Vegetation Pattern Formation

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- Water scarcity

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    - Seeds fall near plants

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The Article

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- Ising model describes local interactions
- *But requires careful tuning of parameters!*
- **New step:** Linearize Ising model
- **Result:** Model gives rise to Power-law clustering of vegetation (without careful tuning!)

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Research Questions

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- How big are the effects of the global and local rules, respectively, on power-law clustering?

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- How robust is power-law clustering across different vegetation densities?
- How big are the effects of the global and local rules, respectively, on power-law clustering?
- Under which circumstances do percolating clusters appear?

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- **Speed.** At each step 20% of all cells are updated

The Model: Local Interaction Rule

Local tree density ρ at cell (i, j) computed over neighborhood radius M:

$$\rho_{i,j} = \frac{\sum_{\text{neighbors}} \left(\frac{d_{\text{min}}}{d}\right)^k \cdot x_{\text{neighbor}}}{\sum_{\text{neighbors}} \left(\frac{d_{\text{min}}}{d}\right)^k}$$

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Parameters:

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Pareto-type weighting: nearby trees matter more than distant ones

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Probability of growth (empty $\rightarrow$ vegetated):

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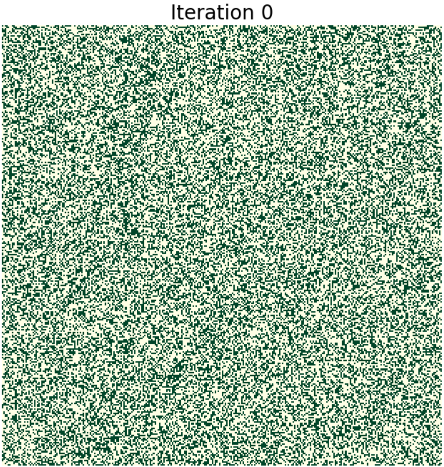
Interpretation:

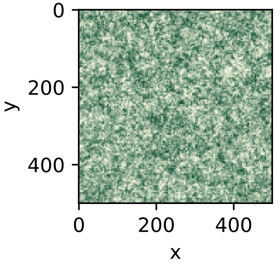
- High local density $\rho \rightarrow$ more likely to grow, less likely to die
- $f < f^*$ (below carrying capacity) $\rightarrow$ system-wide growth bias
- $f > f^*$ (above carrying capacity) $\rightarrow$ system-wide death bias

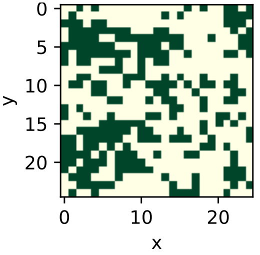
Animation



Analysis: Measuring Clustering

- **Data**: 2D grid that evolves in time





Analysis: Measuring Clustering

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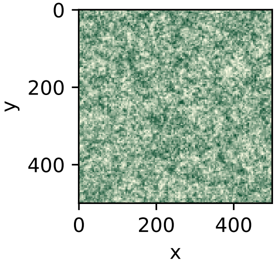
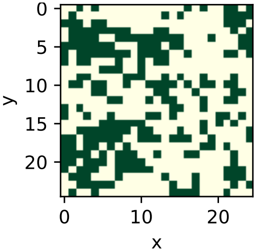
- We are interested in **cluster formation**
- `scipy.ndimage` $\rightarrow [1, 1, 1, 1, 12, 68, 1, 1, \ldots]$





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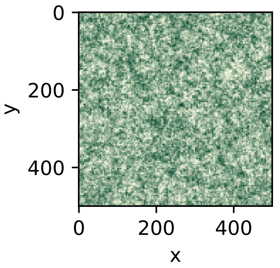
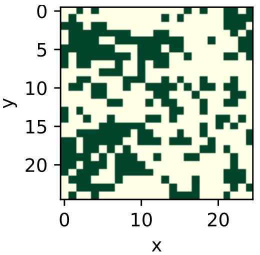
- We are interested in **cluster formation**
- `scipy.ndimage` $\rightarrow [1, 1, 1, 1, 12, 68, 1, 1, ...]$
- `powerlaw` $\rightarrow$ probability distribution
- **Q: Are the cluster sizes distributed according to a power law?**





Cluster Size Distribution

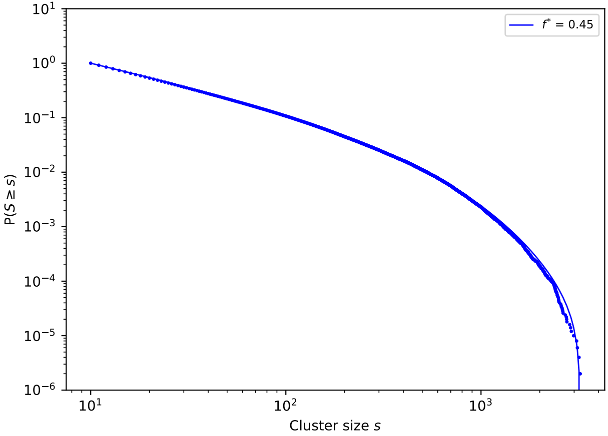
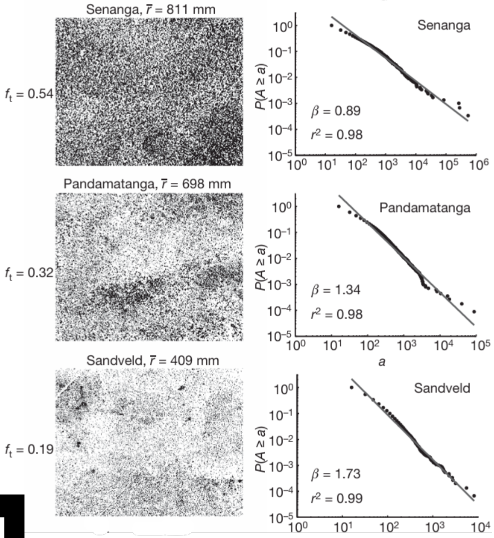


Figure: Cluster size distribution for $f^* = 0.45$, $M = 20$, $k = 3$.

Lucky Sampling or True Self-Organization?



Power law scaling persists across many tree densities in the Kalahari desert. The model should show this too.

Cluster Size Distributions Across Varying Vegetation Densities

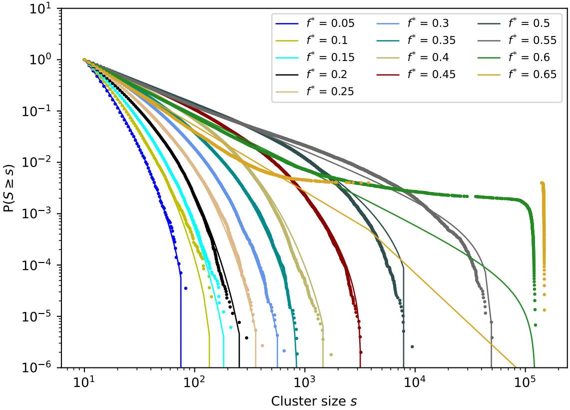


Figure: Cluster size distribution for $f^* \in [0.05, 0.65], M = 20, k = 3$.

Quantitative Analysis: Fit Statistics

- `powerlaw` package provides `distribution_compare`
- Input: two distributions (null and alternative) and data $\rightarrow$ output: log-likelihood ratio $R \sim \ln\left(\frac{L_1}{L_0}\right)$

Quantitative Analysis: Fit Statistics

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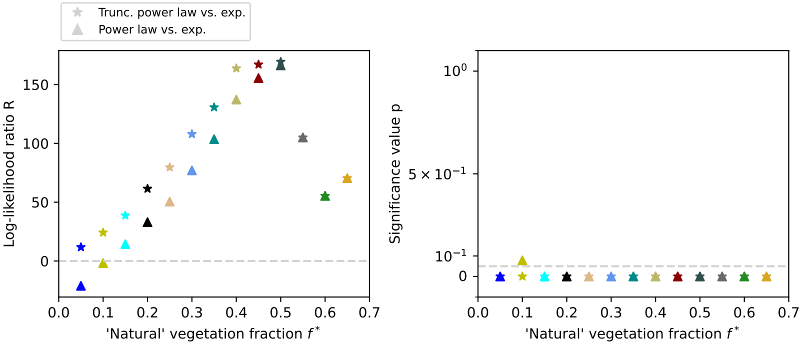
- Focus: (truncated) power law versus exponential

Fit Statistics



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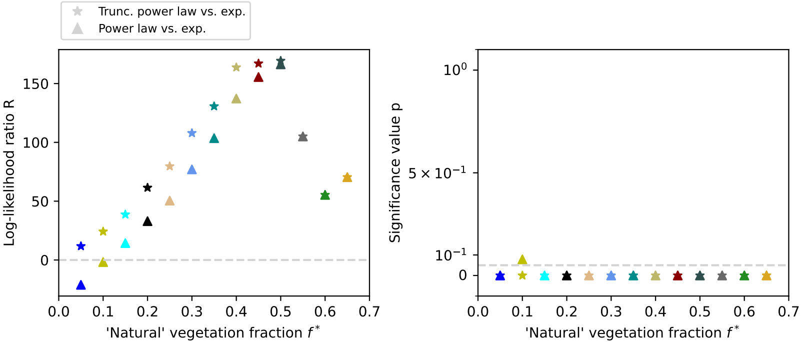

Figure: Fit statistics of the previously shown cluster size distributions.

(Truncated) power law outperforms exponential distribution for almost all parameters → **self-organization**!

Local vs global update rules

Probabilities are controlled by two different mechanisms:

$$P(o \to t) = \rho + \frac{f^* - f}{1 - f} \tag{1}$$

$$P(t \to o) = (1 - \rho) + \frac{f - f^*}{f} \tag{2}$$

Probability = Local term + + Global term

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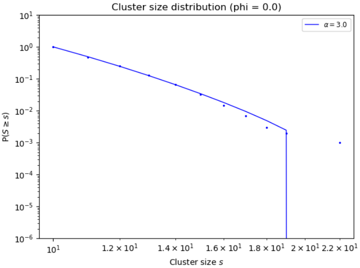
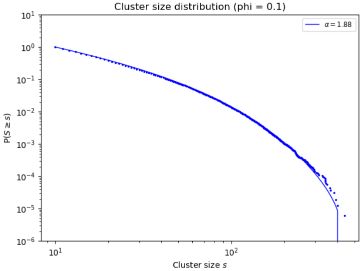
Probability = Local term + + Global term

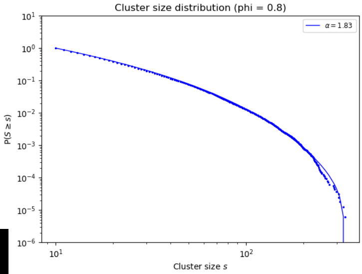
$$\Downarrow$$

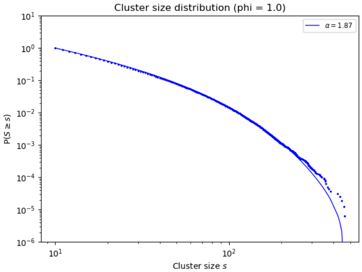
$$P(o \rightarrow t/t \rightarrow o) = \varphi * \text{local term} + (1 - \varphi) * \text{global term} \tag{3}$$

Cluster size distributions









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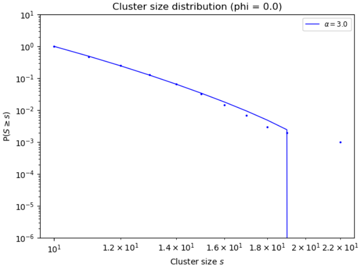
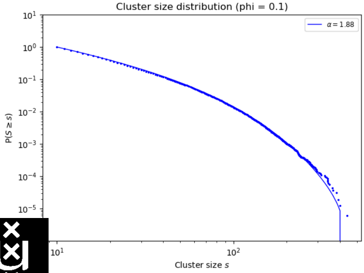
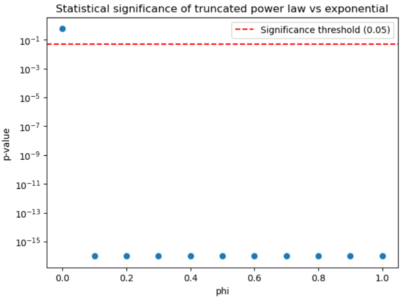




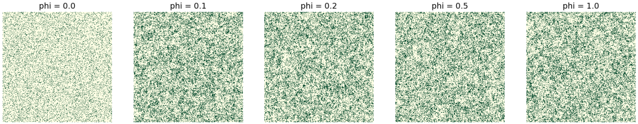
- For $\varphi = 0$, number of clusters is much smaller than for $\varphi > 0$.
- The distribution drops sharply around $s \approx 20$.
- For $\varphi > 0$, the distribution extends up to $s \approx 10^3$.
- Small amount of local facilitation enables more, and larger connected clusters

Statistical test: truncated power law vs exponential



- Only $\varphi = 0$ yields a high p-value ($p \approx 0.587$)
- For all $\varphi > 0$, the p-values are 0
- As soon as local rule term is present, system prefers truncated power-law distribution.

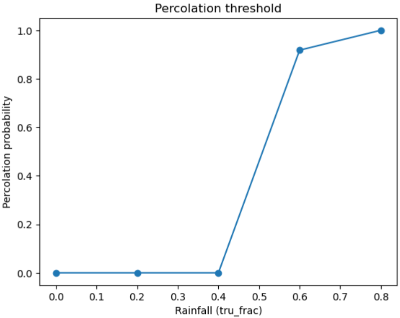
Grids visualizations



- For $\varphi = 0$, vegetation remains more homogeneous with no large clusters.
- For $\varphi > 0$, the spatial patterns look similar across values

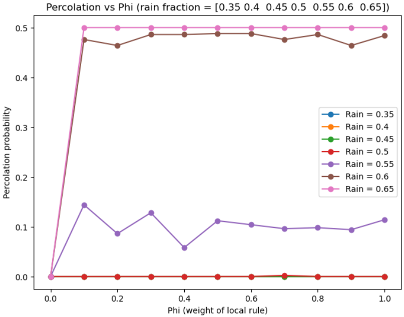
This again shows that the key transition is between **no local rule** and **some local rule**.

Percolation vs rainfall



- Below rainfall $f^* = 0.4$, no percolation occurs.
- Between 0.4 and 0.6, percolation probability rises.
- Above 0.6, percolation is almost guaranteed.
- Percolation is primarily controlled by the **total vegetation density**.

Percolation vs φ (weight of local rule)

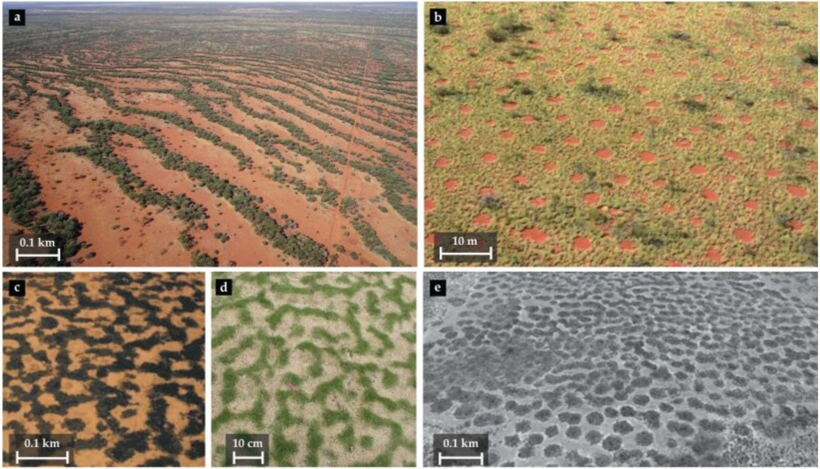


- For $\varphi > 0.1$, the curves stays quite identical.
- Low rainfall (< 0.5): not enough vegetation to percolate, regardless of φ.
- **Notable observation:** for $\varphi = 0$, system never percolates, even at rainfall levels above expected threshold.

Conclusion

- Hypothesis confirmed: local facilitation and global constraints generate realistic vegetation patterns.
- $\varphi = 0 \rightarrow$ homogeneous vegetation; $\varphi > 0 \rightarrow$ large connected clusters emerge.
- Cluster sizes follow a robust truncated power-law across parameters.
- Simple cellular automaton captures key mechanisms without fine-tuning.

The End



Appendix

Fit Statistics: comparing all distributions

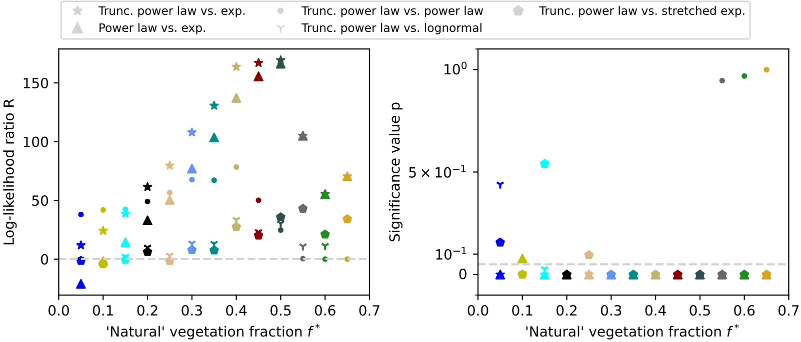
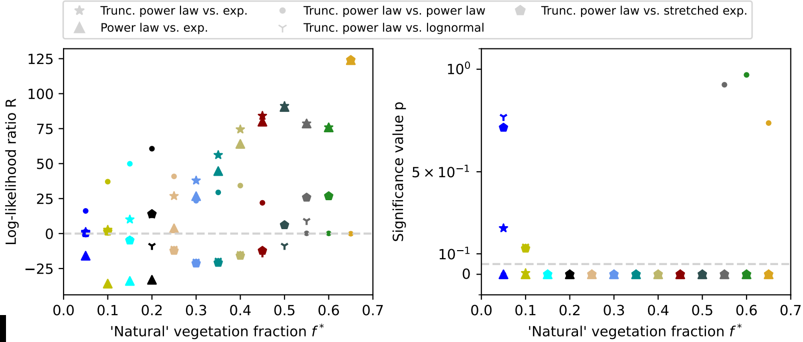


Figure: Fit statistics of all supported distributions in the `powerlaw` library.

Fit Statistics: varying M and k

For a large range of the additional fit parameters M and k, the truncated power law is still the best fit. However, this has a limit, and changing them by a lot (either M=5 or k=1 in following plots) decreases the range of truncated power law clustering, and increases the range of exponential scaling.



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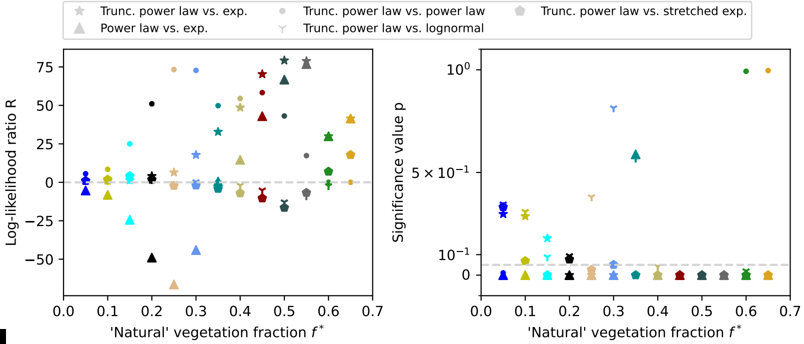

References

Scanlon TM, Caylor KK, Levin SA, Rodriguez-Iturbe I. *Positive feedbacks promote power-law clustering of Kalahari vegetation.* Nature. 2007;449(7159):209-12.