One-Way ANOVA (part 2)

Lecture 15 Emma Ning, M.A.

Between-subject Tests

t Test

One-way ANOVA

How many **groups** can I compare?

2

More than 2

What is our **test statistic**?

t statistic

Fratio

What is our measure of **effect size**?

Cohen's d

η[∠] (eta-squared)

From our last lecture...

From our last lecture...

Source	SS	df	MS	F
Between	60	3		
Within	200	50		
Total				

Our goal is to get to the **last column** (our F-ratio)

Take your best guess!

Red pill, blue pill, and green pill to treat pain.

ANOVA result:
$$F(2, 35) = 5.27, p < .05$$

Average levels of pain rating: $m_p = 5.46$, $m_g = 6.02$, $m_g = 9.34$

Red pills cause more side effects than blue pill.

Which colored pill should you take?

Take your best guess!

ANOVA result: F(2, 35) = 5.27, p < .05

Average levels of pain rating: $m_R = 5.46$, $m_R = 6.02$, $m_C = 9.34$

It's hard to tell because:

- Green pill is a lot more different than red or blue. So the significant ANOVA result is probably from that.
- But you don't know if red is better than blue. Even though the pain rating is better (difference of 0.56), we do not know whether this difference is bigger than within-group variability.

TODAY'S PLAN









Learning objectives

- Understand and can explain the logic of one-way ANOVA then follow-up tests.
- Describe situations in which follow-up tests are necessary.
- Conduct a one-way ANOVA and Tukey's post-hoc test (with a given HSD value) and report the results in APA style.
- **Know and understand** why we need **assumptions** for ANOVA, similar to the assumptions of t-tests we talked about before

01

Why Do A Follow-Up Test?

Recap

One-way ANOVA asks the question: "At least one of these things is not like the others"



Important: A One-Way ANOVA does not tell us which group is different. It only tells us whether a difference exists somewhere.

Recap

Well that's too bad...

"At least one of these things is not like the others"

If a scientist cannot point out where the difference is, they are not making a very convincing argument, right?

So today, we will learn how to find where the difference(s) is/are.

Important: A One-Way ANOVA does not tell us which group is different. It only tells us whether a difference exists somewhere.

How to find where the difference is

ONE-WAY ANOVA

SIGNIFICANT

(p < 0.05)

You will do follow-up tests.

Note: they are sometimes called follow-up tests, or post-hoc tests.

NOT SIGNIFICANT

(p > 0.05)



(you do not need to do any additional tests)

How to find where the difference is

ONE-WAY ANOVA

Why do we only do follow-up tests when the ANOVA is significant?

(p < 0.05)

You will do follow-up tests

Note: they are sometimes called follow-up tests, or post-hoc tests.



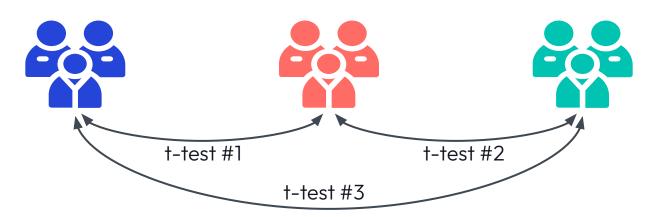
(you do not need to do any additional tests)

There are 2 main reasons

- 1. If the overall ANOVA is not significant, that means we don't have evidence that any group differs from the others. And if we don't have evidence that any difference exists, then there's no reason to go looking for which groups differ.
- 2. If the ANOVA is significant, it gives us a reason to look for specific group differences. But if you do it the other way around running all the pairwise t-tests first and then deciding which groups differ you inflate your Type I error rate and increase the risk of drawing a false conclusion.

Recap: Family-wise Error Rate

Family-wise error rate (FWER) is the probability of making one or more false discoveries, or type I errors when performing multiple hypotheses tests.



Family: 3 tests

Alpha: 0.05

FWER: $1 - (1 - 0.05)^3 = 0.143$

(You do not need know how to calculate FWER)

If we conduct 3 separate t-tests, and when there's <u>no real</u> <u>effect</u>, there's about a 14% chance that we'll make at least one false positive — that is, find a "significant" result just due to chance or random noise.

discoveries, or type I errors when performing multiple hypotheses tests.

Think of it like a metal detector.

A metal detector beeps when it senses something under the surface. If it doesn't beep, you don't dig. If it does, then you investigate further to find out what's there.

02 Follow-up Tests

Many Types of Follow-Up Tests

There are many types of follow-up tests, most with esoteric names based on the people who invented them.

But they all have the same goal: **to figure out where the difference(s) are**, once you've found a significant ANOVA.

Another thing they have in common: they're all **conservative** — meaning they try to keep your Type I error rate low when you're making multiple comparisons.

Tukey's HSD

Fisher's LSD

The Scheffè Test

Bonferroni's correction

Tukey's HSD (honestly significant difference)

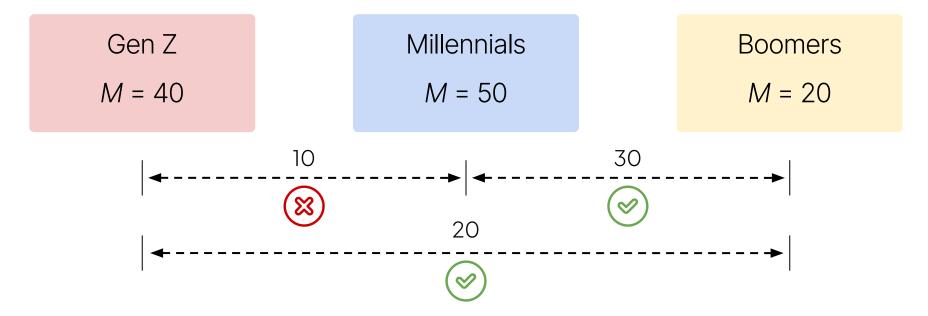
Tukey's HSD test allows you to compute a **single value** that determines the **minimum difference between group means** that is necessary for significance.

- You use this if you want to compare all possible pairs of means (pairwise comparisons).
- Tukey's test requires that the sample size (n) be the same for all conditions.

For this class, you need to be able to **interpret** the mean differences with a given HSD value. You will not be expected to calculate the HSD value.

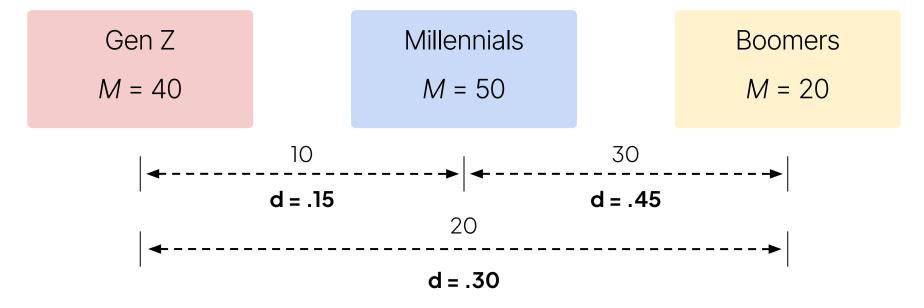
Tukey's HSD Example

Let's say that we get an HSD value of **15.** That means that the differences in the means between two groups must **exceed 15** for there to be a significant difference.



Tukey's HSD (Effect Size)

Because these are basically t tests, you will get an effect size (**Cohen's d**) when you do each test. You interpret this the same way you did before.



How is a post-hoc test different from a regular independent-samples t-test?

Since the follow-up/post-hoc test basically compares the conditions in pairwise, why use fancy tests instead of independent samples t-test?

Tukey's HSD controls the familywise error rate — it adjusts for the fact that you're making multiple comparisons (e.g., A vs B, A vs C, B vs C).

Thus, post hoc tests adjust their calculations based on the number of comparisons, which raises the threshold for significance, and makes it harder to mistakenly call a difference "significant."

Worked Example

NHST STEPS (One-way ANOVA + Follow Up)

- **STEP 1:** State **hypotheses**.
- → STEP 2: Find Fcrit (and draw critical region).
- → STEP 3: Calculate F-ratio using source table.
- ★ STEP 4: Make your decision (reject, fail to reject).
- **◆ STEP 5:** If significant, conduct a **follow-up test** (e.g., Tukey's).
- STEP 6: Write results in APA style.

Note: this is a new step in our process!

Worked Example

Let's imagine we want to compare math achievement scores amongst urban, suburban, and rural students in Illinois. We collect data from 30 students from each type of school and then compare their average math score (0 - 100 scale). I plan to conduct a one-way ANOVA and a Tukey's post-hoc test (if needed).

Rural	Suburban	Urban
M = 65	M = 75	M = 60
SS = 1500	SS = 1700	SS = 1300
n = 30	n = 30	n = 30
N = 90 SS	petween = 500 T	ukey's HSD = 8





NULL HYPOTHESIS

Hο

The math scores are the same across the three types of schools

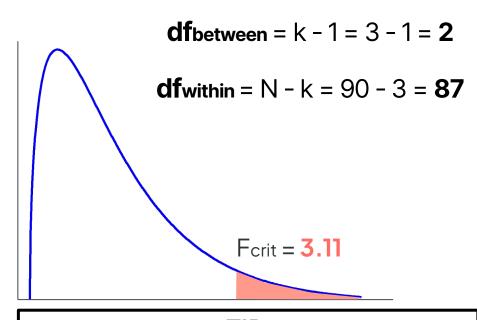
ALTERNATIVE HYPOTHESIS

H

The math scores are the **different** in <u>at least one</u> type of school

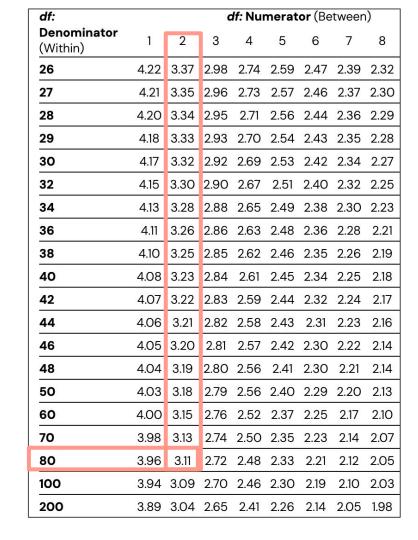






TIP:

Round down if your number is not there!



Source	SS	df	MS	F
Between	(given)	k – 1	SS _b /df _b	MS_b/MS_w
Within	ΣSS inside each group	N – k	SS_w/df_w	
Total	$SS_b + SS_w$	N – 1		
	k = number of groups		N = <u>total</u> sample siz	re

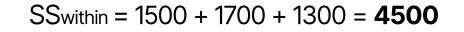


Source	SS	df	MS	F
Between	500	2	_	
Within	?	87		
Total				

Remember, you have already calculated your **df**between and **df**within, and **SS**between was provided in the problem.



Source	SS	df	MS	F
Between	500	2		
Within	4500	87		
Total				







Source	SS	df	MS	F
Between	500	2		
Within	4500	87		
Total	5000	89		

TIP 1: Add down to get your SStotal and dftotal.



Source	SS	df	MS	F
Between	500	2	250	
Within	4500	87	51.72	
Total	5000	89		

TIP 2: Divide across to get your MSbetween and MSwithin.



Source	SS	df	MS	F
Between	500	2	250	4.83
Within	4500	87	51.72	
Total	5000	89		

TIP 3: **Divide down** (MSbetween/MSwithin) to get your F-ratio!







Source	SS	df	MS	F
Between	500	2	250	4.83
Within	4500	87	51.72	
Total	5000	89		

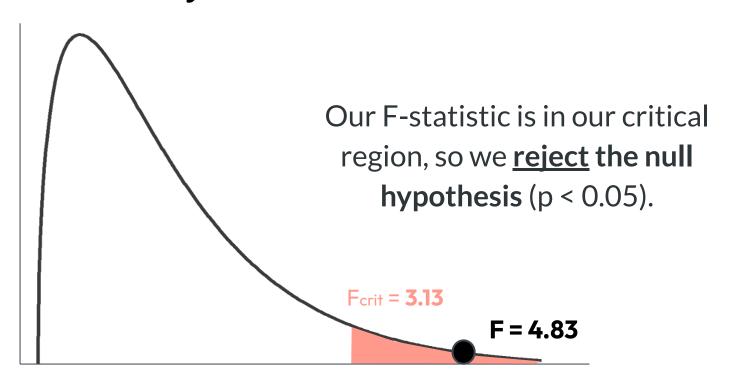
Always calculate your **effect size (η²)** as well.

$$\eta^2 = SS_{between}/SS_{total} = \frac{500}{5000} = 0.10$$





→ STEP 4: Make your decision.







Rural	Suburban	Urban		
M = 65	M = 75	M = 60		
Tukey's HSD = 8				

Rural vs. Suburban: 75 - 65 = **10**





Significant (more than 8)

Suburban vs. Urban: 75 - 60 = **15**





Significant (more than 8)

Rural vs. Urban = 65 - 60 = 5





Non-Significant (less than 8)

(tip: you can always subtract the higher number from the lower number)



→ STEP 6:Report Results in APA style

A one-way ANOVA revealed a significant difference in math achievement amongst rural, suburban, and urban conditions, F(2,87) = 4.83, p < 0.05, $\eta^2 = .10$, with a medium effect size.

dfbetween p-value effect size (eta-squared)
$$F(2, 87) = 4.83, p < .05, \eta^2 = .10$$
dfwithin F-statistic a





★ STEP 6: Report Results in APA style

A one-way ANOVA revealed a significant difference in math achievement amongst rural, suburban, and urban conditions, F(2,87) = 4.83, p < 0.05, $\eta^2 = .10$, with a **medium** effect size.

A Tukey's HSD test revealed that suburban students had significantly higher math achievement than rural students. Suburban students also had higher math achievement than urban students. However, **rural** and **urban** students did not significantly differ.

(tip: remember to state all pairs of meaningful comparisons)

Depressive symptoms among Mexican medical students: High prevalence and the effect of a group psychoeducation intervention

Link to study

- Since research has shown that medical students show higher depression levels than the general population, and psychoeducation intervention can be potentially beneficial, researchers at a university in Mexico city decided to investigate whether psychoeducation would be effective for their medical students.
- Recruited medical students from 1st to 8th semesters, measured depression using the Beck Depression Inventory (BDI)

2.4. Statistical analysis

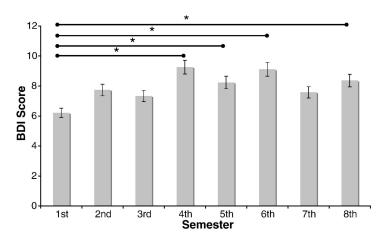
Always run descriptive analyses and take a close look at your data before running any analyses!

First, descriptive analyses were done for each year (prevalence of depressive symptoms, actual psychiatric treatment and request for psychiatric treatment).

Second, bivariate association tests between BDI scores on the one hand and gender, semester and psychoeducation on the other were performed. Where continuous BDI scores were used, one-way ANOVA was used; Pearson χ^2 test was used to analyze categorical (dichotomous: yes/no) BDI results based on the cutoffs described above.

ANOVA & Follow-up Results

In a one-way ANOVA with semester as factor and BDI as the dependent variable, a highly significant effect of semester was found: F(7, 1945)=5.74, p<0.0001. Post-hoc tests of all pairwise differences (with Tukey correction for multiple comparisons) showed that depression scores during the fourth (8.9±8.5), fifth (8.3±7.8), sixth (9.0±7.8) and eighth (8.7±8.0) semesters were all significantly higher than depression scores during the first semester (6.2±6.5).



ICA 15

Your are working with a mental health team at UIC team to see if average stress levels differ between sophomores, juniors, and seniors. You collect data from 50 students from each class and record their stress levels (1-10 scale, higher = more stressed). Conduct a one-way ANOVA and Tukey's HSD using all 6 NHST steps. Show all your work on the board. (assume $\alpha = 0.05$)

Sophomores	Juniors	Seniors
M = 6.5	M = 7.5	M = 7.1
SS = 650	SS = 750	SS = 700
n = 50	n = 50	n = 50
N = 150 SS b6	etween = 250 Tuk	xey's HSD = 0.5

Source	SS	df	MS	F
Between	(given)	k – 1	SS _b /df _b	MS_b/MS_w
Within	ΣSS inside each group	N – k	SS_w/df_w	
Total	$SS_b + SS_w$	N – 1		
	k = number of groups		N = <u>total</u> sample siz	re

F TABLE

df within			(d f betv	veen (numer	ator)				df within				dfbe	tween	(nume	erator)			
(denominator)	1	2	3	4	5	6	7	8	9	10	(denominator)	1	2	3	4	5	6	7	8	9	10
1	161	200	216	225	230	234	237	239	241	242	26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39	27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.30	2.25	2.20
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78	28	4.20	3.34	2.95	2.71	2.56	2.44	2.36	2.29	2.24	2.19
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	29	4.18	3.33	2.93	2.70	2.54	2.43	2.35	2.28	2.22	2.18
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74	30	4.17	3.32	2.92	2.69	2.53	2.42	2.34	2.27	2.21	2.16
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	32	4.15	3.30	2.90	2.67	2.51	2.40	2.32	2.25	2.19	2.14
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63	34	4.13	3.28	2.88	2.65	2.49	2.38	2.30	2.23	2.17	2.12
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34	36	4.11	3.26	2.86	2.63	2.48	2.36	2.28	2.21	2.15	2.10
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13	38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97	40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.07
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86	42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.06
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76	44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67	46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.14	2.09	2.04
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60	48	4.04	3.19	2.80	2.56	2.41	2.30	2.21	2.14	2.08	2.03
15	4.54	3.68	3.29	3.06	2.90	2.79	2.70	2.64	2.59	2.55	50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.02
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10	2.04	1.99
17	4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55	2.50	2.45	70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.01	1.97
19	4.38	3.52	3.13	2.90	2.74	2.63	2.55	2.48	2.43	2.38	80	3.96	3.11	2.72	2.48	2.33	2.21	2.12	2.05	1.99	1.95
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	100	3.94	3.09	2.70	2.46	2.30	2.19	2.10	2.03	1.97	1.92
22	4.30	3.44	3.05	2.82	2.66	2.55	2.47	2.40	2.35	2.30	200	3.89	3.04	2.65	2.41	2.26	2.14	2.05	1.98	1.92	1.87
23	4.28	3.42	3.03	2.80	2.64	2.53	2.45	2.38	2.32	2.28	400	3.86	3.02	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85
24	4.26	3.40	3.01	2.78	2.62	2.51	2.43	2.36	2.30	2.26	1,000	3.85	3.00	2.61	2.38	2.22	2.10	2.02	1.95	1.89	1.84
25	4.24	3.38	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24	∞	3.84	2.99	2.60	2.37	2.21	2.09	2.01	1.94	1.88	1.83

Note: The critical values in this table are for p = .05





NULL HYPOTHESIS Ho

ماله ماله ماله ما

The <u>stress levels</u> are the **same** across the three years/classes

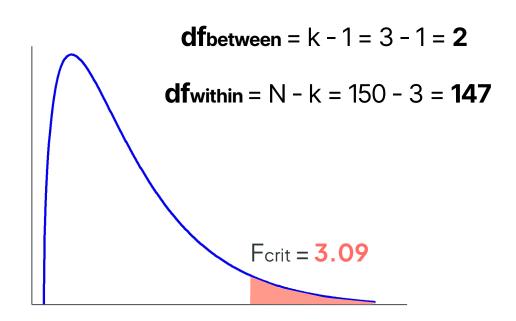
ALTERNATIVE HYPOTHESIS

 H_1

The <u>stress levels</u> are the **different** in at least one of the years/classes







TIP:

Round down if your number is not there!

df:	df: Numerator (Between)							
Denominator (Within)	1	2	3	4	5	6	7	8
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.30
28	4.20	3.34	2.95	2.71	2.56	2.44	2.36	2.29
29	4.18	3.33	2.93	2.70	2.54	2.43	2.35	2.28
30	4.17	3.32	2.92	2.69	2.53	2.42	2.34	2.27
32	4.15	3.30	2.90	2.67	2.51	2.40	2.32	2.25
34	4.13	3.28	2.88	2.65	2.49	2.38	2.30	2.23
36	4.11	3.26	2.86	2.63	2.48	2.36	2.28	2.21
38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18
42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17
44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16
46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.14
48	4.04	3.19	2.80	2.56	2.41	2.30	2.21	2.14
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13
60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07
80	3.96	3.11	2.72	2.48	2.33	2.21	2.12	2.05
100	3.94	3.09	2.70	2.46	2.30	2.19	2.10	2.03
200	3.89	3.04	2.65	2.41	2.26	2.14	2.05	1.98



→ STEP 3A: Calculate F-ratio using source table.

Source	SS	df	MS	F
Between	250	2	125	8.75
Within	2100	147	14.29	
Total	2350	149		

Remember, you have already calculated your **df**between and **df**within, and **SS**between was provided in the problem.







Source	SS	df	MS	F
Between	250	2	125	8.75
Within	2100	147	14.29	
Total	2350	149		

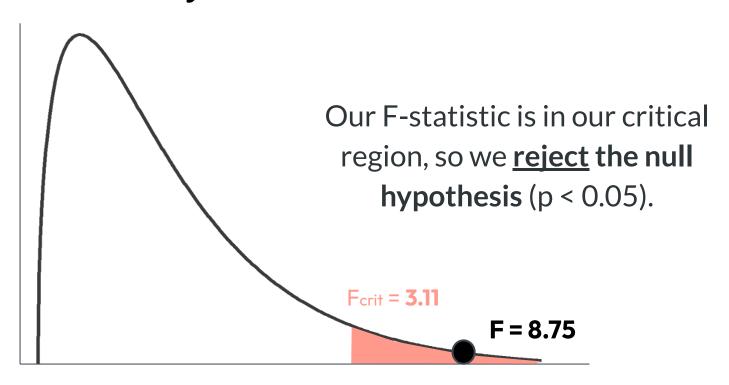
<u>Always</u> calculate your **effect size (η²)** as well.

$$\eta^2 = SS_{between}/SS_{total} = \frac{250}{2100} = 0.11$$





→ STEP 4: Make your decision.







Sophomores	Juniors	Seniors						
M = 6.5	M = 7.5	M = 7.1						
Tukey's HSD = 0.5								

Sophomores vs. Juniors: 7.5 - 6.5 = 1.0





Significant (more than 0.5)

Juniors vs. Seniors: 7.5 - 7.1 = 0.4 Not Significant (less than 0.5)





Sophomores vs. Seniors: 7.1 - 6.5 = 0.6





Significant (more than 0.5)





★ STEP 6: Report Results in APA style

A one-way ANOVA revealed a significant difference in stress levels amongst sophomores, juniors, and seniors, F(2,147) = 8.75 p < 0.05, η^2 = .11, with a **medium** effect size.

A **Tukey's HSD** test revealed that **juniors** reported significantly <u>higher</u> stress levels than **sophomores**, and **sophomores** reported significantly higher stress than **seniors**. However, stress levels in juniors and seniors did not significantly differ.

ANOVA Assumptions & Wrap Up

Four Assumptions for ANOVA

- Normality: The samples are drawn from a normally distributed population.
- Independence: Each sample is independent of the other samples.
- **Equal Variances:** The variance within each groups should be about the same.
- Continuous DV: The dependent variable must be interval/ratio.

While we will not learn to test these assumptions, you should know be able to <u>explain</u> why we need assumptions, and know that there are tests to test these assumptions.

Assumptions are necessary to ensure that the inferences we draw are sound and trustworthy.

In Lecture: Independent Samples t-test, Part 2, we discussed what each assumption means and how they help ensure that our conclusions from the sample will generalize appropriately to the population.

We make these assumptions for the same reasons we talked about in that lecture.

analyses.

Always run descriptive statistics and be critical when you are examining your data before analyses.

One group's sum of squares looks surprisingly large? That's a red flag. Could it be an outlier? A data entry mistake? Or maybe the group comes from a totally different population?

There's rarely a single "correct" answer — but your sharp eyes and critical thinking will reveal more than you realize.

analyses.

Key Takeaways

- Follow-up tests for one-way ANOVA
 - Why we need them
 - Types of follow-up tests
 - How to conduct a Tukey's HSD
- Research example
- ANOVA assumptions

