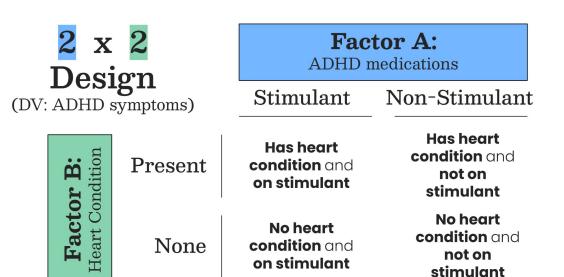
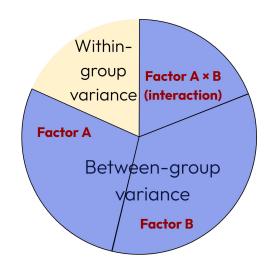
Factorial ANOVA (part 2)

Lecture 17 Emma Ning, M.A.

Recap: From our last class

• Factorial ANOVA (2*2, 2*3, ...)

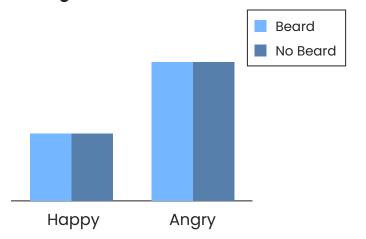




Recap: Visually identify main effects & interaction



"Eyeball" Method





"Table" Method

		Fact o Treat	Marginal Means	
		CBT	Mindful	Ivicalis
Factor B:	In-person	30	50	40
Modality	Online	40	60	50
Marginal Means		35	55	

Recap: Visually identify main effects & interaction

Going back to our example from last class...

If you're deciding which ADHD medication to recommend, just seeing an interaction between medication type and heart condition isn't enough.

You need to statistically test if your effect is real, like what we have been doing all along with NHST.

Нарру

Angry

TODAY'S PLAN









Learning objectives

- Conduct a two-way (two-factor) ANOVA, including measures of effect size, for both main effects and the interaction.
- Complete a two-way ANOVA **source table** (when given initial numbers).
- **Report and interpret** the results of a two-way ANOVA are reported in APA style.

01 Key Concepts

Let's think with what we already know...

Since our goal is to statistically test whether the main effects and interactions are real, we know based on our experience so far that:

- We need some kind of **statistic** to make **decisions** with!
 - In independent and paired t-tests, we used the t-statistic
 - In one-way ANOVA, we used the F-statistic

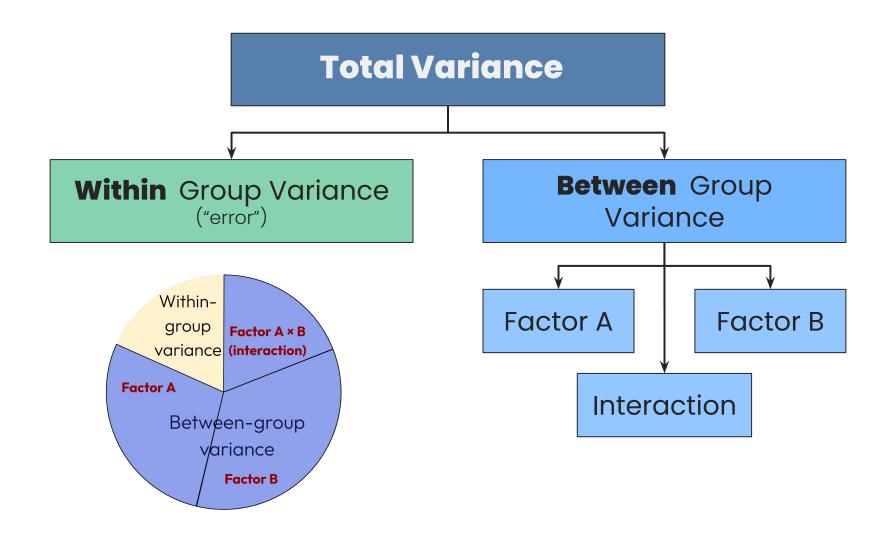
Okay, we are getting somewhere!
Since factorial ANOVA is just an extension of one-way ANOVA, we should probably still use the F-statistic

Factorial ANOVA is just an extension of One-way ANOVA

This is the one-way ANOVA source table:

Source	SS	df	MS	F
Between	Within- group variance	k – 1	$SS_{_b}/df_{_b}$	MS _b / MS _w
Within	Between-group variance	N – k	SS_w/df_w	
Total	SS _b + SS _w	N – 1		

These will probably change!

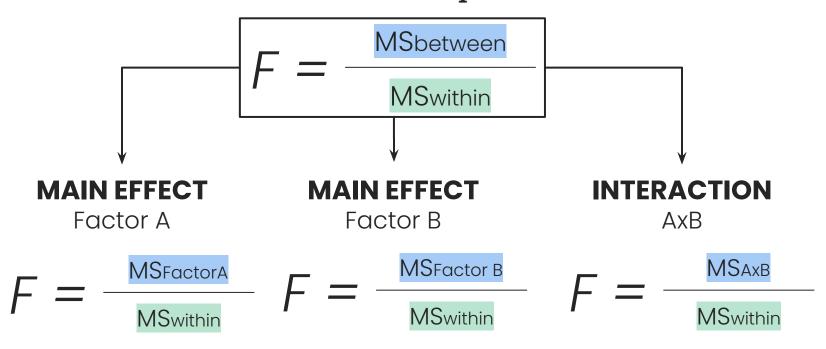


Total Variance



Three F-Ratios

Because of between-group variance is now **split into three**, we need to calculate **three separate F-ratios**

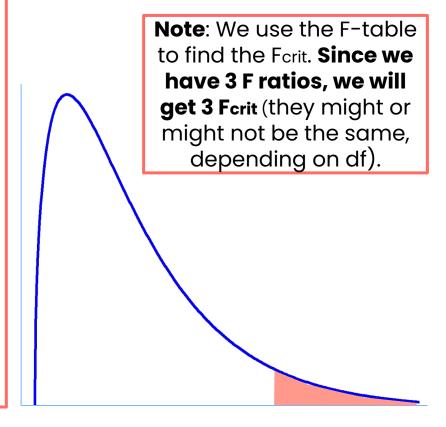


For *each* of the three F-ratios, we compare the calculated F-ratio to the F_{crit} to determine whether the main effects or interaction are "statistically significant."

This is like doing 3 separate NHST tests!
Our null: there is no effect
Our alternative: there is an effect

Use an F-Table to find your critical value (FCRIT)

df within	df between (numerator)									
(denominator)	1	2	3	4	5	6	7	8	9	10
1	161	200	216	225	230	234	237	239	241	242
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.70	2.64	2.59	2.55
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49



Effect Size

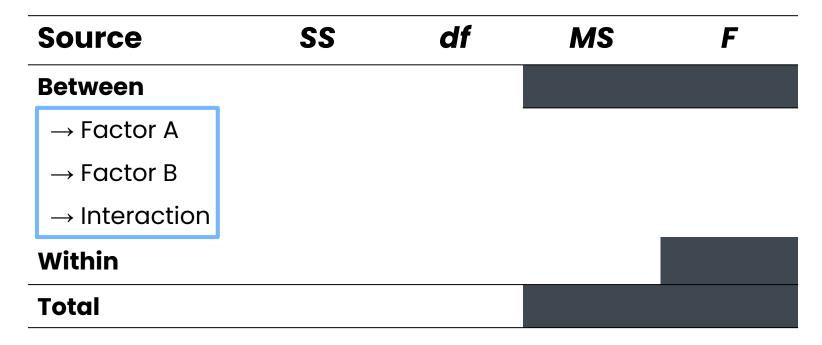
For a two-factor ANOVA, we compute <u>three</u> separate values for η^2 (eta-squared): one measuring the percentage of the variance explained by 1) the main effect for factor A, 2) the main effect for factor B, and 3) the interaction.

MAIN EFFECT Factor A Factor B INTERACTION

Factor B AXB

$$\eta_{p}^{2} = \frac{\text{SSA}}{\text{SSA} + \text{SSwithin}} \quad \eta_{p}^{2} = \frac{\text{SSB}}{\text{SSB} + \text{SSwithin}} \quad \eta_{p}^{2} = \frac{\text{SSAXB}}{\text{SSAXB} + \text{SSwithin}}$$

We call these *partial eta-squared* (η^2_p). There's also regular **eta-squared** (η^2). Since reporting one is sufficient, we won't go into the calculation for η^2 .



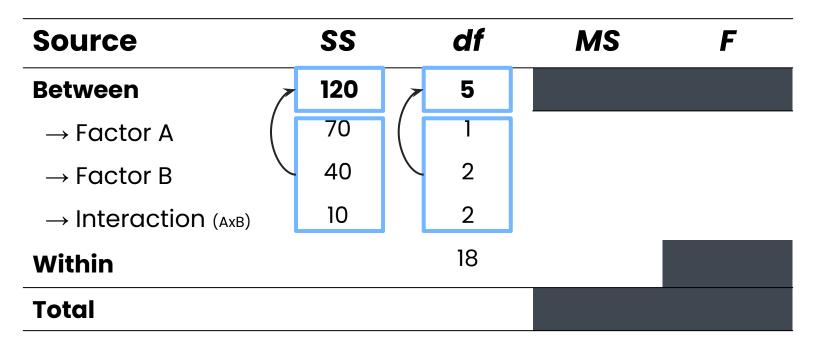
Our between group variance is now split into 3 parts

Source	SS	df	MS	F
Between		k – 1		
→ Factor A		k _A – 1		
→ Factor B		k _B – 1		
\rightarrow Interaction (AXB)		df _A x df _B		
Within		N – k		
Total		N - 1		

TIP 1: Start with df calculations (k = # of <u>cells</u>, k_A = # of levels for Factor A, k_B = # of levels for Factor B, N = <u>total</u> sample size)

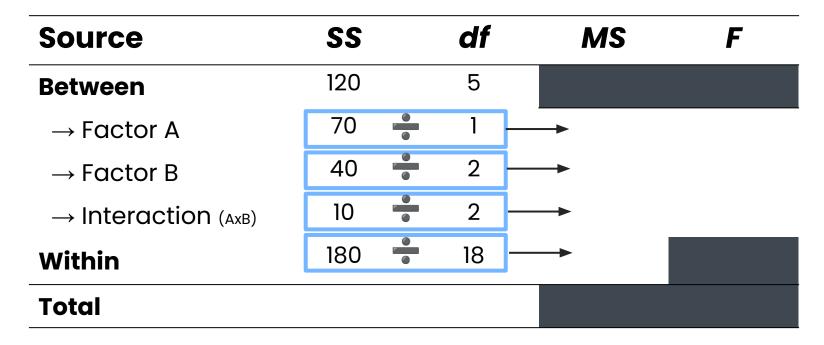
Source	SS		df	MS	F
Between		7	5		
→ Factor A			1		
→ Factor B		\	2		
→ Interaction (AXB)		L	2		
Within			18		
Total					

TIP 2:
$$\frac{\text{SSbetween} = SSA + SSB + SSAXB}{\text{dfbetween} = dfA + dfB + dfAXB}$$

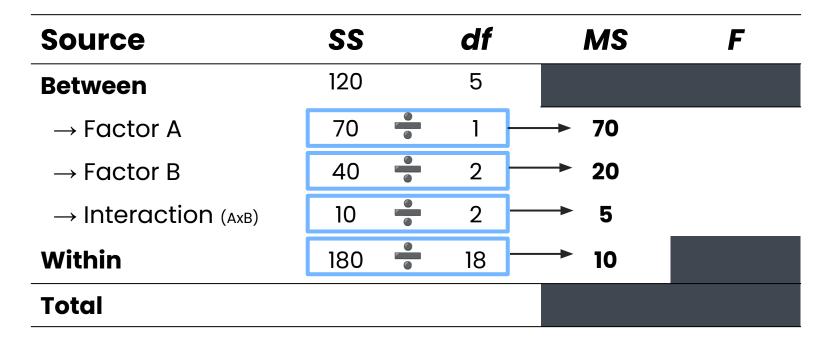


TIP 2:
$$\frac{\text{SSbetween} = SSA + SSB + SSAXB}{\text{dfbetween} = dfA + dfB + dfAXB}$$

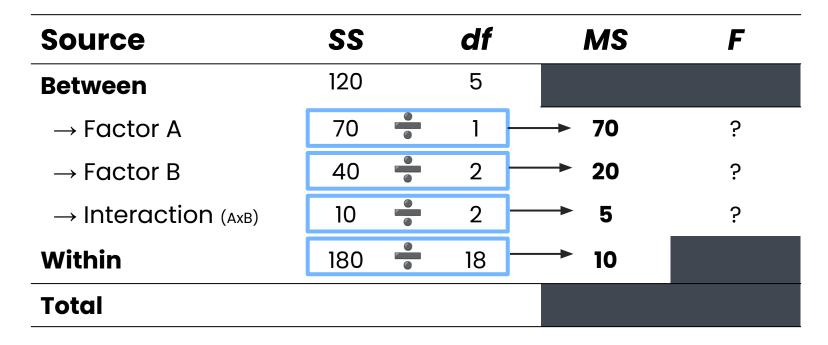
Source	SS	df	MS	F
Between	120	5		
→ Factor A	70	1		
→ Factor B	40	2		
\rightarrow Interaction (AXB)	10	2		
Within	180	18		
Total	300	23		



TIP 4: Divide across to get your four MS values.



TIP 4: **Divide across** to get your four MS values.



TIP 4: Divide across to get your four MS values.

Source	SS	df	MS	F
Between	120	5		MSA/MSB/MSAXB
→ Factor A	70	1	70	are your 3 numerators
→ Factor B	40	2	20	
→ Interaction (AXB)	10	2	5	
Within	180	18	10	MSwithin is your
Total				denominator

TIP 5: Divide MSA, MSB, and MSAxB by MSwithin (three calculations)

Source	SS	df	MS	F
Between	120	5		
→ Factor A	70	1	70	7
\rightarrow Factor B	40	2	20	2
\rightarrow Interaction (AXB)	10	2	5	0.5
Within	180	18	10	
Total				

TIP 5: Divide MSA, MSB, and MSAxB by MSwithin (three calculations)

Practice: Factorial ANOVA

Source	SS	df	MS	F
Between	75	3		
\rightarrow Factor A	50	1		
\rightarrow Factor B	20	1		
\rightarrow Interaction				
Within	500	100		
Total				

Practice: Factorial ANOVA

Source	SS	df	MS	F
Between	75	3		
→ Factor A	50	1	50	10
→ Factor B	20	1	20	4
\rightarrow Interaction	5	1	5	1
Within	500	100	5	
Total	575	103		

Example Problem

A psychologist wants to know if the effectiveness of learning format (video vs. podcast) depends on the age of the learner. They conduct a 2 (**Age**: Teens vs. Adults) × 2 (**Format**: Video vs. Podcast) factorial ANOVA. Their dependent variable (DV) is performance on a comprehension quiz about the material. The psychologist recruits 24 participants, evenly distributed across the 4 cells of the study design (6 participants per condition).

		Factor A: Age		
		Teens	Adults	
Factor B:	Video	85	65	
Format	Podcast	90	80	

Source	SS	df	MS	F
Between				
\rightarrow Factor A (age)	25			
\rightarrow Factor B (format)	75			
→ Interaction (AXB)	40			
Within	100			
Total				

Where shall we start?

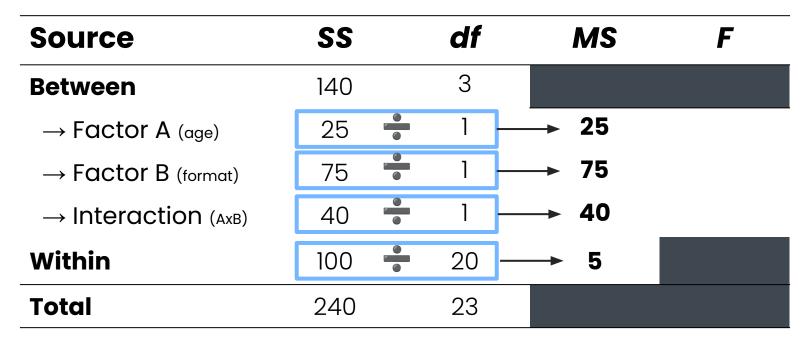
Source	SS	df	MS	F
Between		k - 1		
\rightarrow Factor A (age)	25	k _A - 1		
$\to Factor\ B\ ({\sf format})$	75	k _B - 1		
\rightarrow Interaction (AXB)	40	df _A x df _B		
Within	100	N - k		
Total		N - 1		

Start with **df** calculations (**k** = # of <u>cells</u>, **k**_A = # of levels for Factor A, **k**_B = # of levels for Factor B, **N** = <u>total</u> sample size)

Source	SS	df	MS	F
Between		3		
\rightarrow Factor A (age)	25	1		
\rightarrow Factor B (format)	75	1		
→ Interaction (AXB)	40	1		
Within	100	20		
Total		23		

Start with **df** calculations (**k** = # of <u>cells</u>, **k**_A = # of levels for Factor A, **k**_B = # of levels for Factor B, **N** = <u>total</u> sample size)

Source	SS	df	MS	F
Between	140	3		
\rightarrow Factor A (age)	25	1		
\rightarrow Factor B (format)	75	1		
→ Interaction (AXB)	40	1		
Within	100	20		
Total	240	23		
SSbetween = SSA + SS	SB + SSAXB	SStote	al = SSbetwee	en + SSwithi
SS between = $25 + 75$	+ 40 = 140	SStote	al = 140 + 10	0 = 240



Divide across to get your four MS values.

Source	SS	df	MS F	
Between	140	3	numerat	ors
\rightarrow Factor A (age)	25	1	25	
\rightarrow Factor B (format)	75	1	75	
\rightarrow Interaction (AXB)	40	1	40	
Within	100	20	5	
Total	240	23	denomin	ator

Divide MSA, MSB, and MSAxB by MSwithin (three calculations)

Worked Example

Source	SS	df	MS	F
Between	140	3		
\rightarrow Factor A (age)	25	1	25	5
\rightarrow Factor B (format)	75	1	75	15
\rightarrow Interaction (AXB)	40	1	40	8
Within	100	20	5	
Total	240	23		

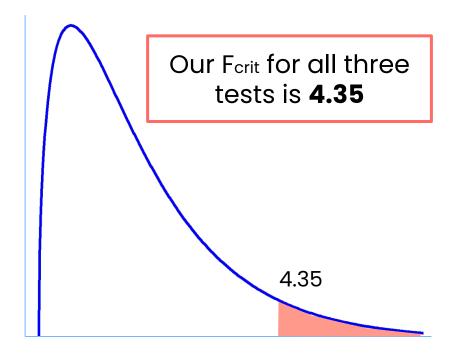
Worked Example

Source	SS	df	MS	F
Between	140	3		
\rightarrow Factor A (age)	25	1	25	5
\rightarrow Factor B (format)	75	1	75	15
→ Interaction (AXB)	40	1	40	8
Within	100	20	5	
Total	240	23	Poor	nuse dfa d

Because dfa, dfb, dfaxb **all equal 1**, we will have the same Fcrit for all three F-ratios.

$\mathbf{F}_{ ext{crit}}$

df within	df between (numerator)									
(denominator)	1	2	3	4	5	6	7	8	9	10
1	161	200	216	225	230	234	237	239	241	242
2	18.51	9.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78
4	7.71	5.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63
8	5.32	1.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.70	2.64	2.59	2.55
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55	2.50	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.55	2.48	2.43	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.52	2.45	2.40	2.35



For *each* of the three F-ratios, we compare the calculated F-ratio to the F_{crit} to determine whether the main effects or interaction are "statistically significant."

Make Your Decisions

 $F_{crit} = 4.35$

Source	SS	df	MS	F
Between	140	3		
\rightarrow Factor A (age)	25	1	25	5
\rightarrow Factor B (format)	75	1	75	15
\rightarrow Interaction (AXB)	40	1	40	8
Within	100	20	5	
Total	240	23		

Main Effect (Factor A)
5 > 4.35
significant

Main Effect (Factor B)
15 > 4.35
significant

Interaction 8 > 4.35 significant

Effect Size

Source	SS	df	MS	F
Between	140	3		
\rightarrow Factor A (age)	25	1	25	5
\rightarrow Factor B (format)	75	1	75	15
\rightarrow Interaction (AXB)	40	1	40	8
Within	100	20	5	
Total	240	23		

Source	SS
Between	140
→ Factor A (age)	25
→ Factor B (format)	75
→ Interaction (AXB)	40
Within	100
Total	240

Effect Size

MAIN EFFECT

Factor A

$$\eta^2_{p} = \frac{\text{SSA}}{\text{SSA} + \text{SSwithin}}$$

$$\eta^2_{p} = \frac{25}{25 + 100}$$

$$= .20$$

MAIN EFFECT

Factor B

$$\eta^2_{p} = \frac{\text{SSB}}{\text{SSB} + \text{SSwithin}}$$

$$\eta^2_{p} = \frac{75}{75 + 100}$$

$$= .43$$

INTERACTION

AxB

$$\eta^2_{\ p} = rac{ extsf{SSaxB}}{ extsf{SSaxB}} + extsf{SSwithir}$$

$$\eta^2_{p} = \frac{40}{40 + 100}$$

Interpretation Template

(example for **factor A**)

$$F(1, 20) = 5.00, p < .05, \eta^2 = .20$$

$$f(1, 20) = 5.00, p < .05, \eta^2 = .20$$

Interpretation Template

A _ × _ factorial ANOVA was conducted to examine the effects of **[Factor A]** and **[Factor B]** on **[DV]**.

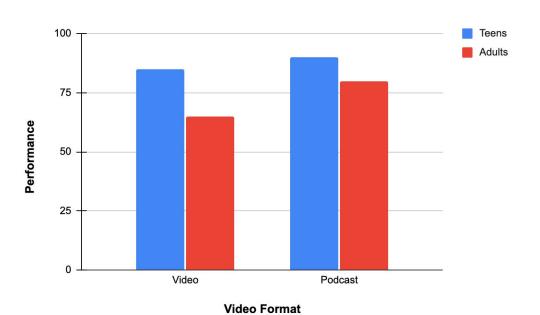
There [was/was not] a main effect of **[Factor A]**, F(?,?) = ?, p (< or >) .05 and [was/was not] main effect of **[Factor B]**, F(?,?) = ?, p (< or >) .05.

There [was/was not] a significant **interaction** between [Factor A] and [Factor B], F(?,?) = ?, p (< or >) .05

Interpretation

A **2 × 2** factorial ANOVA was conducted to examine the effects of *age* and *learning format* on *comprehension scores*. There was a significant **main effect of age**, F(1, 20) = 5.00, p < .05 and a significant **main effect of format**, F(1, 20) = 15.00, p < .05. There was also a significant **interaction** between age and format, F(1, 20) = 8.00, p < .05.

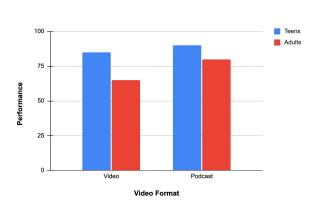
Means Plot



Interpretation, plain English

<u>If</u> there is a significant interaction, you should <u>only</u> write a plain-English interpretation when a means plot is provided. Without the plot, simply report the result in APA format (as shown on the previous slide).

Interactions are tricky to interpret without visualizing them, so only interpret them when you can see the means plot.



Overall, participants performed better when listening to the podcast than when watching the video, and teens performed better than adults. However, the effect of format depended on age: teens' performance stayed about the same across formats, whereas adults performed worse when switching from podcast to video.

ICA 17

You conduct a study with a 2 x 2 factorial design examining the effect of personality (factor A) and caffeine (factor B) on mood. You split group $\bf N=84$ participants into groups of 21 per cell/condition. (assume $\alpha=.05$)

Using the formulas and the tips you learned, complete the table, calculate the effect size, and state decide whether there is a ...

 Main effect for factor A (pers) Main effect for factor B (caff) 	sonality)? eine)?		Facto Person	
• Interaction?			Introvert	Extravert
	Factor B:	0mg	25	35
	Coffeire			

Jameine

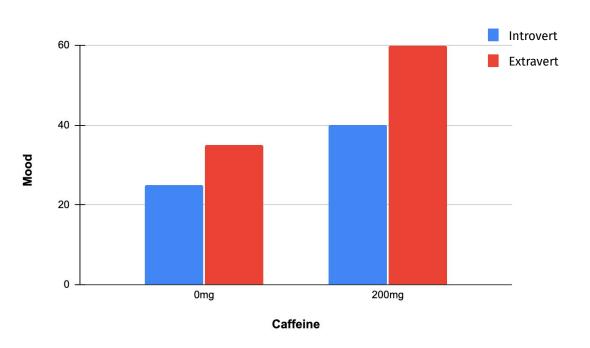
200mg

40

60

From our group means, we also get the means plot

		Facto Person	
		Introvert	Extravert
Factor B:	0mg	25	35
Caffeine	200mg	40	60



ICA 17 Source Table

Source	SS	df	MS	F
Between	100			
\rightarrow Factor A (personality)	10	1		
\rightarrow Factor B (caffeine)	50	1		
→ Interaction (AXB)			_	
Within	400	80		
Total		83		

df within			C	if betv	veen (numer	ator)				df within				df be	tween	(nume	erator)			
(denominator)	1	2	3	4	5	6	7	8	9	10	(denominator)	1	2	3	4	5	6	7	8	9	10
1	161	200	216	225	230	234	237	239	241	242	26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39	27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.30	2.25	2.20
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78	28	4.20	3.34	2.95	2.71	2.56	2.44	2.36	2.29	2.24	2.19
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	29	4.18	3.33	2.93	2.70	2.54	2.43	2.35	2.28	2.22	2.18
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74	30	4.17	3.32	2.92	2.69	2.53	2.42	2.34	2.27	2.21	2.16
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	32	4.15	3.30	2.90	2.67	2.51	2.40	2.32	2.25	2.19	2.14
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63	34	4.13	3.28	2.88	2.65	2.49	2.38	2.30	2.23	2.17	2.12
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34	36	4.11	3.26	2.86	2.63	2.48	2.36	2.28	2.21	2.15	2.10
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13	38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97	40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.07
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86	42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.06
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76	44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67	46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.14	2.09	2.04
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60	48	4.04	3.19	2.80	2.56	2.41	2.30	2.21	2.14	2.08	2.03
15	4.54	3.68	3.29	3.06	2.90	2.79	2.70	2.64	2.59	2.55	50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.02
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10	2.04	1.99
17	4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55	2.50	2.45	70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.01	1.97
19	4.38	3.52	3.13	2.90	2.74	2.63	2.55	2.48	2.43	2.38	80	3.96	3.11	2.72	2.48	2.33	2.21	2.12	2.05	1.99	1.95
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	100	3.94	3.09	2.70	2.46	2.30	2.19	2.10	2.03	1.97	1.92
22	4.30	3.44	3.05	2.82	2.66	2.55	2.47	2.40	2.35	2.30	200	3.89	3.04	2.65	2.41	2.26	2.14	2.05	1.98	1.92	1.87
23	4.28	3.42	3.03	2.80	2.64	2.53	2.45	2.38	2.32	2.28	400	3.86	3.02	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85
24	4.26	3.40	3.01	2.78	2.62	2.51	2.43	2.36	2.30	2.26	1,000	3.85	3.00	2.61	2.38	2.22	2.10	2.02	1.95	1.89	1.84
 25	4.24	3.38	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24	∞	3.84	2.99	2.60	2.37	2.21	2.09	2.01	1.94	1.88	1.83

ICA 17 Source Table

Source	SS	df	MS	F
Between	100	3		
\rightarrow Factor A (personality)	10	1	10	2 🕱
\rightarrow Factor B (caffeine)	50	1	50	10
→ Interaction (AXB)	40	1	40	8
Within	400	80	5	
Total	500	83		

Our F_{crit} for all three tests is **3.96**

Source	SS
Between	100
→ Factor A (personality)	10
→ Factor B (caffeine)	50
→ Interaction (AXB)	40
Within	400
Total	500

Effect Size

MAIN EFFECT

Factor A

Factor B

AxB

$$\eta^2_{p} = \frac{\text{SSA}}{\text{SSA} + \text{SSwithin}}$$

$$\eta^2_{p} = \frac{\text{SSB}}{\text{SSB} + \text{SSwithin}}$$

$$\eta^2_{p} = \frac{\text{SSaxb}}{\text{SSaxb}} + \text{SSwithin}$$

$$\eta^2_{p} = \frac{10}{10 + 400}$$

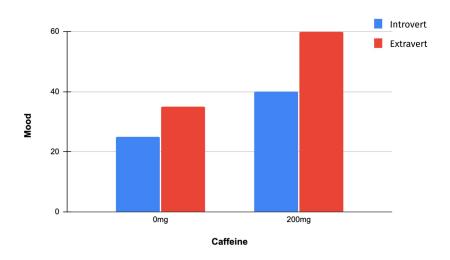
$$= .02$$

$$= .11$$

Interpretation

A **2 × 2** factorial ANOVA was conducted to examine the effects of *personality* and *caffeine* on *mood*. There was a significant **main effect of caffeine**, F(1, 80) = 10.00, p < .05, but no significant **main effect of personality**, F(1, 80) = 2.00, p > .05. There was a significant **interaction** between personality and mood, F(1, 80) = 8.00, p < .05.

Interpretation, plain English



Overall, participants reported better moods when they consumed caffeine, and extraverts tended to have higher moods than introverts, though the difference was not significant. However, caffeine **boosted mood more strongly** for extraverts than for introverts.

Exam 2 Information & Wrap Up

SOME EXAM 2 KEY TOPICS

Make sure you know these concepts well!

Independent-samples t-test
Paired-samples t-test
One-way ANOVA
2-way (factorial) ANOVA

Calculation of t-statistic &/ F-ratio

Calculation of effect size & 95% CI

Interpretation

Type 1 & Type 2 errors, Power

Between vs within subjects design & pros & cons

One-way ANOVA follow-ups Interaction interpretation

Note: see learning objectives for each lecture as a "study guide"

EXAM 2 INFORMATION & FORMAT

- Exam will cover content on lectures 9 17
- Exam 2 will occur in class and will last **75 minutes**
- You are allowed one double-sided <u>hand-written</u> page of notes
- It will be mostly short-answer questions
- You will be given a formula sheet and t-table & F-table
- You should **show all your work**, including formulas
- Round to <u>two</u> decimal places (e.g., 10.21, 120.43)

Bring your calculator!!!

Announcements

- Shelby is giving this Thursday's lecture!
 - She will give you lots of examples on menopause
- No lab this Friday, I will post the practice exam on Friday, you can work on it on your own, and the following Tuesday's lecture is for Q&A
- No homework this week. Focus on treating the Practice Exam as an actual exam