2176 - Lattice Point

Asia - Taipei - 2000/2001

A lattice point is a point (x, y) in the 2-dimensional xy-plane with $x, y \in Z$, where Z be the set of integers. Let

$$P(r) = \{(x, y) | x^2 + y^2 \leq r^2, (x, y) \text{ is a lattice point in the } xy\text{-plane}\}$$

and we denote D(r) be the number of elements in P(r). For each lattice point (x, y) in the xy-plane, let

$$S(x, y) = \{(u, v) | x \leq u \leq x + 1, y \leq v \leq y + 1\}$$

and

$$B(r) = \{(x, y) | x^2 + y^2 \leq r^2, x \text{ and } y \text{ are real numbers}\}$$

Then it is easy to verify that when $r > \sqrt{2}$

$$B(r - \sqrt{2}) \subset \bigcup_{(x,y)\in P(r)} S(x, y) \subset B(r + \sqrt{2})$$

We know that

$$Area(\bigcup_{(x,y)\in P(r)} S(x, y)) = \sum_{(x,y)\in P(r)} Area(S(x, y)) = \sum_{(x,y)\in P(r)} 1 = D(r).$$

Hence

$$\pi(r - \sqrt{2})^2 < D(r) < \pi(r + \sqrt{2})^2$$

This implies

$$\pi(1 - \frac{\sqrt{2}}{r})^2 < \frac{D(r)}{r^2} < \pi(1 + \frac{\sqrt{2}}{r})^2$$

It yields

$$\lim_{r\to\infty} \frac{D(r)}{r^2} = \pi$$

So if we can calculate $D(r)$ for a large r, then we can estimate the value of π.

The following C function can be used to calculate the value of $D(r)$ withing a reasonable aumount of time when r is a small integer, say e.g., $1 \leq r \leq 10,000$.

```
long D(long r)
{   long x,y,count=0;
    for(x=-r;x<=r;x++)
        for(y=-r;y<=r;y++)
            if(x*x+y*y<=r*r)
                count++;
    return count;
}
```

Is is easy to obtained $D(1) = 5$, $D(2) = 13$, $D(3) = 29$, and $D(10000) = 314159053$ using this program. Recall that $\pi = 3.14159....$ Your task is to find $D(r)$ for a large r within a reasonable aumount of time.

Input

There are five lanes in the input file, the kth line contain an integer $n_k (1 \leq n_k \leq 100,000,000)$.

Output

List integer n_k in line $2k - 1$ and the value of $D(n_k)$ in line $2k$ for $k = 1, 2, 3, 4, 5$.

Sample Input

```
1
2
3
10000
100000000
```

Sample Output

```
1
5
2
13
3
29
10000
314159053
100000000
31415926535867961
```

Taipei 2000-2001