



## 2177 - Coin Exchange

Asia - Taipei - 2000/2001

Let  $a_1, a_2, \dots, a_n$  be  $n$  relatively prime positive integers. A positive integer  $k$  has a representation by  $a_1, a_2, \dots, a_n$  if there exist non-negative integers  $x_1, x_2, \dots, x_n$  such that  $k = x_1 a_1 + x_2 a_2 + \dots + x_n a_n$ . The linear Diophantine problem of Frobenius is to determine the largest integer  $g(a_1, a_2, \dots, a_n)$  with no such representation.

The linear Diophantine problem of Frobenius has very practical application. It is equivalent to the problem of coin exchange. Given sufficient supply of coins of denominations  $a_1, a_2, \dots, a_n$ , determine the largest amount which cannot be formed by means of these coins.

Mathematical problems, such as this, are usually difficult to solve. We shall consider a simple case in which  $n = 2$ . In addition to  $g(a_1, a_2)$ , we shall also want to find  $n(a_1, a_2)$  the number of positive integers that cannot be represented by  $a_1$  and  $a_2$ .

Write a program to compute  $g(a_1, a_2)$  and  $n(a_1, a_2)$ . The following theorem may help you in designing your program.

Theorem: A positive number  $k = qa_1 + r$ ,  $0 < r < a_1$ , can be represented by  $a_1, a_2, \dots, a_n$  if and only if  $k \geq t_r$ , where  $t_r$  is the smallest positive integer which has the same residue  $r$  modulo  $a_1$  and can be represented by  $a_2, a_3, \dots, a_n$ .

### Input

The input file will consist of one or more cases. Each case contains two positive integers  $a_1$  and  $a_2$  in a line. The product of  $a_1$  and  $a_2$  is less than  $2^{32}$ . A line containing two zeros follows the last case, and terminates the input file.

### Output

For each case  $a_1$  and  $a_2$  in the input file, the output file should contain a line with two numbers  $g(a_1, a_2)$  and  $n(a_1, a_2)$  separated by a blank.

### Sample Input

```
3 5
23 20
0 0
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### Sample Output

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7 4
417 209
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