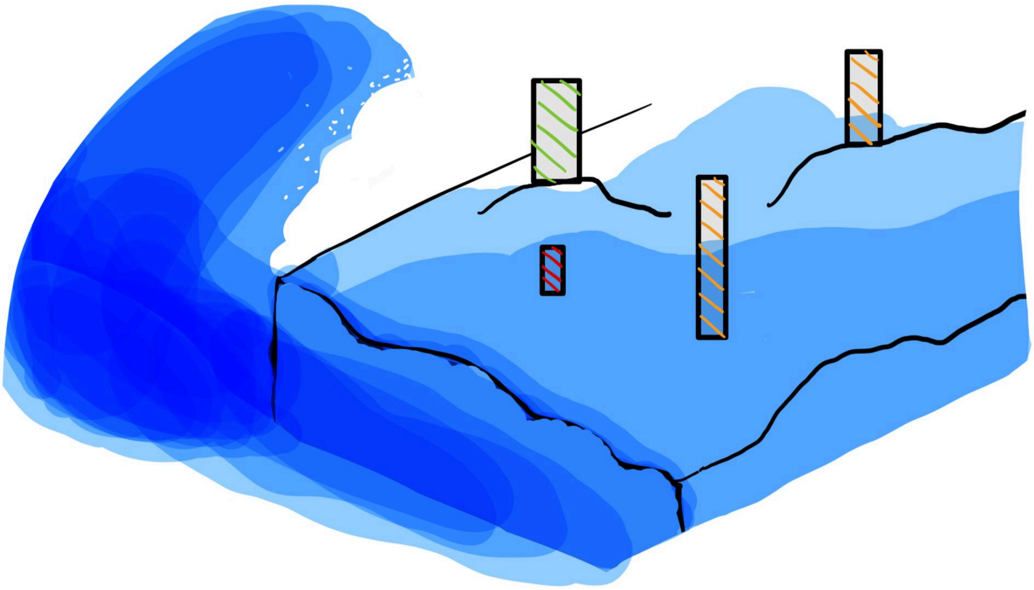
Project report

Computational methods and tools



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Computational methods and tool

1) *Introduction*

The project we have decided to undertake, focuses on modeling a tsunami with the goal of visualizing infrastructures on a coastal region and categorizing them according to whether they are completely submerged, partially submerged, or protected from the tsunami in question.

In our project, we chose to model the coastal terrain and the tsunami randomly to keep our simulation as general as possible and avoid focusing on any specific existing area. This could allow potential clients to use our model to have a better understanding of the risk of flood in the area they desire to study.

2) *Data Collection*

To realistically simulate a tsunami wave striking a coastal terrain, data collection was a crucial step. In this case study, it was necessary to gather data on the sizes tsunamis can reach, as well as the elevation of coastal terrains :

Tsunami wave

Using a database from NOAA [1] (National Oceanic and Atmospheric Administration), a scientific and administrative agency of the United States dedicated to the study of oceans, the atmosphere, and environmental conditions, we were able to classify tsunamis into three categories :

- *Tsunamis in the open sea,* which often go unnoticed as their height typically does not exceed 1 to 2 meters.
- *Tsunamis near the coasts,* where the wave height increases due to the slowing of the water and the compression of the waves. The height of these waves can reach 10 to 30 meters, or even higher depending on local conditions [3].
- *Exceptional tsunamis,* which can be much higher. One notable example is the tsunami in Lituya Bay (Alaska, 1958), caused by a landslide, which generated a massive wave of 254 meters in a confined bay (such cases are extremely rare and localized). Tsunamis caused by major undersea earthquakes, such as the 2004 Indian Ocean tsunami or the 2011 Tohoku tsunami in Japan, reached heights of 30 to 40 meters in some affected areas.

Coastal land

Regarding the size of coastal lands, thanks to the Topographic website [2], an online platform that allows the visualization of topographic maps and provides detailed information on altitude, which we were able to utilize. Similar to tsunamis, we observed various coastal lands with different altitudes:

- *Flat coastal lands:* these are coastal areas with low altitudes, generally between 0 and 10 meters above sea level, such as beaches, deltas, and coastal plains like those in Northern Europe or the Gulf of Mexico.
- *Hilly or gently sloping coastal lands:* these are areas with moderate altitudes, ranging from 10 to 100 meters above sea level (for example: certain regions of the Mediterranean coast, the shores of Southeast Asia, or some rock formations in North America).
- *Steep or mountainous coasts:* these are areas with high altitudes exceeding 100 meters, sometimes reaching several hundred meters or even kilometers. Examples include the steep cliffs of the California coast or the coastal mountains of Southeast Asia.
- *Volcanic islands and rocky formations:* this category includes terrains with very high altitudes. Some coastal areas are formed by volcanoes or coastal mountains, often exceeding 1,000 meters or more.

In this project we chose a maximum altitude of 100 meters for most of our results.

3) *Models and functions used*

During the development of our program, we used several simplified equations derived from physical and mathematical equations, particularly for terrain and wave modeling.

The first step of our program is based on the modeling of the coastal terrain, which we wanted to be generated randomly. To achieve this, we created a main function to which we added trigonometric functions (sine and cosine) in order to create altitude variations and produce natural topographic patterns, as well as to add complexity to the terrain. Thus, for a 2D grid of coordinates (X, Y), the trigonometric functions we used create altitude variations by combining three types of waves.

Main function for terrain altitude modeling :

$$z = altitude_{base} + variations_{terrain} \cdot \frac{altitude_{base}}{hauteur_{max}}$$

where $altitude_{base} = hauteur_{max} \cdot log(1 + (distance \cdot facteur_{lissage})$. Here, the $facteur_{lissage}$ plays a crucial role in terrain generation, as it controls the progression of altitude based on the distance from the origin. Depending on the chosen value of the factor, it will impact the progression more or less significantly. For a low value (0.001), the progression will be very smooth and slow, thereby creating a natural and gradual altitude increase, closely resembling coastal terrain.

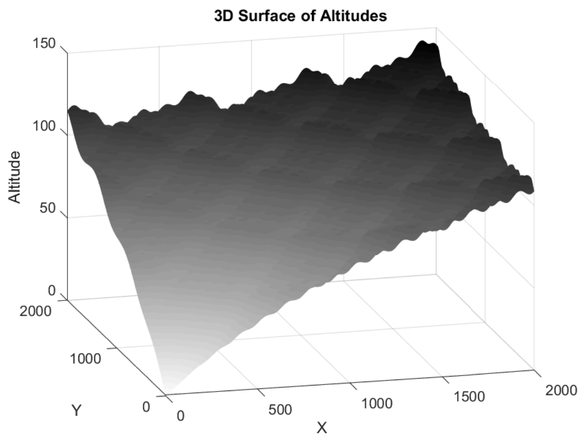
In addition, the distance is calculated as the distance of each point from the point (0,0) :

$$distance = \sqrt{x^2 + y^2}$$

In concrete terms, this function creates a base altitude that increases gradually with the distance from the origin point, using a logarithmic progression (which produces a slowed growth) and is bounded by the maximum height (set to 100 meters). The goal is to simulate a topography where the altitude gradually increases from the origin towards the inland.

Trigonometric functions added to create reliefs with $variations_{terrain}$ ***:***

$5 \cdot sin(x \cdot 0.01) \cdot cos(y \cdot 0.01)$	$3 \cdot sin(x \cdot 0.02) \cdot sin(y \cdot 0.02)$	$2 \cdot cos(x \cdot 0.05) \cdot sin(y \cdot 0.05)$
This function calculates large, low-frequency undulations. The function has a maximum amplitude of $\pm$ 5 but a very low frequency of 0.01, which creates broad and smooth variations. Additionally, the combination of sin(x) and cos(y) produces a 2D wave pattern. In our modeling, this function simulates very gradual wave-like motions that resemble large-scale geological deformations, such as expansive hills or plateaus.	This function calculates medium-frequency undulations (or variations). The function has a maximum amplitude of $\pm$ 3 and an intermediate frequency of 0.02. Moreover, the use of sin(x) and sin(y) generates a more complex pattern. In our modeling, this function will simulate medium-sized terrains, create closer undulations, and represent intermediate-sized hills.	This last function calculates small variations at high frequency. Indeed, we can see that the maximum amplitude is 2 and the frequency is 0.05 (a high frequency compared to the other two functions). This function has the effect of simulating fine terrain details and representing small hills, rocks, or terrain variations in the model.



The use of sine and cosine functions is well-suited for terrain modeling because they allow for generating variations that are more or less smooth (depending on the amplitude and frequency), regular, and controllable at different scales, making it possible to simulate natural reliefs such as waves, hills, or mountains. [4]

<u>Figure: Modeling of terrain with a maximum altitude of 100 meters</u>

Another important modeling aspect in our project is that of the tsunami wave striking our previously modeled coastal terrain. For this modeling, we chose to use an exponential function. This use of the exponential in calculating wave heights offers several interesting advantages, including natural decay. Indeed, exponential decay realistically models natural phenomena; in physics and geology, many processes follow an exponentially decaying curve (e.g., wave damping, energy dissipation, or the propagation of disturbances in a medium: "*A wave propagating through a material is subject to energy dissipation due to internal friction. This phenomenon, commonly referred to as material damping, is characterized by an exponential decay of the wave amplitude.*" [5]).

Another advantage to consider is progressive reduction. Unlike linear decay, the exponential function rapidly reduces the value at the beginning and then gradually slows the reduction, allowing for a smoother transition. These advantages are thus highly relevant and reinforce the use of the exponential function in our model. [6] (*equation 5, page 37*).

function for wave simulation :

$$wave_{height} = initial_{wave\ height} \cdot facteurHauteur \cdot (1.0 - distanceNormalisee) \cdot (1.0 - 0.5 \cdot reductionPente) \cdot exp(-\alpha)$$

where $\alpha = tauxDeCroissance \cdot distanceNormalisee$ and $reductionPente = \frac{1.0}{1.0 + exp(-sensibilitePente * |pente|}$

function parameters:

$tauxDeCroissance$	$sensibilitePente$	$facteurHauteur$
In our function, this parameter aims to control the rate at which the wave height decreases. Generally, the higher the value, the faster the wave diminishes. In this case, the value is set to 0.02, which is a low value, resulting in a very gradual reduction. This rate represents the "friction" or "dissipation" of the wave's energy in our simulation.	The parameter $sensibilitePente$ is the parameter in our function that measures the influence of the slope on the reduction of the wave. This parameter plays an important role in the variable *renductionPente*, which is a sigmoid function (functions commonly used for their ability to model progressive transitions or saturation behaviors) that transforms the slope. It is observed that the higher the value, the more quickly the slope influences the wave height. Thus, this parameter simulates how the terrain affects the wave propagation. Indeed, a steep slope can reduce the wave height more rapidly.	This parameter serves as a global adjustment factor for the model's sensitivity and helps compensate for approximations and local variations. Essentially, it is a multiplier applied to the wave height, allowing the wave to be amplified. Since the parameter is set to 2.0, it potentially doubles the initial wave height.

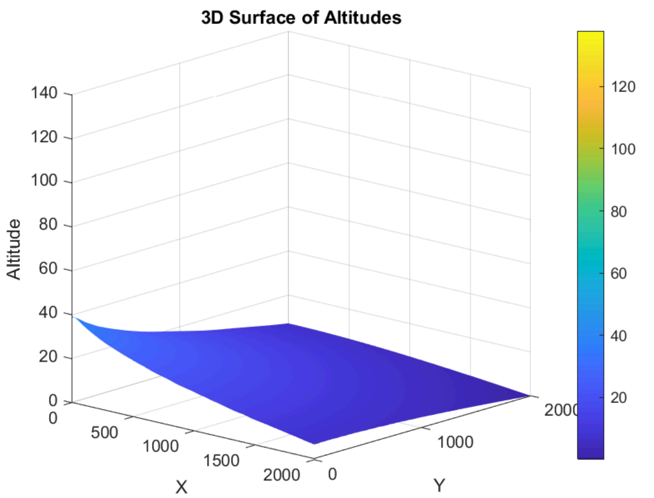


Figure: Modeling of a wave with an initial height of 44.25 meters

The function is therefore designed to create a more nuanced and realistic model of wave height reduction. Its main characteristics take into account: the gradual reduction of wave height with distance, the consideration of the terrain slope, and exponential decay. It allows for the calculation of wave heights for each point along a coastal region, considering both the distance from the origin and the local terrain slope. Additionally, parameters such as $tauxDeCroissance$ (decay rate), $sensibilitePente$ (sensitivity to slope), and $facteurHauteur$ (height factor) helps refine the wave height calculation to match specific environmental conditions.

Finally the last model we used is the one for the classification ; we created three classes based on the difference between the infrastructures height and the wave height on top of the coast.

$$difference = infrastructure_{height} - wave_{height}$$

As the infrastructure is modeled as a 3D cylinder, each point forming the base might not be experiencing the same submersion (cf our wave height changes in space), so to make the comparison more accurate we used the minimum and maximum difference for each infrastructure in the classification process. We defined a threshold based on the proportion of vertical height submerged to create the class. We limited our classification to water height, other parameters exists to make a more precise diagnostic. [7]

$$seuil = infrastructure_{height} / 10$$

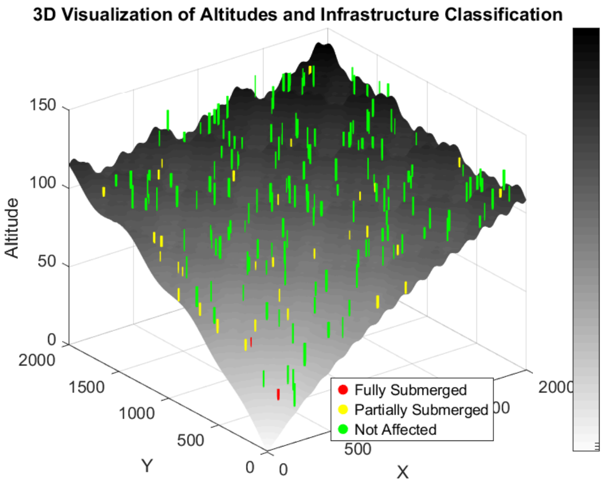
Classification criterion	***Class***
$difference_{max} < 3 * seuil$ OR $difference_{min} < 3 * seuil$	Infrastructures that will be lost in case of the tsunami generated , either fully submerged or more than ⅔ of the height of at least one of the point forming the infrastructure is under water
$3 * seuil =< difference_{max} < 6 * seuil$ OR $3 * seuil =< difference_{min} < 6 * seuil$	Infrastructures that are at high risk still but that could be saved if the wave only submerged between ⅓ and ⅔ of their height.
$difference_{max} >= 6 * seuil$ OR $difference_{min} >= 6 * seuil$	Infrastructures that are considered safe, with less than ⅓ impacted from the wave.

4) Discussion of the Results

We can observe the results of our wave propagation by visually comparing the proportions of the different classes of infrastructures. We saw that depending on the initial parameters we fixed before running the model, the outcome changed drastically :

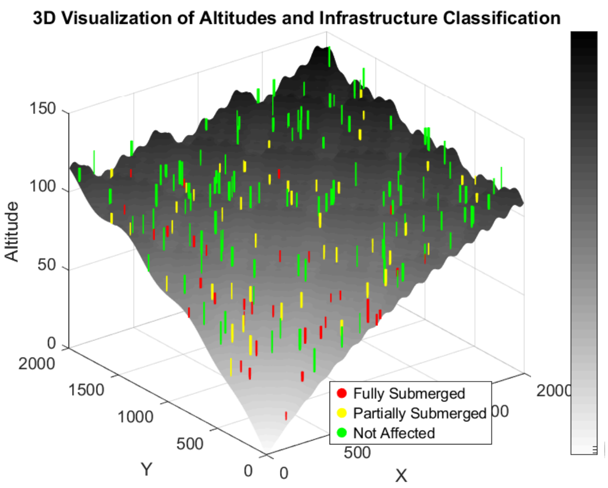
I) For different initial wave height range :

For those next four plots we generated altitudes with a maximal height fixed at 100m and a sample of 200 infrastructures



This plot was generated for an initial_wave_height chosen randomly in a range between 5-10 m , more precisely this one was about 7,37m when reaching the coast at point (0,0).
We can see that the infrastructures at high risk are a minority in this map and very close to the edge of the coast.
We still have a few infrastructures partially submerged but there are principally the ones with a small height which can explain why we observe them so far away from the coast.

Plot I . 1 of a coastal region affected by a wave of 7,37m

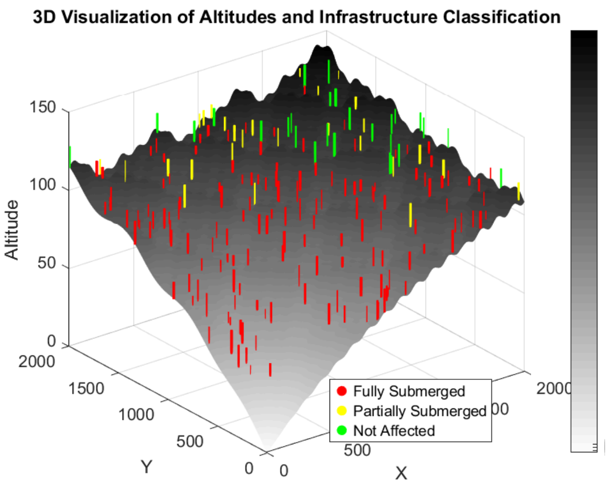


For this second plot we used a range between 10-30 m, which is the typical range of tsunami heights in history. [1] The initial_wave_height was actually 14,75m for this plot so two times higher than the first one.

We can see the increase of the infrastructures classified as fully submerged, they are still localised mainly near the origin of our grid. The few far away from the edge have very little height.
The two other categories' proportions haven't changed much, but we observe more partially submerged far away from the origin.

Plot I . 2 of a coastal region affected by a wave of 14,75m

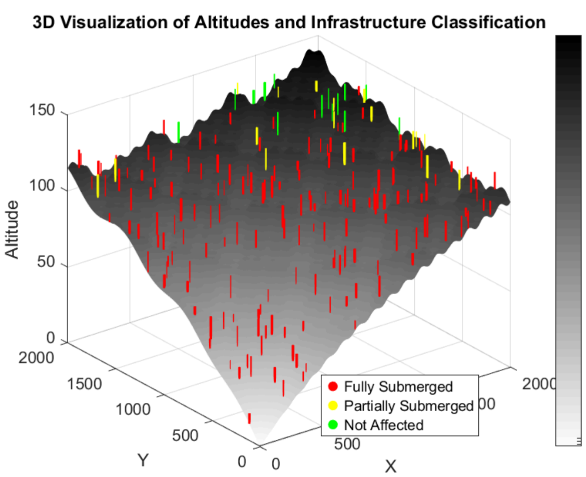
Those two first plots represent the most frequent tsunami events in history. [1] Now let's look at some more extreme cases that have happened in the past and whose frequency may intensify in the future.



This represents a more extreme case where we choose a range between 30-40m. The initial_wave_height was 44,25m for this plot.

We can see the drastic difference with the two first plots, the class fully submerged representing the infrastructures who have more than ⅔ of their heights under water are in majority.
Only a few buildings are not affected by this tsunami event those are the ones the farest from the coast

Plot I . 3 of a coastal region affected by a wave of 44,25m



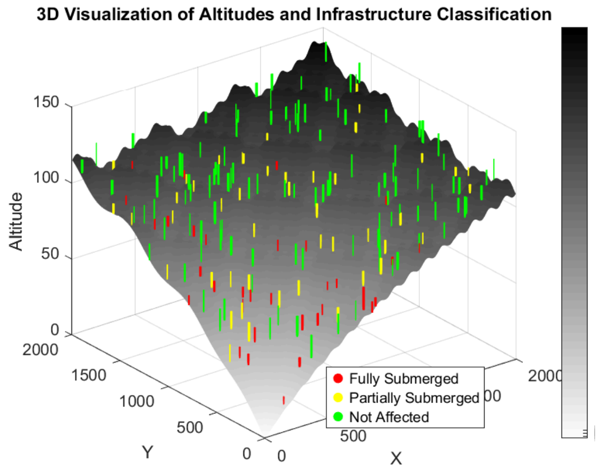
For this last plot the range was fixed between 40-70m, with an initial wave height at 59m. (This extreme case isn't unrealistic, *there was for example a tsunami that happened in the USA in 1964, the wave reached 51,8m[1])*

In this most extreme case we can see that almost all infrastructures have at least ⅔ of their height under water while the not affected class can be counted by eyes as their number has much decreased.

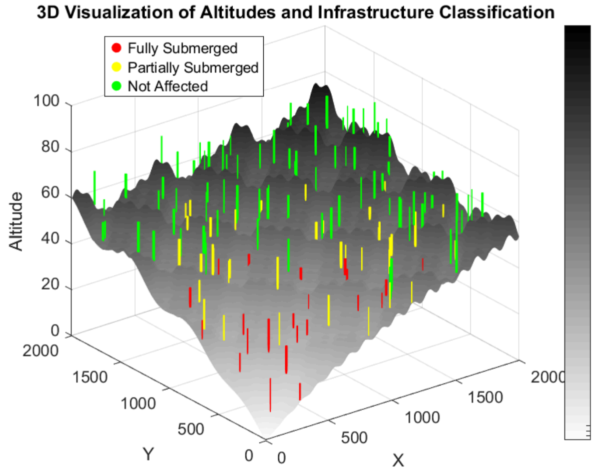
Plot I . 4 of a coastal region affected by a wave of 59m

II) For different simulation of coastal region :

After comparing for the same coastal region 4 different categories of wave height we can now try to see how the shape of our coastal region influences our model. For the same initial_height_wave range (10-30m) we can try to change the maximum altitudes in the coast generation.



Plot II .1 of a coastal region affected by a wave



Plot II .2 of a coastal region affected by a wave

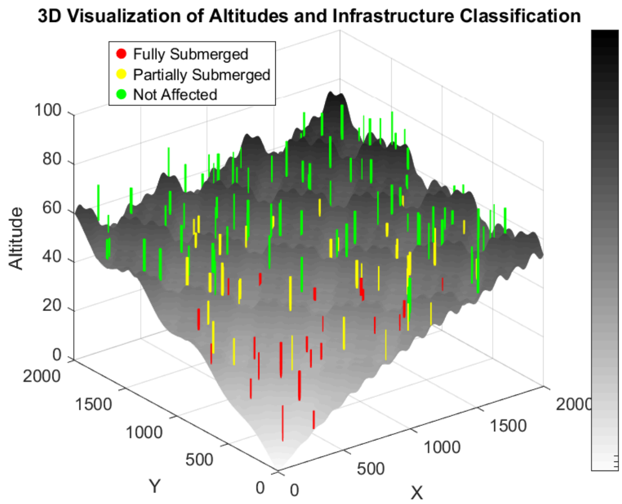
maximum altitudes	100m	50m
initial_wave_height	**14,75 m**	**14,87 m**
description	We already described this plot in ***result I .1***	We can see that the infrastructures classified as fully submerged are localised more distinctively next to the origin of the plot. but overall there is not a significant difference in the proportions of the class.

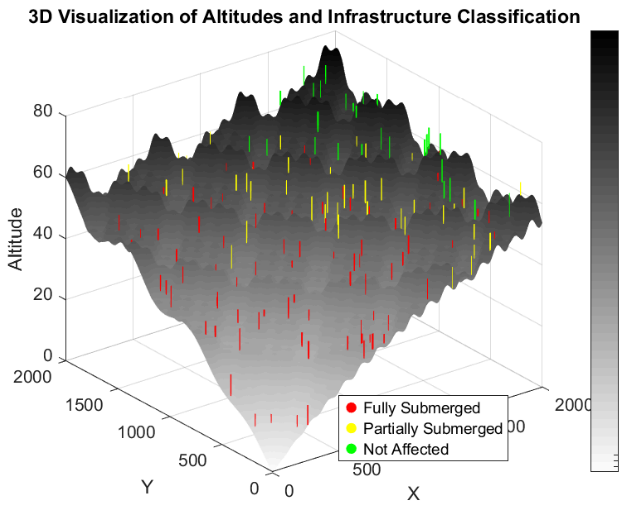
To understand these observations, we have to remember that our wave propagation model is based on the initial wave height but also on the slope between points in our altitudes_matrix. Our wave behaviour stays consistent when we only change the maximum altitudes our land can reach.

III) For different infrastructures heights range :

We can see that on this Plot II.2 with smaller altitudes the dimension of our infrastructures doesn't seem very appropriate *(those infrastructures were chosen with a random height between 5-15m)* we could try to modificate the dimensions of the cylinder to see the distinction between a urban area and a more rural area *(country village established on a coastal region)*

We will keep the altitudes of plot II.2 and the wave height of **14,87m** for this last comparison.





Plot III .1 of a coastal region affected by a wave

Plot III .2 of a coastal region affected by a wave

cylinder range height	5-15m	2-8m
range for cylinder base radius	3-9m	2-5m
description	We already described this plot in ***result II .2***	By simply changing the infrastructure dimensions, we can see the important difference in the proportion for the fully submerged class and in echo the not affected one became the minority class.

5) Conclusion

Limit to our modelisation :

The limits we fixed for the altitudes in our code (100m or 50m in our results), aren't respected due to our choice of functions to model in a more realistic way the coastal region.
(cf part 2 : model 1.) The trigonometric functions add some positive and negative variations to our $hauteur_{max}$ and the second part of our equation also helps increase the altitudes without a well defined boundary.

We choose to model a coast starting with an altitude close to sea level (without a tsunami) and a gentle increasing slope. If the model was to be applied to a coastal region starting with a cliff for example, our wave model might overestimate the penetration of the wave inland. In reality a steep cliff can break the wave more violently by absorbing its energy[8], a behaviour not captured by our current equation.

Finally we could say that our model doesn't really take into consideration enough factors to be realistic : we neglected the velocity of the wave and other physical obstacles like soil porosity, vegetation or even the encounter of other water bodies. We also limited ourselves to the study of one wave, while waves come successively at the coast.

summary of our results :

The plots generated on Matlab show us how our model can play with different parameters to encapsulate a large panel of situations. We found out expected behaviour such as : increasing the initial wave height can generate more infrastructures at risk. Depending on whether we are in a city with tall buildings, or a small town, the risk also changes. Finally our model would allow potential clients to visually identify dangerous areas to help them decide where to start a construction project or even to choose a safe place to live in.

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