isomorphic (i.e., governed by the same set of rules). The reader is encouraged to derive the solution to each problem, along with the isomorphism between them. The solutions to both problems are given in Supplementary Section S1.

also displayed strong zero-shot performance on letter string analogies [14, 15], four-term verbal analogies [16], and identification of analogies between stories [17, 18]. These results provide a rich characterization of the emergent analogical reasoning capabilities in current LLMs, suggesting that they may already possess one of the defining features of human reasoning.

2 Results

We evaluated the language model GPT-3 on a set of analogy tasks, and compared its performance to human behavior. GPT-3 is a transformer-based [20] language model containing 175 billion parameters, trained on a web-based corpus of natural language consisting of over 400 billion tokens [11, 21]. We used the text-davinci-003 instance of this model, which was trained using two objectives. The first training objective was the standard predictive objective used to train language models, in which, given a string of natural language, the model is trained to predict the token that will most likely appear next. The key feature distinguishing this more recent instance of GPT-3 from its predecessors was the use of an additional alignment objective, intended to improve its ability to accurately respond to human-generated prompts. This was accomplished using a reinforcement learning (RL) approach, in which a separate reward model is first trained to predict human ratings for pairs of human-generated prompts and language-model responses, and this reward model is then used to fine-tune GPT-3 through RL.

Our primary evaluation focuses on a novel text-based matrix reasoning task that we designed to emulate the structure and complexity of Raven’s Standard Progressive Matrices (SPM). The task is illustrated in Figure 1. We also evaluated GPT-3 on letter string analogies [14, 15] and four-term verbal analogies [16], and performed a qualitative evaluation on a more naturalistic problem-solving task that requires forming an analogy between two stories [17, 18].

a	Constant	b	Distribution-of-3	c	Progression
	$\begin{bmatrix} 5 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 9 \end{bmatrix}$		$\begin{bmatrix} 6 \end{bmatrix} \begin{bmatrix} 2 \end{bmatrix} \begin{bmatrix} 4 \end{bmatrix}$		$\begin{bmatrix} 3 \end{bmatrix} \begin{bmatrix} 5 \end{bmatrix} \begin{bmatrix} 7 \end{bmatrix}$
	$\begin{bmatrix} 5 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 9 \end{bmatrix}$		$\begin{bmatrix} 2 \end{bmatrix} \begin{bmatrix} 4 \end{bmatrix} \begin{bmatrix} 6 \end{bmatrix}$		$\begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 3 \end{bmatrix} \begin{bmatrix} 5 \end{bmatrix}$
	$\begin{bmatrix} 5 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} ? \end{bmatrix}$		$\begin{bmatrix} 4 \end{bmatrix} \begin{bmatrix} 6 \end{bmatrix} \begin{bmatrix} ? \end{bmatrix}$		$\begin{bmatrix} 5 \end{bmatrix} \begin{bmatrix} 7 \end{bmatrix} \begin{bmatrix} ? \end{bmatrix}$
d	Two rules	e	Three rules		
	$\begin{bmatrix} 7 & 1 \end{bmatrix} \begin{bmatrix} 8 & 9 \end{bmatrix} \begin{bmatrix} 6 & 3 \end{bmatrix}$		$\begin{bmatrix} 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \end{bmatrix}$		
	$\begin{bmatrix} 6 & 9 \end{bmatrix} \begin{bmatrix} 7 & 3 \end{bmatrix} \begin{bmatrix} 5 & 1 \end{bmatrix}$		$\begin{bmatrix} 7 & 1 & 2 \end{bmatrix} \begin{bmatrix} 7 & 3 & 3 \end{bmatrix} \begin{bmatrix} 7 & 6 & 4 \end{bmatrix}$		
	$\begin{bmatrix} 5 & 3 \end{bmatrix} \begin{bmatrix} 6 & 1 \end{bmatrix} \begin{bmatrix} ? \end{bmatrix}$		$\begin{bmatrix} 2 & 6 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} ? \end{bmatrix}$		
f	OR (aligned)	g	OR (permuted)		
	$\begin{bmatrix} 7 \end{bmatrix} \begin{bmatrix} 7 & 4 \end{bmatrix} \begin{bmatrix} 4 \end{bmatrix}$		$\begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 7 & 1 \end{bmatrix} \begin{bmatrix} 7 \end{bmatrix}$		
	$\begin{bmatrix} 9 & 7 \end{bmatrix} \begin{bmatrix} 9 & 7 & 4 & 8 \end{bmatrix} \begin{bmatrix} 4 & 8 \end{bmatrix}$		$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 0 & 7 & 1 \end{bmatrix} \begin{bmatrix} 7 & 5 \end{bmatrix}$		
	$\begin{bmatrix} 9 \end{bmatrix} \begin{bmatrix} 9 \end{bmatrix} \begin{bmatrix} 8 \end{bmatrix} \begin{bmatrix} ? \end{bmatrix}$		$\begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} 0 & 5 \end{bmatrix} \begin{bmatrix} ? \end{bmatrix}$		

Figure 2: Example problems from Digit Matrices problem set. Text-based problem set based on structure of problems from Raven’s Standard Progressive Matrices. Problems were governed either by one or more *transformation* rules (panels a-e), or by a single *logic* rule (panels f and g). **(a)** *Constant* rule: same digit appears across either rows or columns. **(b)** *Distribution-of-3* rule: same set of 3 digits appears in each row or column, but with order varied. **(c)** *Progression* rule: digits either increase or decrease, by values of 1 or 2, across rows or columns. In the example shown here, digits increase by 2 across rows. **(d)** Two-rule problem, with the left digit in each cell governed by a progression rule, and the right digit in each cell governed by a distribution-of-3 rule. **(e)** Three-rule problem, governed by a constant (left), distribution-of-3 (middle), and progression (right) rule. **(f)** *OR* rule: the set of digits present in a particular row or column are defined as the union of the sets present in the other rows or columns. In the illustrated example, the digits in the second column are formed from the union of the sets in the first and third columns. This example illustrates how the spatial alignment of the corresponding elements can make it easier to intuitively grasp the underlying rule. **(g)** More challenging logic problem governed by same rule (*OR*), but in which the corresponding elements are spatially permuted. Other logic problems were governed either by an *AND* rule or an *XOR* rule (not pictured). The solutions to all example problems are given in Supplementary Section S1.

2.1 Matrix reasoning problems

Figure 2 illustrates the structure of the Digit Matrices problem set. The dataset was structured similarly to the work of Matzen et al. [22], who created, and behaviorally validated, a visual matrix reasoning dataset with the same rule structure and complexity as the original SPM. The Digit Matrices dataset is thus comparable to SPM, but guaranteed to be novel for both humans and LLMs. Digit Matrix problems consisted of either digit transformations (Figures 2a- 2e) or logic problems (Figures 2f- 2g). Transformation problems were defined based on a set of three rules: *constant*, *distribution-of-3*, and *progression*. The constant rule entailed the same digit appearing across either rows or columns (Figure 2a). The distribution-of-3 rule entailed the same set of three digits appearing in each row or column, but with the location permuted (Figure 2b). The progression rule entailed a progressive increase or decrease in value (with increments of either 1 or 2) along rows or columns (Figure 2c). Transformation problems consisted of one or more rules. When multiple rules were present, these rules were bound to different spatial locations within each cell (e.g., one rule was bound to the left digit in each cell, and another rule was bound to the right digit).

Logic problems were defined based on a single rule involving a set relation: *OR*, *AND*, or *XOR*. In these problems, the digits present in a particular row or column were determined by applying a set relation to the set of digits present in the other rows or columns. For instance, the problem shown in Figure 2f is defined by an *OR* rule, in which the digits present in the middle column are those that are present in either the left or right columns (i.e., the union of

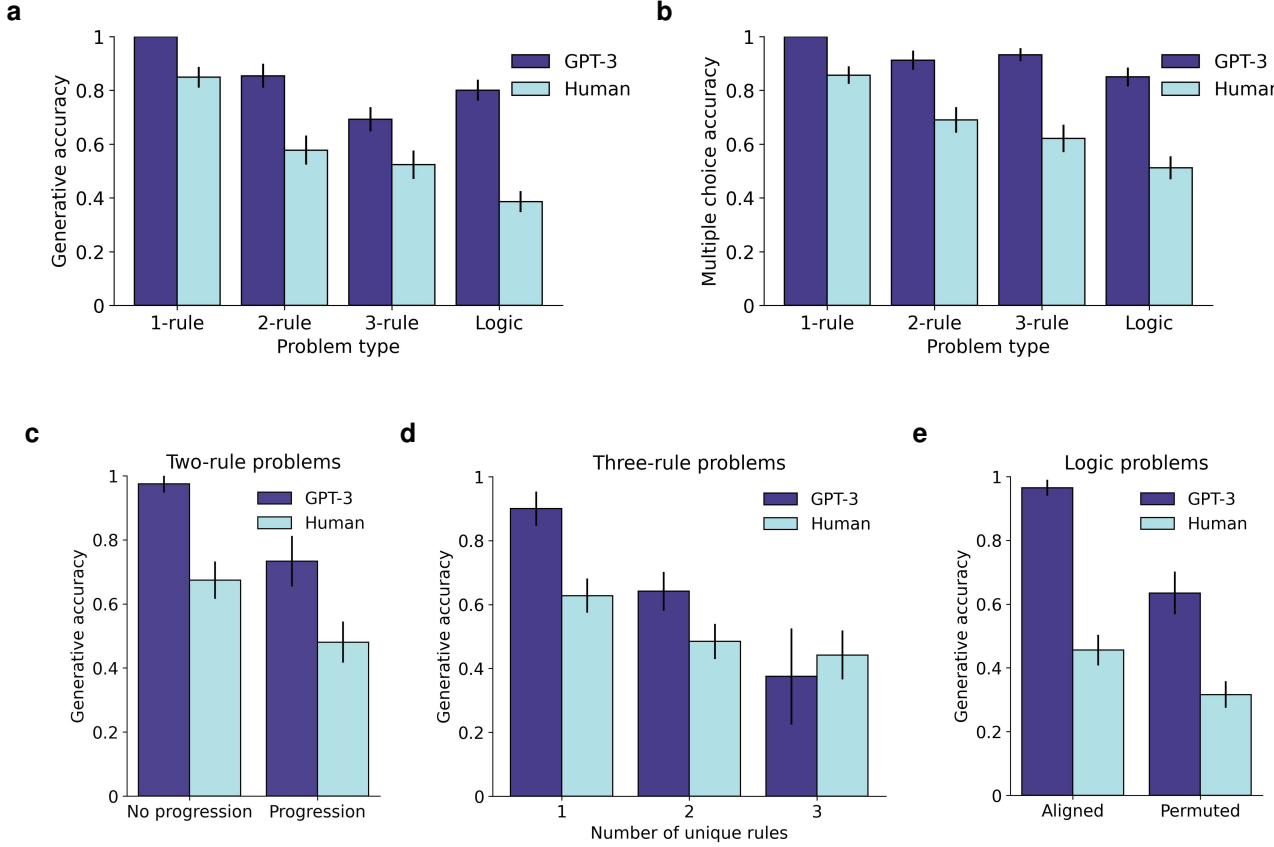


Figure 3: Zero-shot Digit Matrices. GPT-3 matched or exceeded human performance for zero-shot digit matrix problems. (a) Generative accuracy for major problem types, including one-, two-, three-rule and logic problems. (b) Multiple-choice accuracy for major problem types. (c) Two-rule problems with at least one progression rule were more difficult than those without. (d) For three-rule problems, performance was a function of the number of unique rules. (e) Spatially permuted logic problems were more difficult than spatially aligned problems. Human results reflect average performance for $N=43$ participants \pm the standard error of the mean across participants. GPT-3 results reflect average performance over multiple problem instances \pm 95% binomial confidence intervals.

the sets in the left and right columns). In some logic problems, the corresponding elements were spatially aligned (Figure 2f), whereas in others they were spatially permuted (Figure 2g). We hypothesized that spatial alignment would be beneficial when solving the problems via analogical mapping, as it should highlight the isomorphism between analogs [23].

Figure 3 shows performance on the Digit Matrices problems for GPT-3 and human participants ($N=43$, UCLA undergraduates). Human participants were presented with a set of 32 problems presented in random order. These problems were defined by the same set of 32 problem subtypes (particular combinations of one, two, or three rules, or particular logic rules), but the specific problem instances were randomly sampled for each participant. GPT-3 was presented with 40 problem instances from each of the 32 problem subtypes, and solved each one zero-shot (without any fine-tuning or in-context learning). We evaluated analogy performance using two tasks: a generative task (directly filling in the blank cell) and a multiple-choice task (selecting the best answer from a set of 8 choices). For GPT-3, multiple-choice performance was assessed by presenting the problem along with each potential answer choice, and selecting the choice with the highest average log probability.

GPT-3 outperformed human participants on all problem types, both in terms of generative accuracy (Figure 3a; logistic regression, main effect of GPT-3 vs. human participants: $p < 0.0001$), and multiple-choice accuracy (Figure 3b; main effect of GPT-3 vs. human participants: $p < 0.0001$). Despite this difference in overall performance

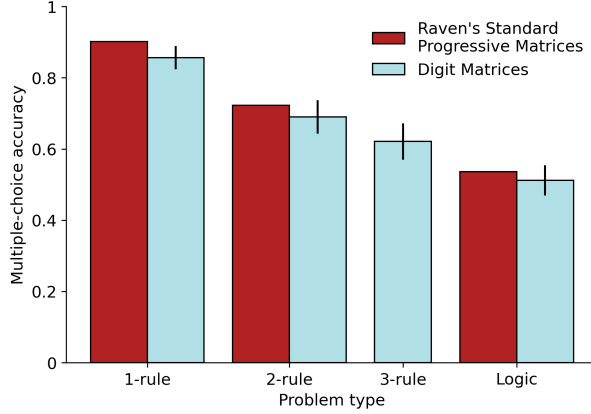


Figure 4: Digit Matrices vs. Raven’s Standard Progressive Matrices (SPM). Digit Matrices and SPM show a similar pattern of performance across problem types (SPM data from [22]).

levels, GPT-3 and human participants showed a similar pattern of performance across problem subtypes (correlation analysis: $r = 0.39$, $p = 0.029$). This was driven both by the pattern of performance across major problem types (one-, two-, three-rule, and logic problems), and by differences within each problem type. Problems with progression rules were more difficult than those without them (Figure 3c; main effect of progression vs. no progression, human participants: $p = 0.002$, GPT-3: $p < 0.0001$); for multi-rule problems, performance was negatively correlated with the number of unique rules in each problem, even when holding constant the number of total rules (Figure 3d; main effect of number of unique rules, human participants: $p = 0.007$, GPT-3: $p < 0.0001$); and logic problems were more difficult when the corresponding elements were spatially permuted vs. aligned (Figure 3e; main effect of spatial alignment, human participants: $p = 0.003$, GPT-3: $p < 0.0001$). These effects replicate well-known characteristics of human analogical reasoning: problems defined by relations (e.g., progression) are typically more difficult than problems defined by the features of individual entities (e.g., constant or distribution-of-3) [22, 24]; problem difficulty is typically driven by the degree of relational complexity, as defined by the number of unique relations [25]; and analogical mapping is easier when a greater number of constraints supports the correct mapping (as is the case in the spatially aligned logic problems) [23]. GPT-3’s pattern of performance thus displayed many of the characteristics of a human-like analogical mapping process.

One potential concern with the Digit Matrices dataset is that its text-based format might not engage the same processes as the visual format employed in traditional matrix reasoning problems. For instance, SPM problems often involve multiple distinct rules bound to different feature dimensions of the same set of objects (as seen in Figure 1a), requiring a greater degree of selective attention, whereas each rule is always bound to a distinct sets of digits in the Digit Matrices. To address this issue, we compared the pattern of performance across problem types for Digit Matrices vs. SPM (using data from [22]). The SPM do not contain three-rule problems, but the pattern of human performance was very similar across one-rule, two-rule, and logic problems (Figure 4), suggesting that the Digit Matrices involve a similar level of problem difficulty as do traditional matrix reasoning problems.

We also investigated how performance might be affected by the problem-solving context. In particular, we hypothesized that progressive presentation of problems in terms of increasing complexity (easy-to-hard: one-rule problems, followed by two-rule problems, and so on) might improve performance relative to random presentation order, by enabling participants to more easily identify the regular spatial structure of the Digit Matrix problems. We were also interested to see whether GPT-3, which has a relatively large context window (2048 tokens), might benefit from such contextual information.

Figure 5a shows the results of this experiment. Human participants ($N=47$, UCLA undergraduates) performed a series of 42 problems in order of increasing complexity, ranging from one-rule to five-rule problems. GPT-3 was presented with a series of these same 42 problem subtypes, in the same order. After answering each problem, it was appended to the context information, along with the multiple-choice answer selected by GPT-3. GPT-3 performed 20 runs on this task, with different problem instances for each of the 42 problem subtypes.

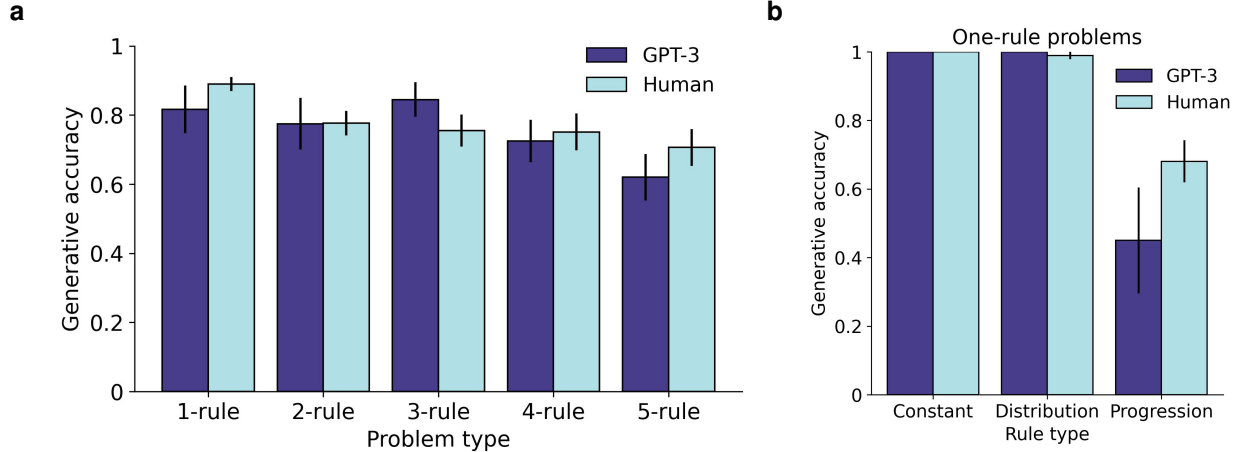


Figure 5: Transfer from few-rule to many-rule problems. (a) When presented with problems in order of increasing complexity, both GPT-3 and human participants were able to generalize the structure inferred from few-rule problems to more complex many-rule problems, resulting in very little decrease in performance even for five-rule problems. (b) Progressive presentation resulted in decreased performance on one-rule problems with progression rules, likely due to a tendency to mistake the progression rule for the distribution-of-3 rule in the previously presented problems.

Both GPT-3 and human participants showed evidence of benefiting from this presentation format, displaying only a very small decrement in performance from one-rule to five-rule problems. To quantify this effect, we compared performance on one- through three-rule problems for zero-shot (or random presentation order) vs. progressive presentation order (increasing complexity). Both GPT-3 and human participants showed a significant interaction between presentation format (progressive vs. zero-shot / shuffled) and problem type (one- to three-rule) (GPT-3: $p < 0.0001$, human participants: $p = 0.029$). This suggests that GPT-3, like human reasoners, was able to identify the regular structure of these problems and use it to compositionally generalize from few-rule to many-rule problems.

Interestingly, GPT-3 performed worse on one-rule problems when presented progressively than when presented zero-shot (compare one-rule performance in Figures 5a and 3a). This effect was driven by lower performance on progression problems (Figure 5b), and likely due to the fact that progression problems were presented following distribution-of-3 problems in this format, leading GPT-3 to mistake a progression rule for a distribution-of-3 rule. Human participants showed a similar effect. GPT-3 was therefore sensitive to contextual information in ways that both improved and impaired its performance, similar to human reasoners. Notably, we found that an earlier version of GPT-3, text-davinci-002, does not show these same contextual effects, suggesting that fine-tuning based on reinforcement learning with human-generated prompts enhanced text-davinci-003’s sensitivity to global contextual information (Supplementary Figure S1).

2.2 Letter string analogies

The letter string analogy domain (Figure 6) was introduced by Hofstadter and Mitchell [14, 15] in order to evaluate computational models of analogical reasoning. This task domain has much in common with our Digit Matrices, in that problems are composed of simple alphanumeric characters, but nevertheless require a significant degree of abstraction to identify an analogy. An informal evaluation performed by Mitchell indicated that GPT-3 was not capable of reliably solving even the most basic letter string analogies zero-shot [26]. Given the similarities between letter string analogies and Digit Matrices, this negative conclusion would seem to be discrepant with the present results. We therefore carried out a more systematic evaluation of GPT-3’s capabilities in the letter string domain.

Figure 6 illustrates the different types of letter string problems that we studied. In the most basic problem (Figure 6a), the letter string ‘a b c’ is transformed to ‘a b d’, involving a successor relation between ‘c’ and ‘d’. In this problem, this successor relation must be mapped on to a distinct set of letters, e.g., by transforming ‘i j k’ to ‘i j l’. Some of the other problems involve various generalizations of this successor relation, by generalizing from letters to numbers (Figure 6c), generalizing from individual letters to groups of letters (Figure 6d), or generalizing to a

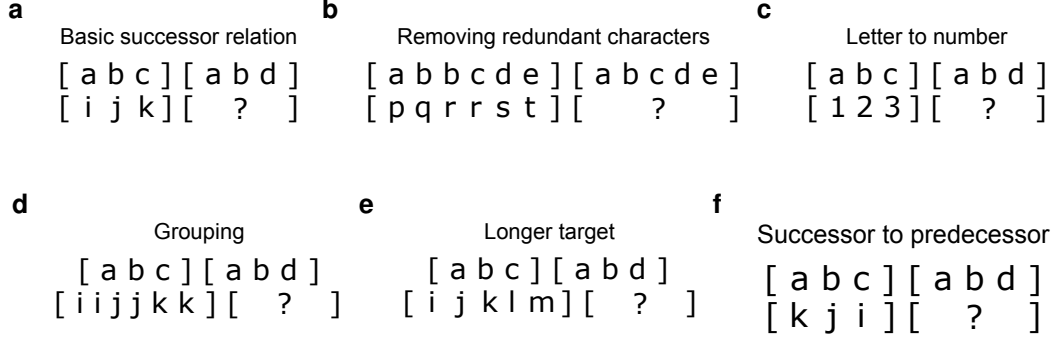


Figure 6: Letter string analogy problems. (a) Basic successor relation: successor relation in source (changing ‘c’ to ‘d’) is applied to new set of letters in target. (b) Removing redundant characters: source transformation involves removal of redundant character pair, target involves different set of letters and different location for redundant pair. (c) Generalization of successor relation from letters to numbers. (d) Generalization of successor relation from individual letters to groups of letters. (e) Generalization to longer target string. (f) Generalization of successor relation to predecessor relation.

longer target string (Figure 6e). We also included problems where the transformation involves removing a redundant character (Figure 6b), and problems that involve generalizing from a successor relation to a predecessor relation (Figure 6f).

We evaluated GPT-3’s zero-shot performance on these problems using a format similar to the Digit Matrices. We also used a prompt (‘Let’s try to complete the pattern.’, similar to [27]) to indicate which task should be performed. We evaluated GPT-3 on multiple instances of each problem type by varying the specific characters used in the target (e.g., using ‘p q r’ instead of ‘i j k’ in Figure 6a).

GPT-3 displayed perfect performance on 4 out of 6 problem types (Figure 7a). On problems with a longer target string, GPT-3 produced the intended answer (applying a successor relation to the final letter in the string) on 11 out of 16 problems (69% accuracy). We also compared GPT-3’s performance on the successor-to-predecessor problem with human behavioral data from a previous study [28]. When presented with this problem zero-shot, GPT-3 identified the successor-to-predecessor mapping (resulting in the response ‘k j h’) more reliably than human participants ($24/66 = 36\%$ of human participants vs. $10/17 = 59\%$ of problems for GPT-3; Figure 7c).

This same behavioral study also showed evidence of a relational priming effect, in which the relation identified in one analogy problem influences the relation identified in a second problem presented immediately afterwards. To test for this effect, rather than solving the successor-to-predecessor problem zero-shot, some participants were first presented with a problem designed to induce the use of a successor relation (Figure 7b). Indeed, for this initial problem, most participants produced a response that employed a successor relation, either transforming the target prompt ‘m r r j j j’ to produce the response ‘m r r j j k’ (applying the successor relation to the final letter in the sequence; 11/74 participants), or transforming it to produce the response ‘m r r k k k’ (generalizing the successor relation from a single letter to a group of letters; 34/74 participants). The relational priming effect can be seen by comparing Figures 7c and 7d. When first presented with the successor context problem, participants were both more likely to employ a successor relation in response to the successor-to-predecessor problem (‘k j j’ response, $43/74 = 58\%$ of participants in Figure 7d vs. $14/66 = 21\%$ of participants in Figure 7c), and less likely to employ a predecessor relation (‘k j h’ response, $9/74 = 12\%$ of participants in Figure 7d vs. $24/66 = 36\%$ of participants in Figure 7c).

We investigated whether GPT-3 would also display this relational priming effect. We first presented GPT-3 with the context problem in Figure 7b (or isomorphic problems with different target letters). Though GPT-3’s responses did not exactly match the distribution of human responses (GPT-3 most often responded with ‘m r r j j k’, whereas human participants most often responded with ‘m r r k k k’), the responses did employ a successor relation for a majority of problems, thus satisfying the conditions to test for relational priming. When subsequently presented with the successor-to-predecessor problem, GPT-3 was both more likely to employ a successor relation (‘k j j’ response, $4/17 = 24\%$ of problems in Figure 7d vs. $0/17$ problems in Figure 7c), and less likely to employ a predecessor relation (‘k j h’ response, $5/17 = 29\%$ of problems in Figure 7d vs. $10/17 = 59\%$ of problems in Figure 7c), thus showing a similar relational priming effect as that seen in human participants.

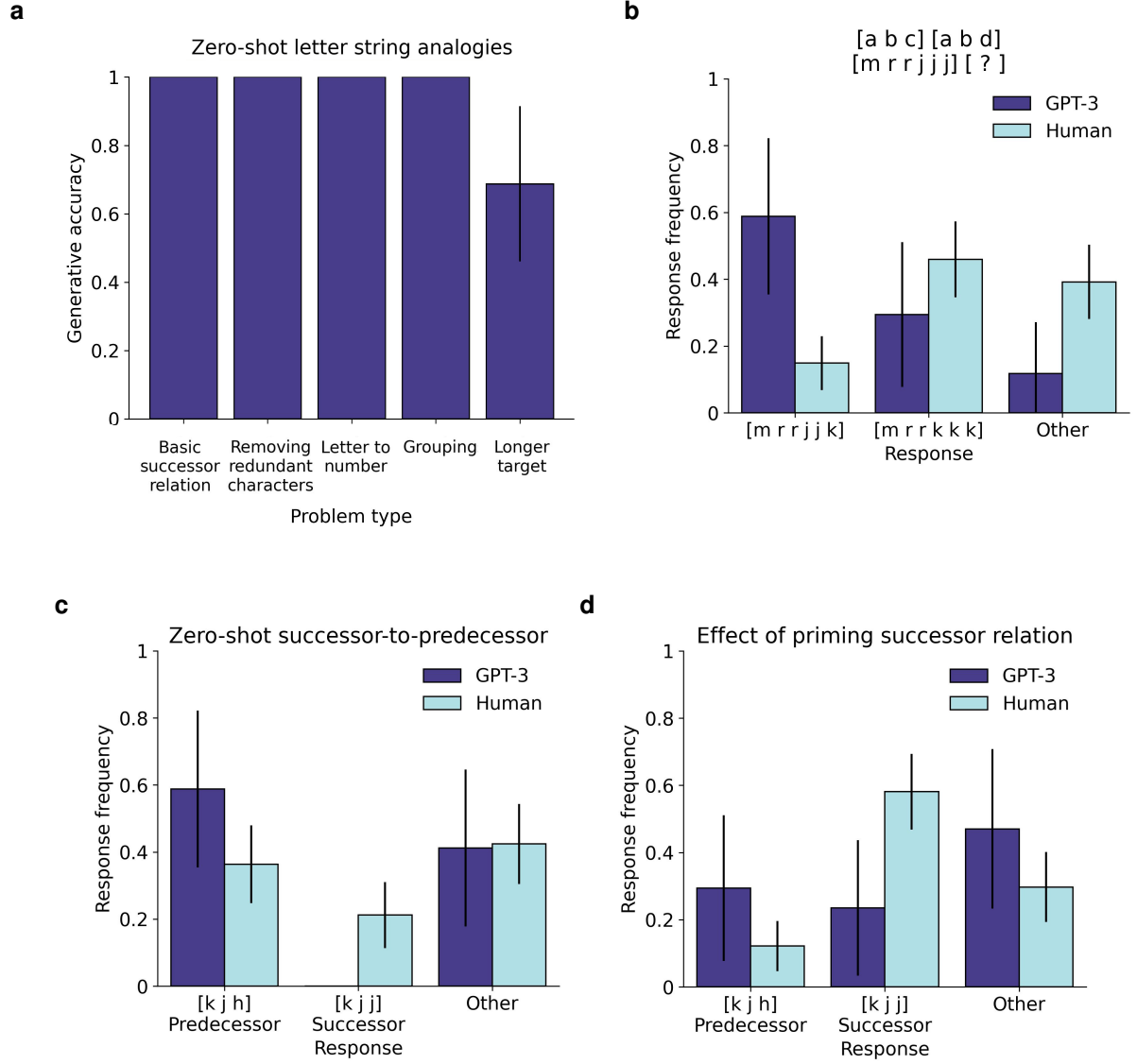


Figure 7: Letter string analogy results. (a) GPT-3 displayed strong zero-shot performance on most problem types. (b) Responses to context problem used to prime successor relation. Though the exact response distribution differed, both GPT-3 and human participants (N=74) produced responses involving a successor relation (either ‘m r j j k’ or ‘m r r k k k’). (c) GPT-3 more reliably identified the successor-to-predecessor mapping (converting the successor relation between ‘a b c’ and ‘a b d’ to a predecessor relation, thus transforming ‘k j i’ to ‘k j h’) than human participants (N=66; data reproduced from [28]). (d) To evaluate relational priming, the context problem in (b) was followed by the successor-to-predecessor problem. In this context, both GPT-3 and human participants were more likely to produce a response involving a successor relation (‘k j j’), and less likely to produce a response involving a predecessor relation (‘k j h’).

Overall, though GPT-3 did not perfectly match the distribution of human responses, it nevertheless displayed strong zero-shot performance on most problem types, a finding that is at odds with the previous results presented by Mitchell [26]. The most significant factor explaining this discrepancy is that our evaluation is based on the most recent instance of GPT-3, text-davinci-003 (not available at the time of the previous evaluation). We found that an earlier instance of GPT-3, davinci, performed significantly worse on our problem set (Supplementary Figure S2). We also found that GPT-3 benefited both from the prompt that we used, and from the problem format adapted from the Digit Matrices (Supplementary Figures S3 and S4). However, even without these changes, the text-davinci-003

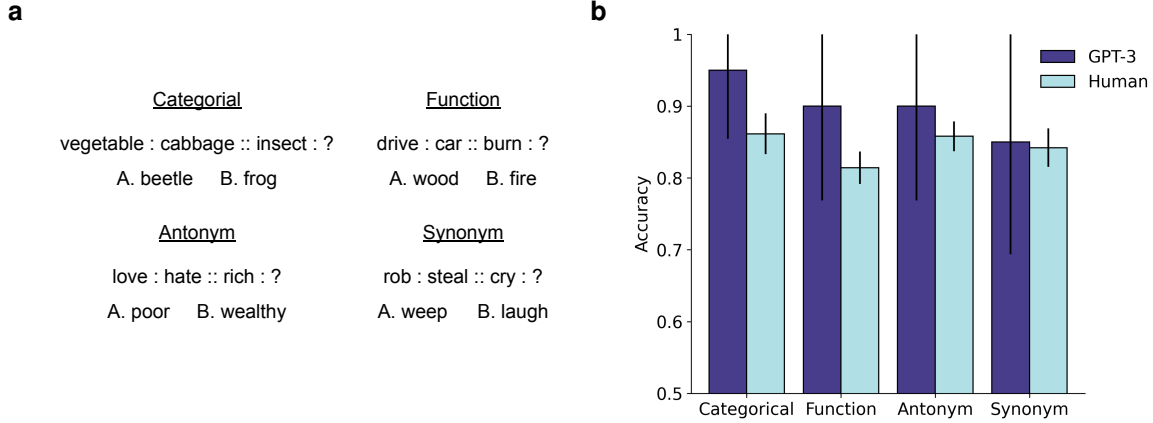


Figure 8: Verbal analogy results. (a) Example problems from the UCLA Verbal Analogy Test. Problems consisted of four relational categories: categorical, function, antonym, and synonym. (b) GPT-3 matched or exceeded human performance (N=57) [16] on these problems.

instance performed significantly better than indicated by the previous evaluation, reliably solving 3 out of 6 problem types zero-shot. It seems probable that the most recent developments in training GPT-3 (notably, adding a secondary alignment objective) were critical in improving the system’s capacity to solve analogy problems zero-shot.

2.3 Four-term verbal analogies

The results of our experiments with the Digit Matrices and letter string analogy problem sets suggest that GPT-3 possesses an emergent capacity for zero-shot analogical reasoning. However, one limitation of these task domains is that they consist of highly constrained, synthetic relations. It is therefore unclear whether GPT-3 is also capable of solving analogies involving richer semantic relations such as those that occur in problems based on natural language. To address this question, we evaluated GPT-3 on a set of four-term verbal analogies that depend on more meaningful relational concepts.

We evaluated GPT-3 on the UCLA Verbal Analogy Test (UCLA-VAT), and compared its performance with human behavioral data from a previous study [16]. UCLA-VAT consists of four-term analogies (i.e., A:B::C:D) defined by relations from four categories (Figure 8a): categorical (B is a member of the category A), function (A is the function of B), antonym, and synonym. There are 20 problems in each category, and each problem presents two options, a correct answer (D) and an incorrect answer (D’). Answer options were created such that D’ was semantically related to C, but did not have the same relationship with C as that between A and B. We evaluated GPT-3 by presenting each potential analogy (A:B::C:D or A:B::C:D’), and choosing the answer based on the log probability assigned to the final term (D or D’).

GPT-3 performed as well or better than human participants (N=57) on all four problem categories (Figure 8b). These results suggest that GPT-3’s analogical reasoning capabilities are not limited to synthetic domains such as the Digit Matrices, but also extend to problems based on more meaningful relational concepts. It is also worth noting that while simpler approaches, such as those based on word embeddings alone [29], are capable of solving some four-term verbal analogy problems, these approaches have been found to underperform on the UCLA-VAT dataset relative to human participants, likely due to the more abstract relations found in this dataset [16]. Earlier instances of GPT-3, and earlier language models (e.g., GPT-2 and BERT), have also displayed poor zero-shot performance on verbal analogies [11, 30], again suggesting that the alignment objective used to fine-tune GPT-3 greatly improved its capacity to solve these problems zero-shot.

2.4 Analogy in natural language problem-solving

Though analogical reasoning is often assessed via well-controlled tasks such as matrix reasoning problems or four-term verbal analogies, it is of interest to cognitive scientists primarily because of the important role that it plays

in real-world reasoning and problem-solving. Human reasoners can exploit analogies between complex real-world inputs such as stories or images, using them to derive solutions to novel problems. Given GPT-3’s ability to process a relatively large amount of text-based input, we examined whether it would be capable of solving more complex naturalistic analogy problems based on stories.

We evaluated GPT-3 using a paradigm developed by Gick and Holyoak [17]. In that paradigm, participants are presented with a target problem in the form of a story. In the original study, Duncker’s radiation problem was used [31]. In that problem, a doctor wants to use radiation to destroy a malignant tumor, but destroying the tumor with a single high-intensity ray will also damage the surrounding healthy tissue. The solution – to use several low-intensity rays that converge at the site of the tumor – is rarely identified spontaneously, but participants are more likely to discover this solution when they are first presented with an analogous source story. In the original study, the source story involved a general who wants to capture a fortress ruled by an evil dictator, but cannot do so by sending his entire army along a single road, which would trigger landmines. The general instead breaks his army up into small groups that approach the fortress from multiple directions, thus avoiding triggering the mines.

We first presented GPT-3 with the target problem in isolation. GPT-3 proposed a solution that involved injecting a radiation source directly into the tumor, rather than identifying the intended solution based on the convergence of multiple low-intensity radiation sources (Supplementary Section S2.1). However, when first presented with the general story, followed by the target problem, GPT-3 correctly identified the convergence solution (Supplementary Section S2.2). GPT-3 was further able to correctly explain the analogy, and to identify the specific correspondences between the source story and target problem when prompted (e.g., general \leftrightarrow doctor, dictator \leftrightarrow tumor, army \leftrightarrow rays).

In a more challenging version of this paradigm, participants were first presented with both the general story, and two other non-analogous stories intended to serve as distractors. In this context, human participants were much less likely to identify the convergence solution. However, when given a prompt to explicitly consider the previously presented stories when trying to solve the radiation problem, participants were often able to correctly identify the analogous general story, and use this analogy to devise the convergence solution. Remarkably, we found that GPT-3 displayed these same effects. When presented with these same distracting, non-analogous stories, GPT-3 no longer identified the convergence solution, instead proposing the same solution that it proposed in response to the radiation problem alone (Supplementary Section S2.3). But when prompted to consider the previous stories, GPT-3 both correctly identified the general story as most relevant, and proposed the convergence solution (Supplementary Section S2.4).

We also evaluated GPT-3 using materials from a developmental study that employed a similar paradigm [18]. In that study, children were tasked with transferring gumballs from one bowl to another bowl that was out of reach, and provided with a number of materials for doing so (e.g., a posterboard, an aluminum walking cane, a cardboard tube), permitting multiple possible solutions. The key result was that when children were first presented with an analogous source story (about a magical genie trying to transfer jewels between two bottles), they were more likely to identify a solution to the target problem that was analogous to the events described in the source story.

When presented with this target problem, GPT-3 mostly proposed elaborate, but mechanically nonsensical solutions, with many extraneous steps, and no clear mechanism by which the gumballs would be transferred between the two bowls (Supplementary Sections S2.5-S2.7). However, when asked to explicitly identify an analogy between the source story and target problem, GPT-3 *was* able to identify all of the major correspondences, even though it could not use this analogy to discover an appropriate solution. This finding suggests that GPT-3’s difficulty with this problem likely stems from its lack of physical reasoning skills, rather than being due to a difficulty with analogical reasoning per se. It is also worth noting that in the original study, this task was presented to children with real physical objects, which likely aided the physical reasoning process relative to the purely text-based input provided to GPT-3. Overall, these results indicate that GPT-3 is capable of identifying analogies between complex naturalistic stories, but its ability to use these analogies for problem-solving is constrained by the content about which it can reason, with particular difficulty in the domain of physical reasoning.

3 Discussion

We have presented an extensive evaluation of analogical reasoning in a state-of-the-art large language model. We found that GPT-3 appears to display an emergent ability to reason by analogy, matching or surpassing human performance across a wide range of problem types. These included a novel text-based problem set (Digit Matrices)

modeled closely on Raven’s Progressive Matrices, where GPT-3 both outperformed human participants, and captured a number of specific signatures of human behavior across problem types. Because we developed the Digit Matrix task specifically for this evaluation, we can be sure GPT-3 had never been exposed to problems of this type, and therefore was performing zero-shot reasoning. GPT-3 also displayed an ability to solve analogies based on more meaningful relations, including four-term verbal analogies and analogies between stories about naturalistic problems.

It is certainly not the case that GPT-3 mimics human analogical reasoning in all respects. Its performance is limited to the processing of information provided in its local context. Unlike humans, GPT-3 does not have long-term memory for specific episodes. It is therefore unable to search for previously-encountered situations that might create useful analogies with a current problem. For example, GPT-3 can use the general story to guide its solution to the radiation problem, but as soon as its context buffer is emptied, it reverts to giving its non-analogical solution to the problem – the system has learned nothing from processing the analogy. GPT-3’s reasoning ability is also limited by its lack of physical understanding of the world, as evidenced by its failure (in comparison with human children) to use an analogy to solve a transfer problem involving construction and use of simple tools. GPT-3’s difficulty with this task is likely due at least in part to its purely text-based input, lacking the multimodal experience necessary to build a more integrated world model [32].

But despite these major caveats, our evaluation reveals that GPT-3 exhibits a very general capacity to identify and generalize – in zero-shot fashion – relational patterns to be found within both formal problems and meaningful texts. These results are extremely surprising. It is commonly held that although neural networks can achieve a high level of performance within a narrowly-defined task domain, they cannot robustly generalize what they learn to new problems in the way that human learners do [6, 33–35]. Analogical reasoning is typically viewed as a quintessential example of this human capacity for abstraction and generalization, allowing human reasoners to intelligently approach novel problems zero-shot. Our results indicate that GPT-3 – unlike any other neural network previously tested on analogy problems – displays a capacity for such zero-shot analogical reasoning across a broad range of tasks.

The deep question that now arises is how GPT-3 achieves the analogical capacity that is often considered the core of human intelligence. One possibility is that, perhaps as a result of the sheer size and diversity of GPT-3’s training data, it has been forced to develop mechanisms similar to those thought to underlie human analogical reasoning – despite not being explicitly trained to do so. The consensus among cognitive scientists working on analogy is that this human ability depends on systematic comparison of knowledge based on explicit relational representations. It is unclear whether and how GPT-3 would implement these processes. Does GPT-3 possess some form of emergent relational representations, and if so, how are they computed? Does it perform a mapping process similar to the one that plays a central role in cognitive theories of analogy [36]?

A few properties of the transformer architecture [20], on which GPT-3 and other large language models are based, are worth considering here. The first is the central role played by *similarity*. Transformers are built on a self-attention operation, which involves explicitly computing the similarity between each pair of vectors in the inputs to each layer. This pairwise evaluation of similarity is also a key feature of cognitive models of analogy, where it provides the primary constraint guiding the process of analogical mapping. In traditional symbolic models [37], this takes the form of literal identity between symbols, but in more recent models [38, 39], a graded similarity function that operates over vector-based inputs is used, much like the self-attention operation in transformers. Second, transformer self-attention employs a form of *indirection*, in which one set of embeddings is used to reference another set of embeddings (i.e., keys vs. values). Cognitive scientists have long hypothesized that this kind of variable-binding plays a central role in analogical reasoning, and abstract reasoning more broadly, as it potentially allows generalization of abstract roles across different contexts [33, 40–44].

But although the mechanisms incorporated into large language models such as GPT-3 may have some important links to building blocks of human reasoning, we must also entertain the possibility that this type of machine intelligence is fundamentally different from the human variety. Humans have evolved to reason within bounds imposed by limited computational power [45]. Thus, we tend to approach complex problems by breaking them into a set of simpler problems that can be solved separately [46], an approach that plays a particularly important role in solving challenging analogy problems such as Raven’s Matrices [19]. It is possible that GPT-3, through sheer computational scale, is able to solve such complex problems in a holistic and massively parallel manner, without the need to segment them into more manageable components.

It must also be noted that, regardless of the extent to which GPT-3 employs human-like mechanisms to perform analogical reasoning, we can be certain that it did not *acquire* these mechanisms in a human-like manner. LLMs receive orders of magnitude more training data than do individual human beings (at least if we consider linguistic inputs alone) [45], and so they cannot be considered as models of the acquisition of analogical reasoning over the

course of human development. Nor can they be considered good models of the evolution of analogical reasoning, as their analogical abilities are derived entirely from being trained to predict human-generated text. Human natural language is replete with analogies, and accurately predicting natural language therefore likely requires an ability to appreciate analogies. But there is no reason to suppose that the same system, absent human-generated inputs, would spontaneously develop a disposition to think analogically, as apparently happened at some point in human evolution [47]. Thus, to the extent that large language models capture the analogical abilities of adult human reasoners, their capacity to do so is fundamentally parasitic on natural human intelligence. Nevertheless, the present results indicate that this approach may be sufficient to achieve human-like reasoning abilities, albeit through a radically different route than that taken by biological intelligence.

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4 Methods

4.1 Code availability

Code for all simulations, along with data from human behavioral experiments, can be downloaded from GitHub.

4.2 GPT-3

We queried GPT-3 in an automated fashion through the OpenAI API. All of our primary simulations employed the text-davinci-003 model instance. We also performed some additional experiments using the davinci and text-davinci-002 instances, as specified in the main text. The temperature was set to 0. We set max_tokens (the parameter controlling the maximum number of generated tokens for a given prompt) to 10 for Digit Matrices, 20 for letter string analogies, 10 for four-term verbal analogies, and 256 for story analogies. All other parameters were set to their default values.

For each prompt, GPT-3 generates a proposed completion (a string of tokens), and assigns log probabilities to each token in the prompt and the completion. We used these log probabilities to evaluate GPT-3 on multiple-choice problems. For each choice in a given problem, we concatenated the problem with the choice, and treated the average log probability assigned to the choice tokens as a score, selecting the answer choice with the highest score. This approach was used for Digit Matrices and four-term verbal analogies.

4.3 Digit Matrices

4.3.1 Dataset

The digit matrix problems consisted of two major problem categories: transformation and logic problems. Transformation problems contained anywhere from one to five rules, while logic problems each contained only a single rule. Transformation problems were defined using a combination of three rule types: constant, distribution-of-3, and progression. The constant rule was defined by the same digit appearing across either rows or columns. The distribution-of-3 rule was defined by the same set of three digits appearing in each row or column, but with the order permuted. The progression rule was defined by a progressive increase or decrease in value, in units of either 1 or 2, across either rows or columns. Logic problems were defined by one of three rules: OR, XOR, and AND. In the OR rule, a particular row or column (e.g., the middle column) contained all entities that appeared in either of the other rows or columns (e.g., the left and right columns). The XOR rule was the same, except that entities appearing in both of the other rows or columns were excluded (e.g., only items that appear in either the left or right column, but not both, will appear in the middle column). In the AND rule, a particular row or column contained only entities that appeared in both of the other rows or columns. For some logic problems, the within-cell spatial

position of corresponding elements was aligned, while in other problems it was permuted. For distribution-of-3 and logic problems, rules applied across both rows and columns, as is the case in the original SPM.

Within each problem type (one- through five-rule and logic problems), there were a number of specific problem subtypes. There were 6 one-rule subtypes, 6 two-rule subtypes, and 10 subtypes for three-rule, four-rule, five-rule, and logic problems. We generated 100 instances of each subtype (except in the case of progression problems, for which there were fewer possible problem instances). The one-rule problem subtypes consisted of a row-wise constant problem, a column-wise constant problem, two distribution-of-3 problems, and two progression problems (one with an increment of 1 and one with an increment of 2). The two- and three-rule problem subtypes consisted of all possible combinations of two or three rules (allowing for the same rule to be used multiple times within each problem). The four- and five-rule problem subtypes were sampled from the set of all possible combinations of four or five rules. There were five spatially aligned logic problem subtypes, and five spatially permuted logic problem subtypes. Three out of each of these five subtypes were OR problems (defined by the row or column in which the set union appeared), and the other two were AND and XOR problems.

For each problem, we also procedurally generated a set of 7 distractor choices, making for a set of 8 total answer choices. Distractors were generated using different methods for the transformation and logic problems. These methods were chosen based on the approach of Matzen et al. [22], who performed an analysis of the answer choices in the original SPM.

For transformation problems, the following methods were used to generate distractors:

1. Sample a random cell from the problem.
2. Sample a random cell from the problem, sample a random digit within that cell, and apply an increment or decrement of either 1 or 2.
3. Start with the correct answer, apply an increment or decrement of either 1 or 2 to a randomly sampled digit.
4. Randomly sample a previously generated distractor for this problem, apply an increment or decrement of either 1 or 2 to a randomly sampled digit.
5. Randomly generate a new answer choice (with the appropriate number of digits given the problem type).

For multi-rule transformation problems, the following additional methods were also used:

1. Start with the correct answer, randomly permute the digits.
2. Sample a random cell from the problem, randomly permute the digits.
3. Randomly sample a previously generated distractor for this problem, randomly permute the digits.
4. Randomly sample digits from multiple cells within the problem and combine.
5. Randomly sample digits from previously generated distractors for this problem and combine.

For logic problems, distractors were generated by sampling from the set of all possible subsets of elements that appeared within the problem, including the empty set (the correct answer was an empty set on some logic problems), but excluding the correct answer. For spatially permuted logic problems, the spatial position of the elements within each distractor was randomly permuted. For spatially aligned logic problems, the order of the elements within each distractor was chosen so as to be consistent with the order that they appeared in the problem.

4.3.2 Evaluating GPT-3

GPT-3 was evaluated on the Digit Matrices by presenting each complete problem as a prompt, including brackets and line breaks, followed by an open bracket at the start of the final cell. For example, the three-rule problem in Figure 1b would be presented to GPT-3 in the following format:

[5 9 3] [8 9 2] [1 9 7]\n[8 4 7] [1 4 3] [5 4 2]\n[1 2 2] [5 2 7] [

GPT-3’s generated responses were truncated at the point where a closing bracket was generated. For logic problems, generated answers were counted as correct if they contained the correct set of digits, regardless of their order. For transformation problems, generated answers were only counted as correct if they contained the correct digits in the correct order.

To evaluate GPT-3’s multiple-choice performance, for each answer choice, the choice was appended to the problem followed by a closing bracket, and presented to GPT-3 as a prompt. The average log probability of the tokens corresponding to the answer choice (not counting the brackets) was computed. The answer choice with the highest average log probability was treated as GPT-3’s selection.

When presenting problems to GPT-3 in order of increasing complexity, the multiple-choice answer selected by GPT-3 was appended to the problem, and the combined problem and answer choice were recursively appended to the prompt for the next problem. This meant that the size of the prompt grew with each problem. For some of the final five-rule problems, the prompt exceeded the size of GPT-3’s context window. When this occurred, problems from the beginning of the context window were deleted until the entire prompt fit within the window. This resulted in the deletion of a few one-rule problems from the beginning of the prompt. For one-rule problems, the two constant problems were presented first, followed by the two distribution-of-3 rules, followed by the two progression problems.

4.3.3 Human behavioral experiment

Human behavioral data was collected in two online experiments. All experiments were approved by the UCLA Institutional Review Board, and all participants provided informed consent. All participants were UCLA undergraduates. Forty-three participants (31 female, 18-35 years old, average age = 21.2 years old) completed the first experiment, and 47 participants (37 female, 18-42 years old, average age = 21.2 years old) completed the second experiment. Participants received course credit for their participation.

In both experiments, participants were first presented with a set of instructions, and a single one-rule example problem involving a constant rule. For each problem, participants first generated a free-response answer, and then selected from the set of answer choices. Problems were presented in a spatially arranged matrix format, as they appear in Figures 1 and 2. Experimental stimuli were created using jsPsych [48].

In the first experiment, participants were presented with one-, two-, three-rule, and logic problems. There were 6 problem subtypes each for the one- and two-rule problems, and 10 problem subtypes each for the three-rule and logic problems, making for 32 problem subtypes in total. Participants received these problem subtypes in random order. Each participant received randomly sampled instances of each problem subtype.

In the second experiment, participants were presented with one- through five-rule problems. There were 6 problem subtypes each for the one- and two-rule problems, and 10 problem subtypes each for the three- through five-rule problems, making for 42 problem subtypes in total. Problems were presented in order of increasing complexity, with all one-rule problem subtypes first, followed by all two-rule problem subtypes, and so on. For one-rule problems, the two constant problems were presented first, followed by the two distribution-of-3 problems, followed by the two progression problems.

4.4 Letter string analogies

We presented letter string analogies to GPT-3 in the same format used for the Digit Matrices, preceded by the prompt ‘Let’s try to complete the pattern:’. We also evaluated GPT-3 without this prompt, and on an alternative format in which problems were presented in the form of a sentence (e.g., ‘If a b c changes to a b d, what should i j k change to?’). For each problem type, we generated multiple instances by varying the letters used in the target. This resulted in 18 basic successor relation problems, 16 problems involving the removal of a redundant character, 6 letter-to-number problems, 18 grouping problems, 16 problems with a longer target, 17 successor-to-predecessor problems, and 17 problems used as context to test for relational priming.

4.5 Four-term verbal analogies

We evaluated GPT-3 on the UCLA-VAT dataset using the standard colon notation (e.g., ‘A : B :: C : D’). Each problem contained two answer choices, the correct answer D, and a distractor D’. For each answer choice, the choice was appended to the problem and presented to GPT-3. The log probability associated with the final term (D or D’) was used to select an answer choice.

4.6 Story analogies

Story analogy problems were entered directly into the OpenAI playground. Materials were taken from [17] and [18]. All prompts and responses are shown in Supplementary Section S2. Each subsection shows the results for a single continuous session, with GPT-3’s responses presented in bold text. Responses were not truncated or curated in any way.

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Supplementary Results

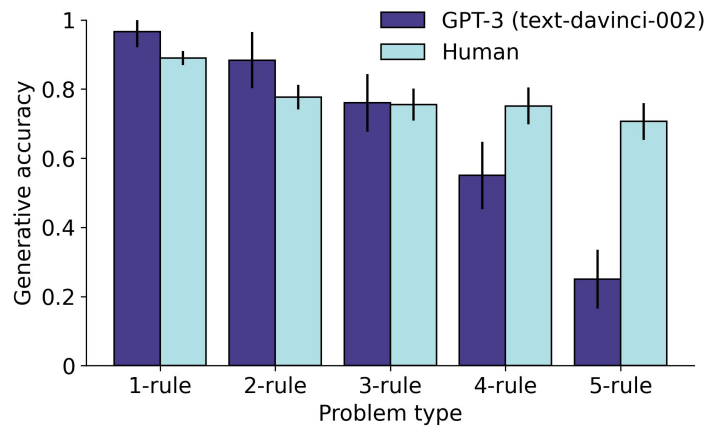


Figure S1: Transfer from few-rule to many-rule Digit Matrix problems in text-davinci-002. An earlier instance of GPT-3, text-davinci-002, did not show the same contextual effects as text-davinci-003. When presented with problems in order of increasing complexity, text-davinci-002 was not able to use this presentation format to generalize from few-rule to many-rule problems, with a significant decrease in performance on four-rule, and especially five-rule, problems.

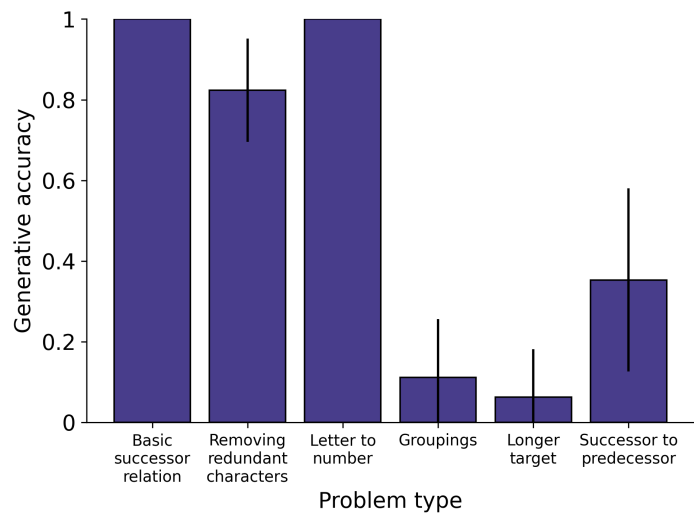


Figure S2: Letter string analogy results for earlier instance of GPT-3 (davinci). Earlier instance of GPT-3, davinci, did not perform as well as text-davinci-003 on letter string analogies.

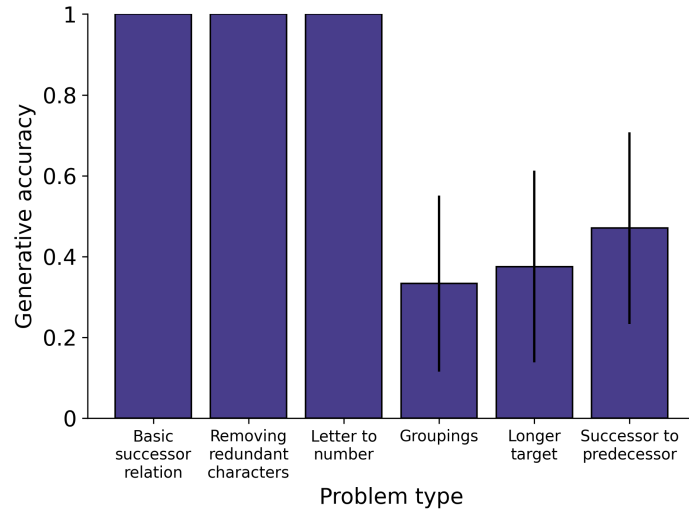


Figure S3: Letter string analogy results without prompt. Text-davinci-003 did not perform as well on letter string analogies when presented without a prompt.

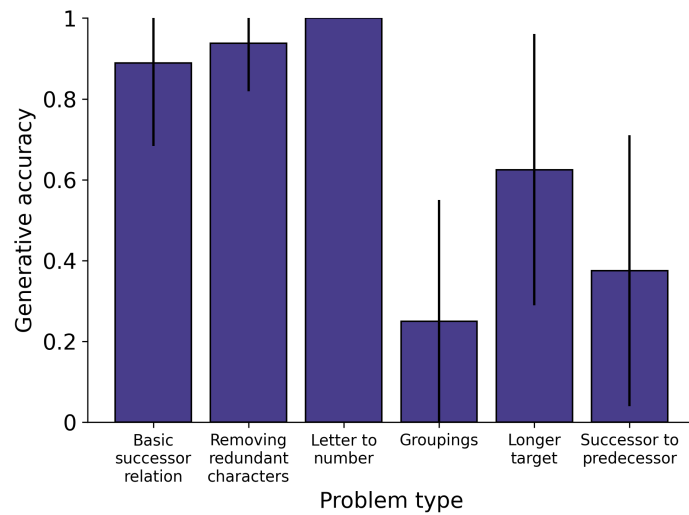


Figure S4: Results for letter string analogies presented in sentence format. Text-davinci-003 did not perform as well on letter string analogies when presented in the form of a sentence ('If a b c changes to a b d, what should i j k change to?').

S1 Solutions to example matrix reasoning problems

The solution to the example visual matrix reasoning problem in Figure 1a is option 5. The solution to the example Digit Matrix problem in Figure 1b is option 7. The solutions to the problems in Figure 2 are presented in Table S1.

Figure	2a	2b	2c	2d	2e	2f	2g
Solution	[9]	[2]	[9]	[4 9]	[2 3 5]	[8]	[5]

Table S1: Solutions to example problems in Figure 2.

S2 Story analogy results

The following sections contain all of the materials used to evaluate analogical reasoning in natural language problem-solving. GPT-3’s responses are presented in bold text.

S2.1 Solution to radiation problem in isolation

The following contains GPT-3’s proposed solution to the radiation problem when presented in isolation.

Target problem:

Suppose you were a doctor faced with a patient suffering from a malignant stomach tumor. The tumor is inoperable, and the patient will die unless it is destroyed. There is a kind of ray that will destroy the tumor if the rays reach it at a high intensity. But unfortunately, such high-intensity rays will also destroy the healthy tissue they pass through on the way to the tumor. At lower intensities the rays will not damage the healthy tissue, but neither will they remove the tumor. How can the doctor use rays to destroy the tumor, while at the same time sparing the healthy tissue?

Solution:

The doctor can use a technique called brachytherapy, which involves placing a radioactive source directly into or near the tumor. This allows the doctor to deliver a high dose of radiation directly to the tumor, while sparing the healthy tissue around it. The radiation source can be placed in the tumor using a catheter, or it can be placed in a sealed container and placed near the tumor. This technique is often used to treat tumors that are inoperable or too close to vital organs to be safely removed.

S2.2 Solution to radiation problem following general story

The following contains GPT-3’s proposed solution to the radiation problem when first presented with the general story, along with responses to followup questions that explicitly probe the analogy between the source story and the target problem.

Source story:

An evil dictator controlled a fortress situated in the center of a small country. Many roads radiated out from the fortress like spokes on a wheel. A general raised an army at the border, vowing to capture the fortress and overthrow the dictator. The general was about to send his entire army down one road to capture the fortress, when he learned that the dictator had mined each road so that although small groups could still pass, a large army would set off an explosion. The general then had a clever idea: he divided his army into small groups and dispatched each group to the head of a different road. Upon his signal, each group charged down a different road. All the groups passed safely to the fortress, where the entire army attacked the fortress in full strength. In this way the general captured the fortress and overthrew the dictator.

Target problem:

Suppose you were a doctor faced with a patient suffering from a malignant stomach tumor. The tumor is inoperable, and the patient will die unless it is destroyed. There is a kind of ray that will destroy the tumor if the rays reach it at a high intensity. But unfortunately, such high-intensity rays will also destroy the healthy tissue they pass through on the way to the tumor. At lower intensities the rays will not damage the healthy tissue, but neither will they remove the tumor. How can the doctor use rays to destroy the tumor, while at the same time sparing the healthy tissue?

Solution:

The doctor could divide the rays into small groups and send each group down a different path to the tumor. By using a lower intensity for each group, the rays will not damage the healthy tissue, but when they all reach the tumor at the same time, the combined intensity will be enough to destroy it. The doctor could also use imaging technology to map out the paths of the rays and ensure that they all reach the tumor at the same time.

Can you identify an analogy between the source story and the target problem?

The analogy between the source story and the target problem is that both involve a situation in which a large force needs to be divided into smaller groups in order to achieve a desired outcome. In the source story, the general divides his army into small groups in order to avoid setting off the mines and capture the fortress. In the target problem, the doctor needs to divide the rays into smaller groups in order to destroy the tumor without damaging the healthy tissue.

Who or what in the target problem is most analogous to the general in the source story?

The doctor in the target problem is most analogous to the general in the source story. Both are faced with a difficult situation and must come up with a clever solution in order to achieve their desired outcome. In the source story, the general divides his army into small groups in order to avoid setting off the mines and capture the fortress. In the target problem, the doctor needs to divide the rays into smaller groups in order to destroy the tumor without damaging the healthy tissue.

Who or what in the target problem is most analogous to the evil dictator in the source story?

The malignant stomach tumor in the target problem is most analogous to the evil dictator in the source story. Both are obstacles that must be overcome in order to achieve the desired outcome. In the source story, the evil dictator must be overthrown in order for the general to capture the fortress. In the target problem, the malignant stomach tumor must be destroyed in order for the patient to be saved.

Who or what in the target problem is most analogous to the army in the source story?

The rays in the target problem are most analogous to the army in the source story. Both are forces that must be divided into smaller groups in order to achieve the desired outcome. In the source story, the army is divided into small groups in order to avoid setting off the mines and capture the fortress. In the target problem, the rays must be divided into smaller groups in order to destroy the tumor without damaging the healthy tissue.

S2.3 Solution to radiation problem with distracting stories

The following contains GPT-3's proposed solution to the radiation problem when first presented with both the general story and two distracting (i.e., non-analogous) stories. GPT-3 did not identify the convergence solution in this case, instead proposing the same solution as it did when presented with the radiation problem in isolation.

Story #1 – The Wine Merchants:

One day a rich man found that his wine cellar was empty. So he sent out messengers to announce a generous offer. The first person to bring the rich man a barrel of wine would be given a brick of solid gold. However, the offer would expire at sundown. Two wine merchants heard the news. Each had a horse-drawn cart loaded with large barrels of wine. They both set out for the duke's palace at once. An hour before sundown they came to a place where the bridge had been washed out by a raging river. The first merchant drove his horses and cart into the flood in a desperate attempt to reach the other side. But the horses were already exhausted and could not fight the current. The cart overturned, and the horses, wine, and driver were washed away. The second merchant tried a different tactic. He poured the wine out of all but one of his barrels, and lashed them together to form a raft; then he loaded the one full barrel, a horse, and himself on top. He set the raft adrift and floated downstream. In a few minutes the raft came to rest on the shore in front of the town where the rich man lived. The merchant disembarked, loaded the wine barrel on the horse, and led it to the rich man's house. He arrived just as the sun was setting, and collected the gold brick as a reward for his efforts.

Story #2 – The General:

An evil dictator controlled a fortress situated in the center of a small country. Many roads radiated out from the fortress like spokes on a wheel. A general raised an army at the border, vowing to capture the fortress and overthrow the dictator. The general was about to send his entire army down one road to capture the fortress, when he learned that the dictator had mined each road so that although small groups could still pass, a large army would set off an explosion. The general then had a clever idea: he divided his army into small groups and dispatched each group to the head of a different road. Upon his signal, each group charged down a different road. All the groups passed safely to the fortress, where the entire army attacked the fortress in full strength. In this way the general captured the fortress and overthrew the dictator.

Story #3 – The Identical Twins:

Once there were identical twins who were continually playing pranks on their family, friends, and teachers. The annual school picnic was always a big event for the twins. There were races and other athletic events in which the twins won lots of prizes. One year a new student arrived who was a star runner. The twins wanted to win the main event: the 2-mile race through the woods behind the school. So they secretly devised a plan which would enable them to outdo the newcomer. The day of the race arrived. Each runner was to pick his own path through the woods to a clearing, where a teacher stood posted to determine the winner. One twin entered the race, while the other excused himself on the grounds that he had hurt his leg in an earlier broadjumping event. The race began and the students rushed into the woods. The twin rushed into the woods and waited until the others had passed out of sight. Then he went back to the school using a path hidden from the picnic area. Shortly after, the other twin, who had been hiding behind a rock near the finish line of the race, burst out and ran into the clearing ahead of the other runners. The teacher named him the winner and marveled at the speed of his running. Next year the twins switched places and thereafter maintained their status on this event.

Target problem:

Suppose you were a doctor faced with a patient suffering from a malignant stomach tumor. The tumor is inoperable, and the patient will die unless it is destroyed. There is a kind of ray that will destroy the tumor if the rays reach it at a high intensity. But unfortunately, such high-intensity rays will also destroy the healthy tissue they pass through on the way to the tumor. At lower intensities the rays will not damage the healthy tissue, but neither will they remove the tumor. How can the doctor use rays to destroy the tumor, while at the same time sparing the healthy tissue?

Solution:

The doctor can use a technique called brachytherapy. This involves placing a radioactive source directly into the tumor. The radiation from the source will be of a high enough intensity to destroy the tumor, but the radiation will not travel far enough to damage the healthy tissue. The doctor can also use a technique called stereotactic radiotherapy. This involves using a machine to deliver a high-intensity beam of radiation to the tumor from outside the body. The beam is focused on the tumor, so that the healthy tissue is spared.

S2.4 Solution to radiation problem with distracting stories and prompt

When presented with distracting stories, GPT-3 did not propose a solution to the radiation problem based on the analogy with the general story. We therefore provided a prompt to encourage GPT-3 to consider the previously presented stories when generating its solution. This same prompt was used in the original human behavioral study [17], where it was found to increase the likelihood that participants would identify the analogous convergence solution. The source stories and target problem were presented in the exact same manner, but the prompt used to elicit GPT-3's solution was changed. When presented with this prompt, GPT-3 correctly identified the convergence solution, and specifically identified the general story as the relevant source.

Solution (in solving this problem you may find that one of the stories you read before will give you a hint for a solution of this problem):

The doctor could use the same tactic as the general in the second story. He could divide the rays into small groups and send each group down a different path to the tumor. The rays would be of low intensity, so they would not damage the healthy tissue, but when they all reached the tumor at the same time, the combined intensity would be enough to destroy it.

S2.5 Solution to gumball problem in isolation

The following contains GPT-3's proposed solution to the gumball problem (from [18]) when presented in isolation (enumeration is part of the response provided by GPT-3).

Target problem:

You are seated in front of a table. Two bowls are on the table, one within your reach and one farther away. The closer bowl contains a number of small gumballs, and the farther one is empty. Also on the table are an aluminum walking cane, a large rectangular sheet of heavy paper (posterboard), a hollow cardboard tube long enough to reach the farther bowl, scissors, string, tape, paper clips, and rubber bands. Using the materials provided, how can you transfer the balls from the filled to the empty bowl without leaving your seat?

Solution:

1. Cut the posterboard into a long strip and tape it to the end of the cardboard tube.
2. Tie one end of the string to the end of the tube and the other end to the handle of the cane.
3. Place the tube in the bowl of gumballs and use the cane to pull the tube and gumballs towards you.
4. Once the tube is close enough, use the paper clips to secure the tube to the edge of the table.
5. Use the rubber bands to secure the tube to the cane.
6. Pull the cane towards you, using the string, and the gumballs will be transferred to the empty bowl.

S2.6 Solution to gumball problem following magic staff story

The following contains GPT-3's proposed solution to the gumball problem when first presented with the 'magic staff story'. This story involves a magical genie who has moved from his old home (a bottle) to a new home (a different bottle), and wants to transport his collection of jewels between the bottles. To do so, he uses a magic staff to pull the new bottle over next to the old bottle and transfers the jewels by hand. When first presented with this story, children most often identify a solution to the gumball problem that involves using the aluminum walking cane to pull the empty bowl over next to the bowl with the gumballs and then transferring the gumballs by hand. GPT-3 was not able to discover this solution, but was able to identify the high-level analogy between the source story and the target problem (though not the specific analogy between the magic staff and the walking cane).

Source story:

Once upon a time there lived a magical genie. He was a very old, wise, and rich genie indeed. One day while he was polishing his home, which was actually a bottle, he decided he would like to find an even bigger and better home to live in. So he began searching far and wide for another bottle. Finally he found the perfect home. It was larger, prettier, and not too far away from his old bottle. The genie was very excited and began moving his belongings right away. But now the genie had a problem. He had a great many beautiful and very precious jewels in his old home. He had to somehow get all the jewels from his old bottle to the new bottle without dropping or losing a single jewel.

After thinking a bit, the genie came up with a wonderful idea. He began searching for his magic staff, or wand. He then commanded his staff to stretch itself from his old home to his new home. Next, the genie tugged and pulled on his magical staff until at last he pulled the new bottle right up next to his old bottle. At once, the genie began gathering his jewels together in his old home and simply dropped them carefully into his new home right next to him. When all his jewels were safely tucked away in his new home, the genie settled in happily. He invited his friend to come in and admire his new home. I'm sure you can still find the genie sitting in his new bigger and better bottle with all his jewels and smiling contentedly even today!

Target problem:

You are seated in front of a table. Two bowls are on the table, one within your reach and one farther away. The closer bowl contains a number of small gumballs, and the farther one is empty. Also on the table are an aluminum walking cane, a large rectangular sheet of heavy paper (posterboard), a hollow cardboard tube long enough to reach the farther bowl, scissors, string, tape, paper clips, and rubber bands. Using the materials provided, how can you transfer the balls from the filled to the empty bowl without leaving your seat?

Solution:

Take the cardboard tube and cut it in half. Place one half of the tube in the bowl with the gumballs and the other half in the empty bowl. Secure the two halves together with the rubber bands. Take the posterboard and cut it into strips. Tape the strips together to form a long rope. Tie one end of the rope to the handle of the walking cane and the other end to the rubber bands. Place the walking cane on the table and hold it in place. Gently pull the cane towards you, and the gumballs will travel through the tube and into the empty bowl. When all the gumballs have been transferred, remove the tube and rope and enjoy your gumballs!

Can you identify an analogy between the source story and the target problem?

The source story and the target problem both involve transferring something from one place to another. In the source story, the genie is transferring his jewels from one bottle to another, and in the target problem, the gumballs are being transferred from one bowl to

another. In both cases, the characters must use creative problem-solving to figure out how to move the items without losing any.

Does this analogy suggest a solution to the target problem?

Yes, the analogy suggests a solution to the target problem. The genie used a magical staff to stretch from one bottle to the other, and the target problem suggests using a rope and a walking cane to transfer the gumballs from one bowl to the other.

Who or what in the target problem is most analogous to the genie's old home in the source story?

The bowl with the gumballs is most analogous to the genie's old home in the source story. Both contain something valuable that needs to be transferred to a new location.

Who or what in the target problem is most analogous to the genie's new home in the source story?

The empty bowl is most analogous to the genie's new home in the source story. Both are the destination for something valuable that needs to be transferred from an old location.

Who or what in the target problem is most analogous to the jewels in the source story?

The gumballs are most analogous to the jewels in the source story. Both are valuable items that need to be transferred from one place to another.

Who or what in the target problem is most analogous to the magic staff in the source story?

The rope and walking cane are most analogous to the magic staff in the source story. Both are used to transfer something valuable from one place to another.

S2.7 Solution to gumball problem following magic carpet story

The following contains GPT-3's proposed solution to the gumball problem when first presented with the 'magic carpet story'. This story has the same setup as the magic staff story – a magical genie wants to transport his jewels from his old bottle to his new bottle – but the genie's solution in this story is to roll up his magic carpet and use it as a bridge between the two bottles. When first presented with this story, children most often identify a solution to the gumball problem that involves rolling up the posterboard and rolling the gumballs through it into the empty bowl. It is interesting to note that children do this despite having access to an already formed cardboard tube, likely due to the analogy between the rolled up posterboard and the rolled up magic carpet. GPT-3 identified essentially this same solution, but added some unnecessary steps, including first cutting the posterboard in half. GPT-3 correctly identified the analogy between the source story and the target problem.

Source story:

Once upon a time there lived a magical genie. He was a very old, wise, and rich genie indeed. One day while he was polishing his home, which was actually a bottle, he decided he would like to find an even bigger and better home to live in. So he began searching far and wide for another bottle. Finally he found the perfect home. It was larger, prettier, and not too far away from his old bottle. The genie was very excited and began moving his belongings right away. But now the genie had a problem. He had a great many beautiful and very precious jewels in his old home. He had to somehow get all the jewels from his old bottle to the new bottle without dropping or losing a single jewel.

After thinking a bit, the genie came up with a wonderful idea. He searched for his magic carpet. Then he commanded it to roll itself up into a long hollow tube. Next the genie commanded his flying carpet to place one end at his old home and the other end at his new home so that it formed a sort of hollow

bridge between the two bottles. Then, the genie very carefully took one jewel from inside his old home and placed it into the opening of his carpet. At once, the jewel began tumbling and rolling through the carpet tube until it reached his new home and plopped safely inside. The genie grinned happily and began rolling all his jewels through the carpet into his new home. In fact, I'm sure you can still find him sitting in his new, bigger and better bottle with all his jewels and smiling contentedly even today!

Target problem:

You are seated in front of a table. Two bowls are on the table, one within your reach and one farther away. The closer bowl contains a number of small gumballs, and the farther one is empty. Also on the table are an aluminum walking cane, a large rectangular sheet of heavy paper (posterboard), a hollow cardboard tube long enough to reach the farther bowl, scissors, string, tape, paper clips, and rubber bands. Using the materials provided, how can you transfer the balls from the filled to the empty bowl without leaving your seat?

Solution:

Take the posterboard and cut it into two pieces. Take one of the pieces and roll it up into a tube. Secure the tube with the rubber bands, tape, and paper clips. Place one end of the tube into the bowl with the gumballs and the other end into the empty bowl. Then, use the walking cane to push the gumballs through the tube until they reach the empty bowl.

Can you identify an analogy between the source story and the target problem?

The source story and the target problem both involve transferring items from one place to another. In the source story, the genie transfers jewels from one bottle to another using a flying carpet. In the target problem, the person transfers gumballs from one bowl to another using a posterboard tube. In both cases, the items are transferred without leaving the original location.

Does this analogy suggest a solution to the target problem?

Yes, the analogy suggests a solution to the target problem. The solution is to use a posterboard tube to transfer the gumballs from one bowl to the other. This is similar to how the genie used a flying carpet to transfer the jewels from one bottle to the other.

Who or what in the target problem is most analogous to the genie's old home in the source story?

The bowl with the gumballs is most analogous to the genie's old home in the source story. Both contain items that need to be transferred to a new location.

Who or what in the target problem is most analogous to the genie's new home in the source story?

The empty bowl is most analogous to the genie's new home in the source story. Both are the destination for items that need to be transferred from an old location.

Who or what in the target problem is most analogous to the jewels in the source story?

The gumballs in the target problem are most analogous to the jewels in the source story. Both are items that need to be transferred from one location to another.

Who or what in the target problem is most analogous to the magic staff in the source story?

The posterboard tube in the target problem is most analogous to the magic carpet in the source story. Both are used to transfer items from one location to another.