Matlab introduction: Session 3

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I. PageRank

Load the file `Adj_matrix.mat` in your Matlab workspace. You should now have a variable `A` in your workspace, which contains a 100x100 matrix. This is an adjacency matrix in a database with 100 webpages that link to each other. Create an m-file that performs the following steps (make your code sufficiently general: the code should give the answer to all steps below, even if you would use a different adjacency matrix with different size, etc.):

- Make a figure of the matrix A. (use `imagesc`).
- Compute the PageRank of each webpage and store them in the variable `PR` (a useful command can be `diag`).
- Scale the vector in `PR` such that the sum of all PageRanks is equal to 1.
- Load the file `solution.mat` which contains the vector in the variable `PR_sol`.
- Check whether `PR_sol`=`PR` by plotting the entries of both vectors on top of each other (in different colors).
- Which webpage has the highest PageRank? Which one has the lowest?
- Compute the same PageRank vector, but now by using the power method (see Lay Section 5.8). Start with a random vector with positive entries, and scale it such that its entries add up to 1. Then apply 100 power iterations. Call the resulting vector `PR_power`. Scale it such that the entries add up to 1.
- Check whether `PR_power`=`PR_sol` by plotting the entries of both vectors on top of each other (in different colors).

II. Dynamical system

In a galaxy far far away, there is a planet with a population of 100 green-colored aliens and 100 blue-colored aliens who live in two different villages which are isolated by a river of lava. One day, the lava river dries up, and the aliens from the two villages can "meet" each other, which results in a baby boom. The next year, the alien government declares that the alien population consists of 115 green-colored aliens and 110 blue-colored aliens. The year after, there are 133 green-colored aliens and 122 blue-colored aliens.

Dr. Gram and Dr. Schmidt, two alien-Nobel prize winners which are experts in linear algebra, figured out that the following linear model describes the evolution of the alien population:

$$\mathbf{x}_{i+1} = A\mathbf{x}_i$$

where A is a 2×2 matrix, and with

$$\mathbf{x}_i = \begin{bmatrix} g_i \\ b_i \end{bmatrix}$$

where g_i and b_i denote the number of green aliens and blue aliens in year i, respectively.

Question 1: Using MATLAB, compute the entries of the matrix A, assuming that the data of the alien population given by the alien government indeed exactly follows the model of Dr. Gram and Dr. Schmidt.

Question 2: What will be the ratio of green versus blue-colored aliens in the limit when $i \to \infty$? (Hint: even when both populations become infinitely large, one can still define a finite ratio between them). Make an m-file `aliens1.m` that uses Matlab to solve this question without computing any iterations of the model of Dr. Gramm and Schmidt. This results in a theoretical analysis of the alien population. Create an m-file `aliens2.m` that computes the evolution of the alien population by computing 40 iterations of the model of Dr. Gramm and Schmidt. This is a computational analysis.

Question 3: You may have noticed that there is a problem with the current model: the population keeps growing out of control. This is because the above data of alien population only incorporates the numbers for alien-child birth (aliens are legally obliged to report newborn aliens to the government, including their skin color). However, due to circumstances, there is no system to track the number of aliens that have died each year, so these are not taken into account.

To improve their model, Dr. Gram and Dr. Schmidt assume that the number of aliens that die (each year) is a fixed percentage α of the current total alien population, where $0 < \alpha < 1$.

What is the correct value for α, if we require the model to predict an alien population that stabilizes when $i \to \infty$ (i.e., without going to ∞ or zero)? Extend the m-file `aliens1.m` to perform the theoretical analysis for this case. Extend the m-file `aliens2.m` to perform the computational analysis for this case.