

# Matlab – Linear Algebra

## *Eigenvalue Decomposition*

1. The MATLAB command **poly(A)** produces the coefficients of the characteristic polynomial of the square matrix  $A$ , beginning with the highest degree term. Find the characteristic polynomial of the following matrices.

(a)  $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

(b)  $A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$

(c)  $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$

2. If we set  $\mathbf{p} = \mathbf{poly}(A)$ , then the command **roots(p)** calculates the roots of the characteristic polynomial of the matrix  $A$ . Use this sequence of commands to find the eigenvalues of the matrices in Exercise 1.
3. The MATLAB command **[V, D] = eig(A)** produces a diagonal matrix  $D$  containing the eigenvalues of  $A$  on the diagonal, and a matrix  $V$  whose columns are the corresponding eigenvectors. Use this command to find the eigenvalues and corresponding eigenvectors of the three matrices in Exercise 1.

4. Let

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

Use MATLAB to find the eigenvalues and corresponding eigenvectors of  $A$ ,  $A^T$ , and  $A^{-1}$ . What do you observe?

5. Let

$$A = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}.$$

We can use MATLAB to diagonalize  $A$  as follows. First compute the eigenvalues and eigenvectors of  $A$ , using the command **[P, D] = eig(A)**. The diagonal matrix  $D$  contains the eigenvalues of  $A$ , and the corresponding eigenvectors form the columns of  $P$ . Verify that  $P$  diagonalizes  $A$  by showing that  $P^{-1}AP = D$ .

6. Follow the procedure outlined in Exercise 5 to show that the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

is *not* diagonalizable.

7. Follow the procedure outlined in Exercise 5 to diagonalize (if possible) the following matrices.

(a)  $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

(b)  $A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$

(c)  $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$

8. Important exercise: learn how to work with (nested) 'for' loops!

Consider the matrix A for which the element  $a(i,j)=1/(i+j+t)$  with  $i,j= 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ .  
Plot the value of the largest eigenvalue of A for t going from 0 to 10 with steps of 0.1.