One-Way ANOVA
(part 1)

Lecture 14

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From our last lecture…

- Paired samples t-test

$$t = \frac{M_D}{s_{MD}}$$

- Comparing all t-tests
- Introduction to ANOVA

E.g., is the wait time different between Gathers, Molly Tea, and Tiger Sugar

TODAY'S PLAN

01

ANOVA Intuitions & Terminology

02

The F Ratio

03

ANOVA Source Table

04

NHST Steps & Worked Example

Learning objectives

- Identify the **circumstances** in which you should **use ANOVA instead of *t* tests** to evaluate mean differences.
- Describe the **terminology** that is used for ANOVA (i.e., factor, level).
- Identify the sources that contribute to the **variance between-treatments** and the **variance within-treatments.**
- Describe the **F-ratio** that is used in ANOVA and **explain** it conceptually.
- **Conduct** a **one-way ANOVA** using the NHST steps.

01

ANOVA Intuitions & Terminology

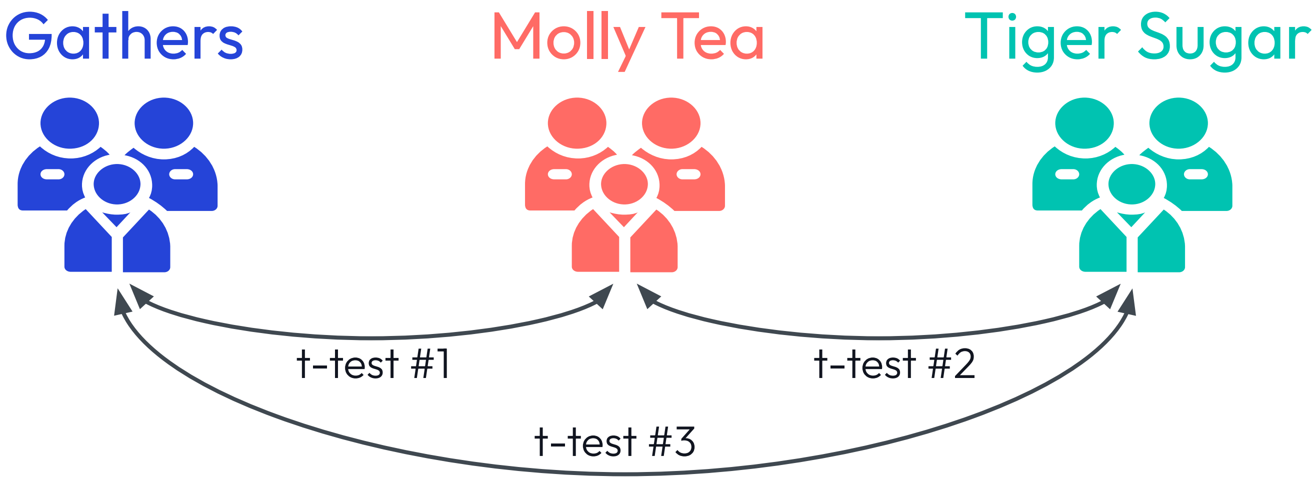
What if we have **more** than two groups?

E.g., is the wait time different between Gathers, Molly Tea, and Tiger Sugar

ANOVA

ANalysis **O**f **VA**riance

We can’t just run a lot of *t* tests



The Family-Wise Error Rate (FWER) is a lot higher than 5% if we do all possible t-tests.
In stats lingo, we call that “<u>inflated Type 1 errors</u>”.

Therefore, when we have more than 2 groups, we use a one-way ANOVA

One-way ANOVA asks the question:
"At least one of these things is not like the others"

Important: A One-Way ANOVA does not tell us which group is different.
It only tells us whether a difference exists somewhere.

Going back to our example

Gathers | Molly Tea | Tiger Sugar

ANOVA answers: “whether at least one shop’s average wait time is different from the rest.”

We don’t know which shop(s) is/are different. We will leave that for the next class. But one-way anova is the necessary first step.

HYPOTHESES FOR ANOVA

NULL HYPOTHESIS

H_0

All the means are the **same**

ALTERNATIVE HYPOTHESIS

H_1

At least one mean is **different** than the others

Key Terms: Factors & Levels

When we use an ANOVA, we typically use the terms **factor** and **level** to help us distinguish our variables and groups.

- **Factor**
  - This is what we call our **independent variable** (IV).
  - This can be something that occur naturally (e.g., race/ethnicity) or something that we manipulate (e.g., treatment condition).
- **Level**
  - These are **groups** *within the same factor*.
  - In an ANOVA, we typically have *three or more levels* for a single factor.

Going back to our example

Gathers

Molly Tea

Tiger Sugar

Factor

Aka independent variable (IV): Bubble tea shops in Chicago

Levels are options of the factor

Levels

3 levels in our example (aka, 3 shops we are comparing)

Dependent variable (DV): Wait time

Going back to our example

Gathers Molly Tea Tiger Sugar

We call this a <u>one-way</u> ANOVA because there's only 1 factor. This is the focus of this class.

If there are 2 factors, we call that <u>2-way</u> ANOVA or <u>factorial</u> ANOVA.

We won't talk about that until a week later.

Factor — Aka indep… Bubble tea shops in Chicago

Levels are <u>options</u> of the factor

Levels — 3 levels in our example (aka, 3 shops we are comparing)

Dependent variable (DV): Wait time

Understanding ANOVA Through Its Name

ANalysis **O**f $\boxed{\textbf{VA}\text{riance}}$

A One-Way ANOVA compares one factor with 3+ levels on a single dependent variable.

We know what variance means.
It is the variability of a variable.

Whose variance are we "analyzing"? →

We are analyzing the DV's variance.

How?

Understanding ANOVA through example

You are working in a lab where researchers investigate the most effective dosage to manage blood glucose levels using insulin injections in individuals with **Type 1 diabetes**.
We randomize our participants to 3 types of dosages: 10, 20, or 30 units per day.

Your goal is to answer: Does the average blood glucose level differ among people given 10, 20, or 30 units of insulin per day?

What's the factor? And how many levels?

Understanding ANOVA through example

You are working in a lab where researchers investigate the most effective dosage to manage blood glucose levels using insulin injections in individuals with **Type 1 diabetes**.
We randomize our participants to 3 types of dosages: 10, 20, or 30 units per day.

Your goal is to answer: Does the average blood glucose level differ among people given 10, 20, or 30 units of insulin per day?

Factor (i.e., IV): insulin injection dosage
Levels: 3
DV: blood glucose level

By now we are familiar with the concept of sampling variation and chance.

Our DV is blood glucose level – there is so much variance in blood glucose level – it varies depends on time of the day we took their blood sample, their previous meal, and of course, the dosage condition the participant is in.

Comparing to Independent Samples t-test

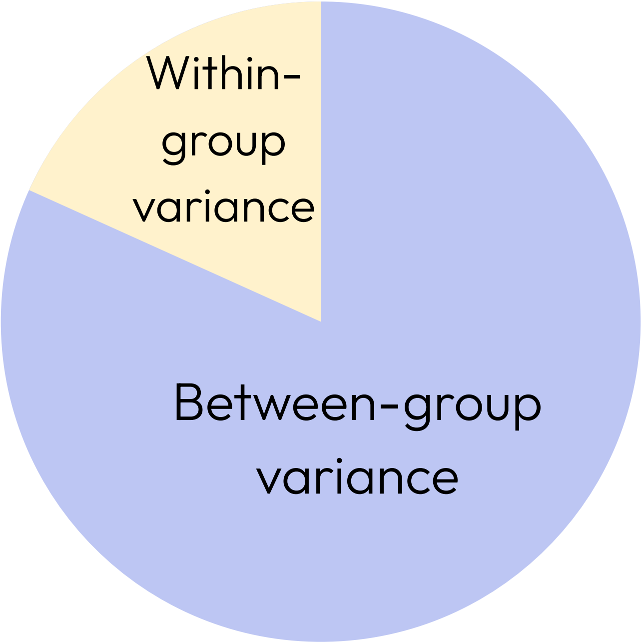
Now we have 3 groups. We can't do M1 - M2 - M3

$$t = \frac{M_1 - M_2}{s_{M_1 - M_2}} = \frac{M_1 - M_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Now we have more than 2 groups...

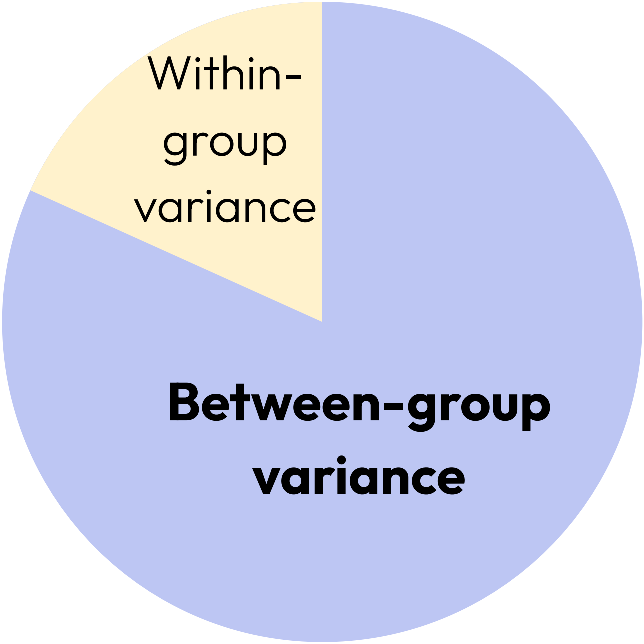
Splitting the total variance

We can think of the total variance in our DV, blood glucose level, as this whole pie.



Between-group variance + Within-group variance = Total Variance in our DV

Splitting the total variance

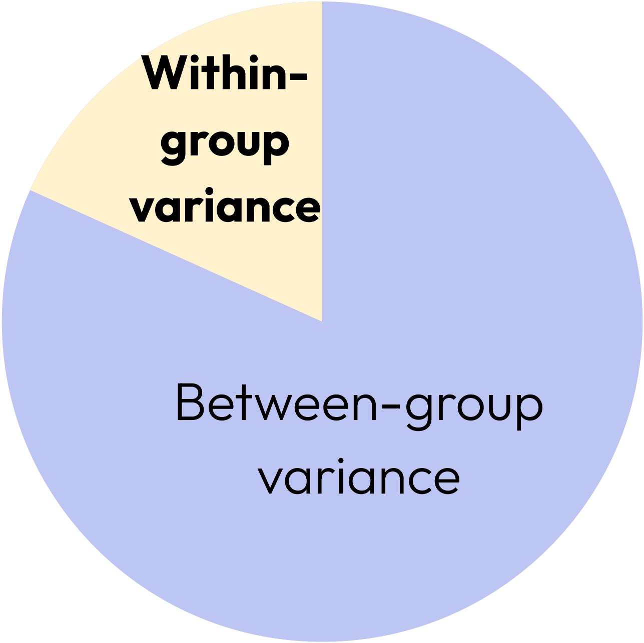


Our IV is insulin dosage. This means the only difference between groups is how much insulin they received (10, 20 or 30).

So if we see differences in average blood glucose levels across these groups, we can start to suspect that dosage might be the reason.

This variation between group means—caused by the different levels of our IV—is what we call **between-group variance**.

Splitting the total variance



Even when participants are assigned the same dosage—say, 20 units/day—their blood glucose levels still vary.

That's because of sampling variation: differences in time of day, what they ate, how their body responds, etc.

This variability within each group is called **within-group variance**—and it reflects the noise or error we can't explain by our independent variable.

02

The F Ratio

How do we make a decision in ANOVA?

How do we know whether the average blood glucose level differ among people given 10, 20, or 30 units of insulin per day?

Do you have any clues? Flashback to last class...

Intuition Behind All t-tests

All t-tests use a signal-to-noise ratio that takes on this form:

$$t = \frac{\text{difference in means}}{\text{Some form of SE}}$$

This ratio is asking the question: **Is the difference I'm observing bigger than what I'd expect just from random noise?**

How do we make a decision in ANOVA?

How do we know whether the average blood glucose level differ among people given 10, 20, or 30 units of insulin per day?

The idea is very similar!

Let's come up with a **signal-to-noise ratio** !

In an ANOVA, instead of a t-statistic, we calculate an **F-Ratio** as our test statistic. This new statistic represents a ratio of the variance between the groups (signal) to the variance within the groups (error).

F-ratio Formula

Don't worry about what MS is yet. For now, you just need to know it represents "variation."

Actual formula:

$$F = \frac{\boxed{MS}_{between}}{MS_{within}}$$

Conceptual formula:

$$F = \frac{\text{Variation between treatments}}{\text{Variation within treatments}}$$

$$F = \frac{\text{Variation between treatments}}{\text{Variation within treatments}}$$

If the between-group variance is **large** compared to the within-group variance (*F* > 1), we have evidence that the groups are **meaningfully (significantly) different**.

If the two variances are **similar** (*F* ≈ 1), then we do likely **do not** have a meaningful difference amongst the groups.

If *F* = 1, then we know there's no difference between groups whatsoever.

Comparing to Independent Samples t-test

We were calculating group mean difference

$$t = \frac{M_1 - M_2}{s_{M_1 - M_2}} = \frac{M_1 - M_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Denominator is **standard error**

$$F = \frac{\text{Variation between treatments}}{\text{Variation within treatments}}$$

Now, both numerator and denominator are **variances**

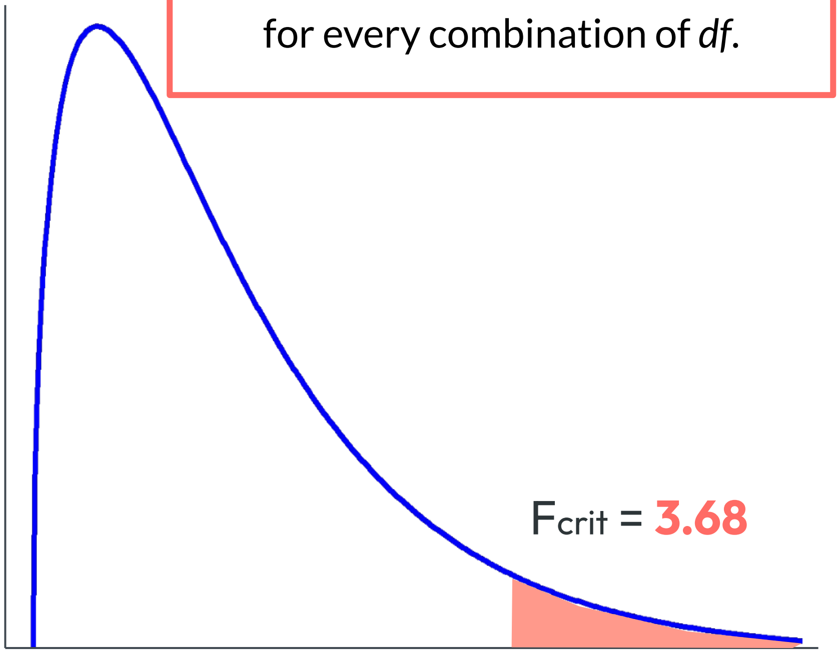
Just like a t-test, we need to use a special table to find a critical value. We call this the F-Table.

We compare the F-statistic we calculate to our critical value to determine if we **reject** or **fail to reject** the null hypothesis.

Use an F-Table to find your critical value (F_{CRIT})

***df*within** (denominator)	***df*between** (numerator) 1	2	3	4	5	6	7	8	9	10
1	161	200	216	225	230	234	237	239	241	242
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.70	2.64	2.59	2.55
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55	2.50	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.55	2.48	2.43	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.52	2.45	2.40	2.35

The F distribution is **only positive** and right-skewed.
There is a different F distribution for every combination of *df*.

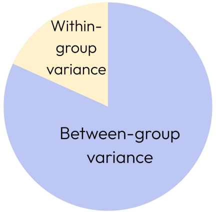


03

ANOVA Source Table

ANOVA SOURCE TABLE

The ANOVA source table helps us organize our calculations. Here are some of the key formulas (you will be given these).

Source	SS	df	MS	F
Between		**k** – 1	SS_b / df_b	MS_b / MS_w
Within		**N** – k	SS_w / df_w	
Total	$SS_b + SS_w$	N – 1		

k = number of groups

N = total sample size

ANOVA SOURCE TABLE

The ANOVA source table helps us organize our calculations. Here are some of the key formulas (you will be given these).

Source	***SS***	***df***	***MS***	***F***
Between	$\Sigma(M\text{-}GM)^2 \times n$	$\mathbf{k} - 1$	SS_b / df_b	MS_b / MS_w
Within	$\Sigma SS_{\text{inside each group}}$	$\mathbf{N} - k$	SS_w / df_w	
Total	$SS_b + SS_w$	$N - 1$		

k = number of groups

N = <u>total</u> sample size

ANOVA SOURCE TABLE TIPS

Here are some very useful tips. You can treat it like a puzzle!

Source	*SS*	*df*	*MS*	*F*
Between	40 +	4 +		
Within	20	10		
Total	?	?		

$SS_{between} + SS_{within} = SS_{total}$

$df_{between} + df_{within} = df_{total}$

ANOVA SOURCE TABLE TIPS

Source	*SS*	*df*	*MS*	*F*
Between	40	4		
	+	+		
Within	20	10		
Total	**60**	**14**		

$SS_{between} + SS_{within} = SS_{total}$

$df_{between} + df_{within} = df_{total}$

ANOVA SOURCE TABLE TIPS

Source	*SS*	*df*	*MS*	*F*
Between	40 ÷	4	?	
Within	20 ÷	10	?	
Total	60	14		

Divide across to get your $MS_{between}$ and MS_{within}

ANOVA SOURCE TABLE TIPS

Source	*SS*		*df*	*MS*	*F*
Between	40	÷	4	**10**	
Within	20	÷	10	**2**	
Total	60		14		

Divide across to get your $MS_{between}$ and MS_{within}

ANOVA SOURCE TABLE TIPS

Source	*SS*	*df*	*MS*	*F*
Between	40	4	10 ÷	?
Within	20	10	2	
Total	60	14		

Divide <u>down</u> to get your F-ratio.

ANOVA SOURCE TABLE TIPS

Source	*SS*	*df*	*MS*	*F*
Between	40	4	10 ÷	5
Within	20	10	2	[redacted]
Total	60	14	[redacted]	[redacted]

Divide <u>down</u> to get your F-ratio.

PRACTICE 1

Solve this on your own or with a partner.

Source	*SS*	*df*	*MS*	*F*
Between	150	3	?	?
Within	50	10	?	
Total	?	?		

PRACTICE 2

Solve this on your own or with a partner.

Source	*SS*	*df*	*MS*	*F*
Between	400	?	?	?
Within	?	16	?	
Total	620	20		

PRACTICE 3 (challenge)

At your table, complete the table.

Source	*SS*	*df*	*MS*	*F*
Between	360	?	20	?
Within	?	16	?	
Total	1360	?		

EFFECT SIZE

η^2 ("eta-squared") represents the **percentage of variance explained by our independent variable (factor)**.

$$\eta^2 = \frac{SS_{between}}{SS_{total}}$$

How to interpret?

Small = 0.01 - 0.059

Medium = 0.06 - 0.13

Large = above 0.13

Example

If we calculated an η^2 of **0.15** for our study, that means the IV explains **15% of the variance** in the DV.

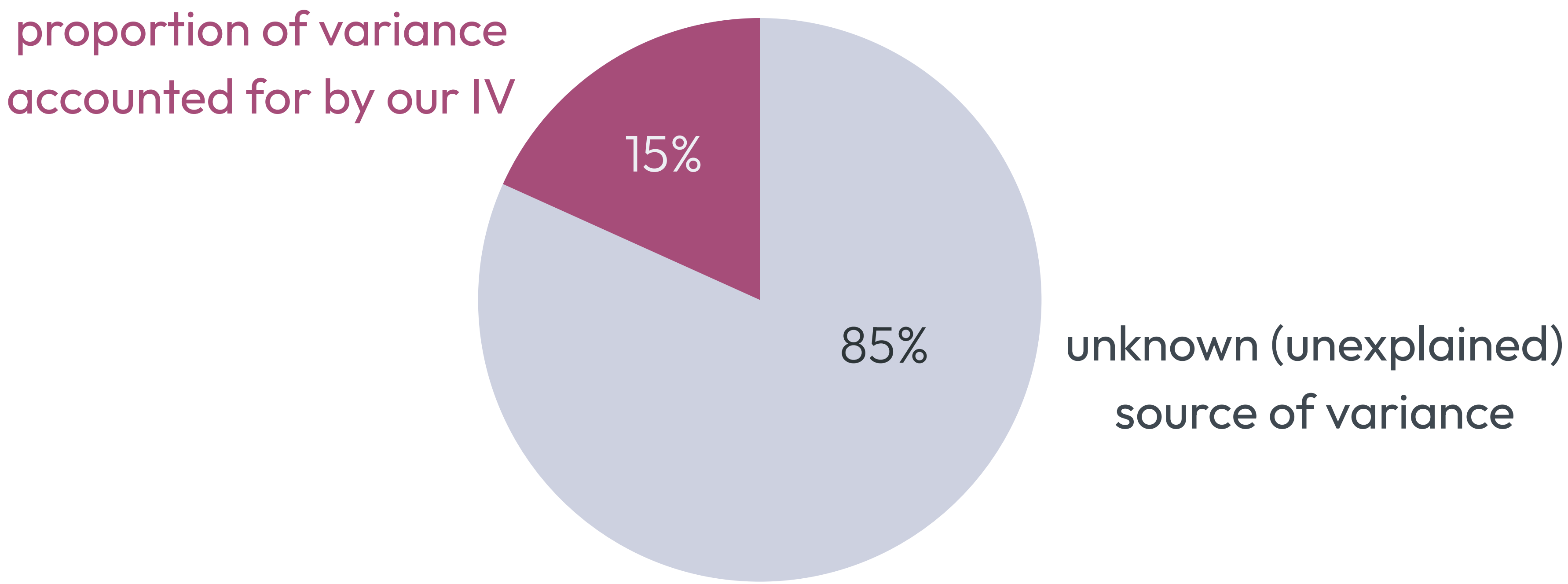
EFFECT SIZE IN THE SOURCE TABLE

Luckily, you just need these two numbers in your source table!

Source	*SS*	*df*	*MS*	*F*
Between	15	2	7.5	3.52
Within	85	40	2.13	
Total	100	42		

$SS_{between} / SS_{total} = 15/100 = .15\ (15\%)$

η^2


proportion of variance
accounted for by our IV
15%
85%
unknown (unexplained)
source of variance

04

NHST Steps & Worked Example

NHST Steps for One-Way ANOVA

1. State your **hypotheses**.
2. Find your **critical value** (using F-table).
3. Calculate your **F-ratio** using the source table.
4. Make your **decision**.
5. Write your results in **APA style**.

Factor (i.e., IV): insulin injection dosage
Levels: 3 (10, 20, or 30 units/day)
DV: blood glucose level

	10 units/day	**20 units/day**	**30 units/day**
Blood Glucose Level (mg/dL)	230	200	160
	245	195	155
	220	210	150
	240	185	165
	235	195	170
Group Mean	234	197	160
SS	370	330	250
Grand Mean	197		
SS_b	13690		

Just to clarify, each participant can only be in one condition and only give one data point (this is a **between-subjects design**)!

Let's copy-paste the important info

10 units/day	20 units/day	30 units/day
M = 234	M = 197	M = 160
SS = 370	SS = 330	SS = 250
n = 5	n = 5	n = 5
N = 15	$SS_{between}$ = 13690	

0

TIP: Calculate your **$df_{between}$** and **df_{within}.**

- $df_{between}$ = number of groups - 1 = k - 1 = **2**
- df_{within} = N - k = 15 - 3 = **12**

NHST Steps for One-Way ANOVA

1. State your **hypotheses**.
2. Find your **critical value** (using F-table).
3. Calculate your **F-ratio** (and effect size) using the source table.
4. Make your **decision**.
5. Write your results in **APA style**.

1 STATE **HYPOTHESES**

NULL HYPOTHESIS

H_0

The blood glucose levels are the **same** across the three dosages

ALTERNATIVE HYPOTHESIS

H_1

The blood glucose levels are **not the same** across the three dosages

Notice how we don't have the mathematical notation here?

NHST Steps for One-Way ANOVA

1. State your **hypotheses**.
2. Find your **critical value** (using F-table).
3. Calculate your **F-ratio** (and effect size) using the source table.
4. Make your **decision**.
5. Write your results in **APA style**.

2 FIND F_{CRIT}

Remember, you need to use your $df_{between}$ and df_{within} to find your F_{crit} in the table.

$k = 3$

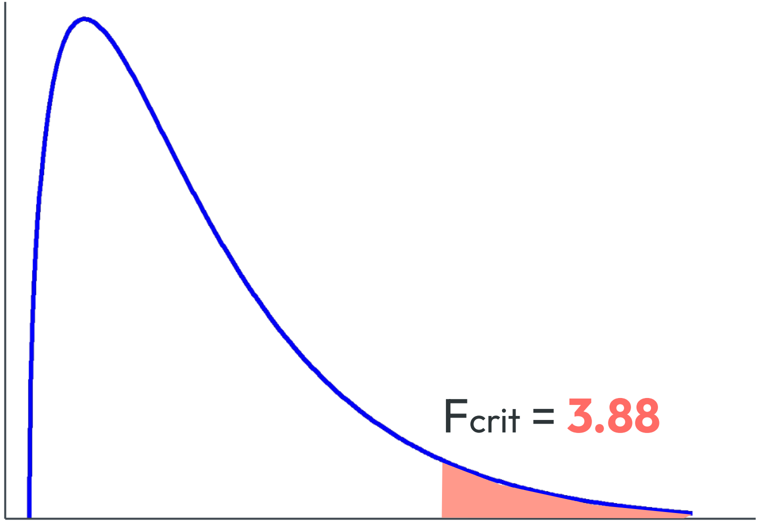
$N = 15$

$df_{between} = k - 1 = 2$

$df_{within} = N - k = 15 - 3 = 12$

df: Denominator (Within)	*df:* Numerator (Between) 1	2	3	4	5	6	7	8	9	10
1	161	200	216	225	230	234	237	239	241	242
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.70	2.64	2.59	2.55
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55	2.50	2.45
19	4.38	3.52	3.13	2.90	2.74	2.63	2.55	2.48	2.43	2.38
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.47	2.40	2.35	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.45	2.38	2.32	2.28
24	4.26	3.40	3.01	2.78	2.62	2.51	2.43	2.36	2.30	2.26
25	4.24	3.38	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24

2 FIND F_{CRIT}



(We need an F-statistic of 3.88 or higher to reject the null)

df: Denominator (Within)	*df:* **Numerator** (Between) 1	2	3	4	5	6	7	8	9	10
1	161	200	216	225	230	234	237	239	241	242
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.70	2.64	2.59	2.55
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55	2.50	2.45
19	4.38	3.52	3.13	2.90	2.74	2.63	2.55	2.48	2.43	2.38
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.47	2.40	2.35	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.45	2.38	2.32	2.28
24	4.26	3.40	3.01	2.78	2.62	2.51	2.43	2.36	2.30	2.26
25	4.24	3.38	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24

NHST Steps for One-Way ANOVA

1. State your **hypotheses**.
2. Find your **critical value** (using F-table).
3. Calculate your **F-ratio** (and effect size) using the source table.
4. Make your **decision**.
5. Write your results in **APA style**.

3 CALCULATE **F-RATIO** (AND EFFECT SIZE)

Source	*SS*	*df*	*MS*	*F*
Between	13690	2		
Within	$\boxed{\Sigma SS_{\text{inside each group}}}$	12		
Total	$SS_b + SS_w$	$df_b + df_w$		

$$SSwithin = \Sigma SS_{\text{inside each group}} = 370 + 330 + 250 = 950$$

3 CALCULATE F-RATIO (AND EFFECT SIZE)

Source	*SS*	*df*	*MS*	*F*
Between	13690	2		
Within	950	12		
Total	?	?		

3 CALCULATE F-RATIO (AND EFFECT SIZE)

Source	*SS*		*df*	*MS*	*F*
Between	13690	÷	2	?	
Within	950	÷	12	?	
Total	14640		14		

3 CALCULATE F-RATIO (AND EFFECT SIZE)

Source	*SS*	*df*	*MS*	*F*
Between	13690	2	6845	?
Within	950	12	79.17	
Total	14640	14		

3 CALCULATE F-RATIO (AND EFFECT SIZE)

Source	*SS*	*df*	*MS*	*F*
Between	13690	2	6845	**86.46**
Within	950	12	79.17	
Total	14640	14		

3 CALCULATE F-RATIO (AND EFFECT SIZE)

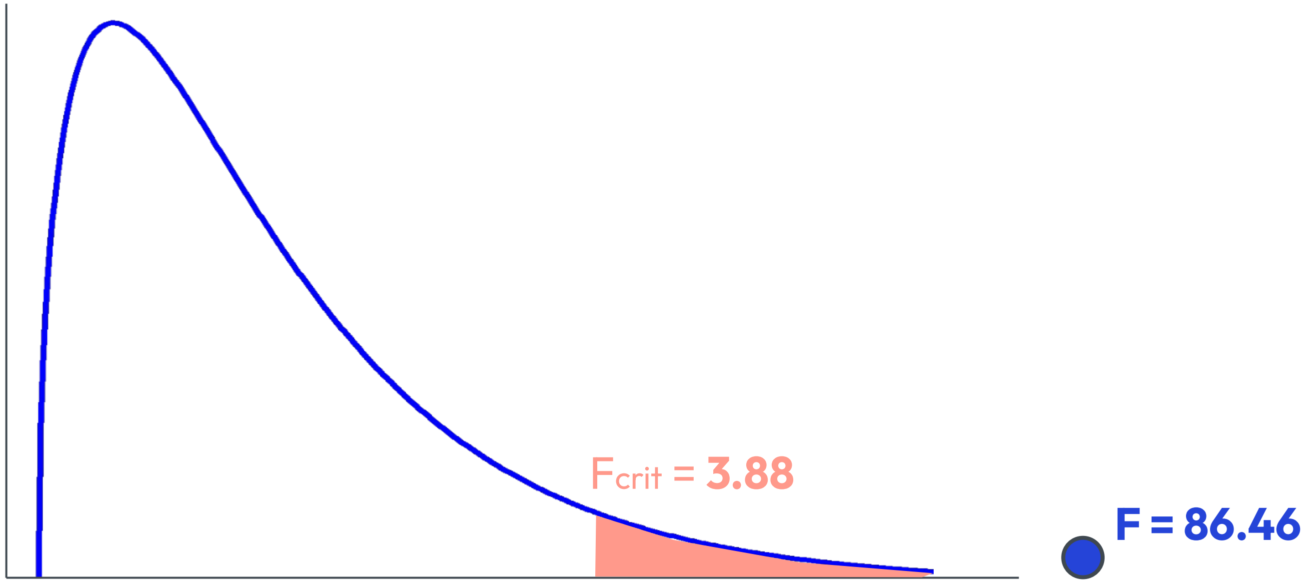
Source	*SS*	*df*	*MS*	*F*
Between	**13690**	2	6845	**86.46**
Within	950	12	79.17	
Total	**14640**	14		

$$\eta^2 = \frac{SS_{between}}{SS_{total}} = \frac{13690}{14640} = 0.94$$

NHST Steps for One-Way ANOVA

1. State your **hypotheses**.
2. Find your **critical value** (using F-table).
3. Calculate your **F-ratio** (and effect size) using the source table.
4. Make your **decision**.
5. Write your results in **APA style**.

4 MAKE YOUR DECISION



Our F-statistic is in our critical region, so we **reject the null hypothesis.**

NHST Steps for One-Way ANOVA

1. State your **hypotheses**.
2. Find your **critical value** (using F-table).
3. Calculate your **F-ratio** (and effect size) using the source table.
4. Make your **decision**.
5. Write your results in **APA style**.

5

WRITE RESULTS IN APA STYLE

"A **one-way ANOVA** revealed a **significant __difference__** in **blood glucose levels** amongst the **10 units/day, 20 units/day, 30 units/day insulin injection dosages**, $F(2{,}12) = 86.46$, $p < 0.05$, $\eta^2 = .94$, with a **large** effect size."

$$F(\underbrace{2}_{df_{between}}, \underbrace{12}_{df_{within}}) = \underbrace{86.46}_{\text{F-statistic}}, \underbrace{p}_{p\text{-value}} < \underbrace{.05}_{\alpha}, \underbrace{\eta^2}_{\text{effect size (eta-squared)}} = .94$$

Note: A F-value as high as ours will probably never occur in real life – this is just for demonstration.

ICA 14

A clinical psychologist wants to examine whether self-reported stress levels differ by profession. Participants from three job groups—**nurses** , **teachers** , and **software engineers** —rate their daily stress on a scale from 1 (not at all stressed) to 10 (extremely stressed). The psychologist asks you to conduct a one-way ANOVA to compare the average stress levels across the three professions (α = 0.05).

Nurses	Teachers	Software Engineers
M = 7.8	M = 6.5	M = 5.2
SS = 25	SS = 35	SS = 40
n = 15	n = 15	n = 15
N = 45	$SS_{between}$ = 40	

Source	*SS*	*df*	*MS*	*F*
Between	(given)	$k - 1$	SS_b / df_b	MS_b / MS_w
Within	$\Sigma SS_{\text{inside each group}}$	$N - k$	SS_w / df_w	
Total	$SS_b + SS_w$	$N - 1$		

k = number of groups

N = total sample size

F TABLE

*df*within (denominator)	*df*between (numerator) 1	2	3	4	5	6	7	8	9	10
1	161	200	216	225	230	234	237	239	241	242
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.70	2.64	2.59	2.55
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55	2.50	2.45
19	4.38	3.52	3.13	2.90	2.74	2.63	2.55	2.48	2.43	2.38
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.47	2.40	2.35	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.45	2.38	2.32	2.28
24	4.26	3.40	3.01	2.78	2.62	2.51	2.43	2.36	2.30	2.26
25	4.24	3.38	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24

*df*within (denominator)	*df*between (numerator) 1	2	3	4	5	6	7	8	9	10
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.30	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.44	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.54	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.34	2.27	2.21	2.16
32	4.15	3.30	2.90	2.67	2.51	2.40	2.32	2.25	2.19	2.14
34	4.13	3.28	2.88	2.65	2.49	2.38	2.30	2.23	2.17	2.12
36	4.11	3.26	2.86	2.63	2.48	2.36	2.28	2.21	2.15	2.10
38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.07
42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.06
44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05
46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.14	2.09	2.04
48	4.04	3.19	2.80	2.56	2.41	2.30	2.21	2.14	2.08	2.03
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.02
60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10	2.04	1.99
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.01	1.97
80	3.96	3.11	2.72	2.48	2.33	2.21	2.12	2.05	1.99	1.95
100	3.94	3.09	2.70	2.46	2.30	2.19	2.10	2.03	1.97	1.92
200	3.89	3.04	2.65	2.41	2.26	2.14	2.05	1.98	1.92	1.87
400	3.86	3.02	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85
1,000	3.85	3.00	2.61	2.38	2.22	2.10	2.02	1.95	1.89	1.84
∞	3.84	2.99	2.60	2.37	2.21	2.09	2.01	1.94	1.88	1.83

Note: The critical values in this table are for p = .05