B3 HOMEWORK

CHAPTER 1

Problem 22

A certain family has 6 children, consisting of 3 boys and 3 girls. Assuming that all birth orders are equally likely, what is the probability that the three eldest children are the three girls?

The possible number of different birth orders are: $6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1$

The possible number of combinations resulting in the first 3 children being the 3 girls and, consequently, the last three being the 3 boys : $3!\cdot3! = 3\cdot2\cdot1\cdot3\cdot2\cdot1$

The probability of the eldest 3 children being the 3 girls is : $P(\text{3 eldest are the 3 girls}) = \frac{3!\cdot3!}{6!} = \frac{3\cdot2\cdot1}{6\cdot4\cdot5} = \frac{1}{20} = 0.05 = \underline{\underline{5\%}}$

Problem 34

A group of 30 dice are thrown. What is the probability that 5 of each of the values $\{1,2,3,4,5,6\}$ appear?

There are 6^{30} possible outcomes.

There are $30!/(5!)^6$ ways of arranging the dice such that 5 of each value appear

comes from:

- Putting the 6's in position: $\binom{30}{5} = \frac{30!}{25!5!}$
- Putting the 5's in position after: $\binom{25}{5} = \frac{25!}{20!5!}$
- 4's : $\binom{20}{5} = \frac{20!}{15!5!}$
- 3's : $\binom{15}{5} = \frac{15!}{10!5!}$
- 2's : $\binom{10}{5} = \frac{10!}{5!5!}$
- 1's : $\binom{5}{5} = 1$

$$\frac{30!\,\cancel{25!}\,\cancel{20!}\,\cancel{15!}\,\cancel{10!}\,\cancel{5!}}{\cancel{25!}5!\,\cancel{20!}5!\,\cancel{15!}5!\,\cancel{10!}5!\,\cancel{5!}5!} = \frac{30!}{(5!)^6}$$ possible combinations of getting 5 of each value.

$$\text{Prob}(\text{5 of each value from 30 dice throws}) = \frac{30!}{(5!)^6}\cdot\frac{1}{6^{30}} \approx 0{,}0004 = \underline{\underline{0{,}04\%}}$$

CHAPTER 2

Problem 2

A woman is pregnant with twin boys. Twins may be either identical or fraternal, in general 1/3 of twins are born identical. Assumption: identical twins are equally likely to be both boys or both girls while for fraternal twins all possibilities are equally likely.
What is the prob. that the woman's twins are identical (given that they are both boys?)

Bayes' rule : $P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$

Where A = identical twins
B = two boys

$$P(\text{identical twins} \mid \text{two boys}) = \frac{P(\text{two boys} \mid \text{ident. twins})\cdot P(\text{ident. twins})}{P(\text{two boys})} = \frac{P(\text{two boys} \mid \text{ident. twins})\cdot P(\text{ident. twins})}{P(\text{two boys} \mid \text{ident})\cdot P(\text{ident}) + P(\text{two boys} \mid \text{frat})\cdot P(\text{frat})}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{3}}{\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{2}{3}} = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{2}{12}} = \frac{\frac{1}{6}}{\frac{2}{6}} = \frac{1}{2} = \underline{\underline{50\%}}$$

Problem 32

Four Efron dice:

A: 4 4 4 4 0 0
B: 3 3 3 3 3 3
C: 6 6 2 2 2 2
D: 5 5 5 1 1 1

a) $P(A>B) = P(A=4) = \frac{4}{6} = \underline{\underline{\frac{2}{3}}}$

$P(B>C) = P(C=2) = \frac{4}{6} = \underline{\underline{\frac{2}{3}}}$

$P(C>D) = P(C=6) + P(C=2 \wedge D=1) = \frac{2}{6} + \frac{4}{6}\cdot\frac{3}{6} = \frac{2}{6} + \frac{12}{36} = \underline{\underline{\frac{2}{3}}}$

$P(D>A) = P(D=5) + P(D=1 \wedge A=0) = \frac{3}{6} + \frac{3}{6}\cdot\frac{2}{6} = \frac{3}{6} + \frac{6}{36} = \frac{4}{6} = \underline{\underline{\frac{2}{3}}}$

b) Is the event $A>B$ independent of the event $B>C$?

Yes, because B can only ever take the value 3, so $P(A>B) = P(A=4)$ which is independent of B and thus also indep. of $B>C$.

Is the event $B>C$ independent of the event $C>D$?

No, because if $C>D$, then the probability of $B>C$ will be lower, since this would entail a higher prob that $C=6$.

Problem 38

a) 7 door Monty Hall problem

Yes, you should always switch. At the beginning of the game, the probability of getting the car is $\frac{1}{7}$. When Monty opens 3 goat doors, the probability of getting the car for the remaining three doors is $\frac{2}{7}$ for each door. Sticking with the original door will yield a 1/7 probability of getting the car, while switching will yield a 2/7 prob. of getting the car.

b) Generalizing to $n \geq 3$ doors of which Monty opens m goat doors, $1 \leq m \leq n-2$

By the same argument, the probability of getting the car in the first picked door is $\frac{1}{n}$.
Probability of getting the car from switching after Monty opens m doors is:

$$\underline{\underline{P(\text{car}) = \frac{n-1}{n}\cdot\frac{1}{(n-m-1)}}}$$

which is the complementary prob. from choosing the first picked door divided by the number of closed doors left less the first picked door $(n-m-1)$