Matlab exercises (session 2)

Alexander Bertrand, Oct. 2014

Create an m-file 'Matlab_ex2.m' in which you save the solution for all exercises below. E-mail the results to yourself such that you can continue working at home, and such that you can review your solutions for the exam.

<u>Matrix exercises:</u>

1) a) Create the matrices

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

b) Make a for loop that runs over the columns of A, and computes the sum of the inverted elements of that column (i.e. the inverse of the number x is 1/x)

2) a)Try to compute the inverse of A, call it invA . Hint: type 'help inv'.
b)Do the columns of A span R^3? If yes: why? If not: what do they span?
c)Try to compute the inverse of L. What happens?
d)Do the columns of L span R^3? If yes: why? If not: what do they span?

3) a) Confirm that invA is an inverse by creating the identity matrix I with I = A * invA.
b) Confirm that A is the inverse of invA by computing Z as the inverse of invA and check whether Z==A.

4) For the following, assume you don't know the numerical value of the entries of invA (this means you cannot 'hardcode' the entries). Create a matrix invA2 as follows: multiply each element in invA by 4 and add 2 to every element (do this in one command line). Then add 5 to the element in the first row/second column. Then multiply the last row with 7.

5) Find A^T. Call it At .
a) Find At^5 . Is this the same as taking the power of 5 of each entry in At?
b) Find $(A^T)^{-1}$ and compare with invA. What do you see?
c) Perform the following operations on At (using Matlab commands)
-Subtract row 1 from row 3.
-Divide row 2 by 4.
-Then add 2 times row 2 to row 3.
d) Is the resulting matrix in c) in echelon form? If yes: why? If not: perform the additional row operations that are required to obtain this.
e) Is the resulting matrix in c) in reduced echelon form? If yes: why? If not: perform the additional row operations that are required to obtain this.

6) Create a 3 x 1 matrix (or vector) $\boldsymbol{b} = \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \end{bmatrix}$

7) If $A\boldsymbol{x} = \boldsymbol{b}$, find $\boldsymbol{x}$ using the 'inv' command. Find $\boldsymbol{x}$ using the 'rref' command (use 'help rref'). Explain the link between these two methods.

8) Find a linear combination of the columns of A that yields $\boldsymbol{b}$.

9) Create a random matrix B with 3 rows and 100 columns. Random numbers/matrices can be generated in Matlab using commands like 'rand' or 'randn' (check the help file to see how they work).

If each column of B corresponds to a specific choice for $\boldsymbol{b}$ in $A\boldsymbol{x} = \boldsymbol{b}$:

- Can you be sure that this system of equations is consistent for each choice of $\boldsymbol{b}$? Why?
- Compute the solution of x for each choice of $\boldsymbol{b}$ defined by the columns in B using a 'for' loop.
- Do the same without a for loop, i.e., with a single command.

10) Create a random matrix C with 20 rows and 50 columns. Are the columns of C linearly dependent? (you don't need Matlab to check this). If yes, find a linear dependence relation between them.

11) If vector x = [x1 x2 x3], create in one line the vector y = [$x1^3$ $x2^3$ $x3^3$]. (hint: .^)

12) In one command line, create a diagonal matrix whose diagonal elements are those of matrix A. Hint: type 'help diag'

13) Create a submatrix D of the matrix C containing the last 10 rows and the 11th-15th column. Check whether the columns of D are linearly independent.

14) Create two random square matrices of the same size (size larger than 1), call them E and F. Confirm that EF≠FE. Can you define another (non-zero) F such that EF=FE?

15) Create a random 4x4 matrix (use 'rand') and call this matrix G.

a) Create the matrix H as the transpose of G. Is GH=HG? Why (not)?
b) Design a new matrix Y=GH. Compute the transpose of Y and call it Yt.
c) Is Y Yt=Yt Y? Why (not)?
d) Compare Y and Yt, what do you see? Y is an example of a *symmetric matrix* (explain why this name is used).
e) Explain why Y will always be a symmetric matrix when you create Y using the above procedure.

16) Create a submatrix Asub containing the first two rows and first two columns of A. Do not type the entries yourself. Check whether Asub is invertible by computing its determinant (hint: type 'help det').

Graphics exercises

Start with closing all plots (use 'close all'), otherwise you will keep accumulating plots every time you run the m-file.

(a) Draw the graph of the sine function over the interval -4 to 4 by first creating a vector x = $[-4 \quad -3.9 \quad -3.8 \quad \ldots \quad 3.9 \quad 4.0]$ in one command line. Then let y = sin(x). Use the 'plot' command.

(b) Now use the 'plot' command to plot sin(x) in red and cos(x) in blue on the same graph.

(c) Over the same interval in a different graphics window (use 'figure'), plot $e^{-2(x+1)}$ in a blue line dotted with circles, e^{-2x} in a magenta line dotted with stars and $e^{-2(x-1)}$ in a green line dotted with triangles. Use the 'legend' command to label the three curves.

Then make the same plot, but where the circles, stars and triangles are not connected with a line.

(d) Now plot the following parametrically defined curve: Let t run from 0 to 2 by steps of .001. Let x = cos(3t), and y = sin(2t). Plot x versus y.

(e) Give your plot a title. Call it "Cool Parametric Curve". Label the axes of the plot x and y. (hint: title, xlabel, ylabel).

(f) Make a cloud of random points in the x-y plane, where the x-coordinate has a standard normal distribution (zero-mean Gaussian distribution with unit variance), and where the y-coordinate has a zero-mean Gaussian distribution with standard deviation of 3 (hint: use 'randn').
Make the x-axis go from -10 to 10, and the y-axis from -12 to 12.

The following exercises involves functions/curves in 3D space (x-y-z)

(g) Plot the curve C(t) = (3cos(t), sin(t), 1/t) in 3D with $0.1 \leq t \leq 4$. Use the 'plot3' command.

(h) Plot the graph of $z = e^{-2x}e^{-2y}$ over the square $[-2, 2] \times [-2, 2]$. Use the 'meshgrid' and 'mesh' commands.

(i) Redraw the above surface using the 'surf' command instead of 'mesh'. What has changed?