Linear Regression Example

The example below uses a marketing dataset, in order to illustrate a linear regression activity.

Workflow:

1. Preparation
   A. Load the dataset from a `.csv` file and show a short description
   B. set the `random_state` variable to make the experiment *repeatable*
   C. Inspect and eliminate rows with nulls
   D. Use the background information provided by the data owners
      a. create new derived columns
      b. drop the column declared by the experts as non interesting
      c. set the target and split the data into *predicting variables* `X` and *target* `y`
   E. Data exploration
      a. Show the two dimensional scatter plots for all the predicting variables with respect to the target
      b. Show the *p-values* of the target with respect to the variables (it is the probability that a variable has zero coefficient in a linear regression)
2. First experiment: compute the regression on a single predicting variable
   A. Consider a reduced dataset containing the chosen variable and the target
   B. Fit the `LinearRegression` estimator on the training set
   C. Show the statistical significance of the fitted model
   D. Predict the target for the test set using the *fitted* estimator
   E. Compute the regression coefficients and the quality measures: *Root Mean Squared Error (RMSE)* and *coefficient of determination (r2)*
3. Second experiment: compute the regression considering all the predicting variables
   A. Repeat the steps from 2.2 to 2.5
4. Third experiment: use the `DecisionTreeRegressor` with the entire dataset
   A. Fit the tree using the default hyperparameters, in order to find the maximum depth of the unconstrained tree
   B. Use *cross-validation* to find the optimal *maximum depth* of the tree
   C. Fit the tree with the optmal `max_depth`
   D. Predict and show the *root mean squared error*

5. Fourth experiment: use the `RandomForestRegressor`
   A. Repeat steps from 4.2 to 4.4 (for simplicity, we use the maximum `max_depth` found in 4.1)

In [1]:
```
# Code source: Claudio Sartori
# License: BSD 3 clause

import matplotlib.pyplot as plt
import numpy as np
import scipy.stats
from sklearn import datasets, linear_model
from sklearn.metrics import mean_squared_error, r2_score
import pandas as pd
random_state = 94922767 # this will be used to guarantee the repeatability of the experiment
```

Load the dataset from a `.xlsx` file and show a short description

In [2]:
```
# This cell allows full compatibility between execution in Google Colab and in local
try:
  import google.colab.files
  IN_COLAB = True
except:
  IN_COLAB = False
# from google.colab import files
if IN_COLAB:
    uploaded = files.upload()
```

In [3]:
```
# The file must be available in the same directory,
# or uploaded in the Colab environment
# in the execution of the previous cell
data_fn = 'FoodUK2014.xlsx'
# to fill
```

Show a short description of the columns

In [4]:
```
# to fill
```

Out[4]:

	hhsize	quarter	adults_n	children_n	totalexp	SexHRP	month	Gorx	Year	income	AgeHRP	qmeat
count	5114.000000	5114	5114.000000	5114.000000	5114.000000	5114	5114	5114	5114.0	5114.000000	5114.000000	4873.000000
unique	NaN	4	NaN	NaN	NaN	2	12	12	NaN	NaN	NaN	NaN
top	NaN	April to June	NaN	NaN	NaN	Male	February	South East	NaN	NaN	NaN	NaN
freq	NaN	1341	NaN	NaN	NaN	3050	445	736	NaN	NaN	NaN	NaN
mean	2.363707	NaN	1.841807	0.521901	519.898868	NaN	NaN	NaN	2014.0	679.542002	53.802698	10.475023
std	1.244704	NaN	0.743052	0.945622	411.543093	NaN	NaN	NaN	0.0	499.596175	16.187912	8.798118
min	1.000000	NaN	0.000000	0.000000	-246.916821	NaN	NaN	NaN	2014.0	0.000000	17.000000	0.086667
25%	1.000000	NaN	1.000000	0.000000	260.598783	NaN	NaN	NaN	2014.0	306.954000	41.000000	4.452500
50%	2.000000	NaN	2.000000	0.000000	426.977227	NaN	NaN	NaN	2014.0	548.086000	54.000000	8.374167
75%	3.000000	NaN	2.000000	1.000000	651.003763	NaN	NaN	NaN	2014.0	925.652500	67.000000	14.005333
max	9.000000	NaN	7.000000	7.000000	5859.877186	NaN	NaN	NaN	2014.0	2134.090000	80.000000	104.589333

Show the number of rows with nulls

It is computed subtracting the number of rows in the dataset without nulls from the original number of rows

In [5]:
```
# to fill
```

Out[5]:
```
1668
```

Drop rows with nulls

In [6]:
```
# to fill
```

```
After dropping rows with nulls the dataset has 3446 rows
```

Data transformation

- Convert the alphanumeric SexHRP into numeric 0 and 1
  - the `sklearn` machine learning procedures work only with numeric predicting attributes
  - this can be done, for example with the `map` function of pandas series

- Generate two new columns as ratio of other columns
  - 'qmeat_hhsize_ratio' = 'qmeat'/'hhsize'
  - 'income_hhsize_ratio' = 'income'/'hhsize'
  - this is suggested by background information

In [7]:
```
# to fill
```

Use only the columns that the experts consider interesting

['adults_n', 'children_n', 'SexHRP', 'AgeHRP', 'qmeat_hhsize_ratio', 'income_hhsize_ratio', 'uvmeat']

This is suggested by background information

In [9]:
```
# to fill
```

Out[9]:

	adults_n	children_n	SexHRP	AgeHRP	qmeat_hhsize_ratio	income_hhsize_ratio	uvmeat
1	2	2	1	38	1.511250	206.130000	8.813621
2	2	0	1	54	5.890083	135.962500	7.965790
4	3	0	1	64	4.285667	165.346667	5.726323
5	2	2	1	70	8.968250	66.632500	8.451528
7	3	0	1	64	4.079111	134.393333	5.904745

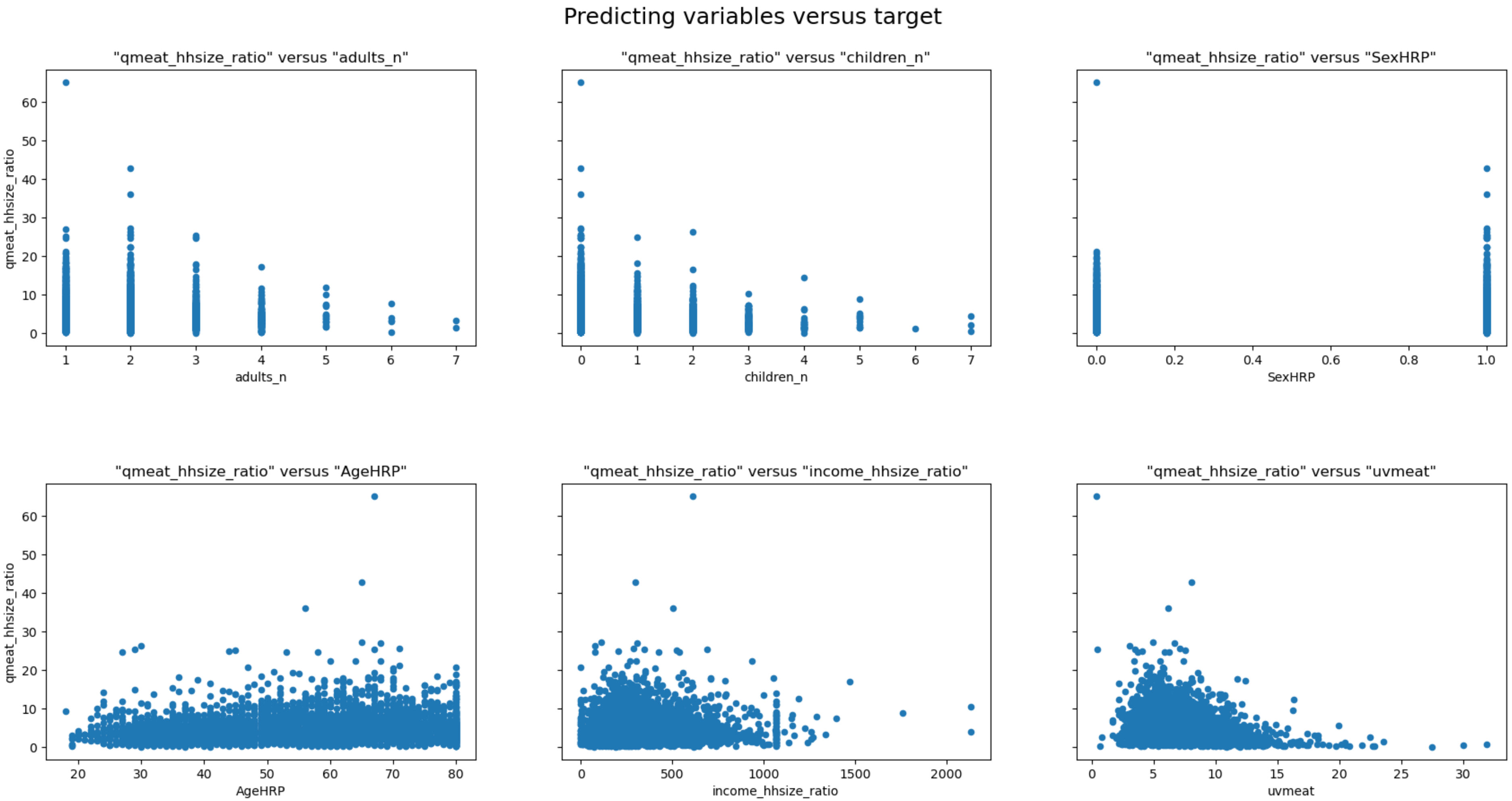
Choose the target and split the data into *predicting variables* X and *target* y

In [10]:
```
target = 'qmeat_hhsize_ratio'
# to fill
```

Show the two dimensional scatter plots for all the predicting variables with respect to the target

Don't worry if your display is slightly different

In [12]:
```
# to fill
```



Show the *p-values* of the target with respect to the variables

you can use the `f_regression` function of Scikit Learn, that returns coefficients and p-values

In [13]:
```
# to fill
```

Out[13]:

	Variable	p-value
0	adults_n	1.415945e-05
1	children_n	1.077386e-30
2	SexHRP	8.429827e-02
3	AgeHRP	1.710126e-21
4	income_hhsize_ratio	1.211099e-03
5	uvmeat	4.789746e-52

In [14]:

```
```

Out[14]:

```
array([ 18.90194414, 135.25964933,   2.981742  ,  91.87247553,
        10.49080033, 238.35533077])
```

Split the data into *train* and *test* and show the sizes of the two parts

Here we set the `random_state` variable to make the experiment *repeatable*

In [15]:

```
# to fill
```

```
Training set and test set have 2412 and 1034 elements respectively
```

Consider a reduced dataset containing the chosen variable and the target

First experiment: Univariate regression

Fit the `linear_model` estimator on the training set and predict the target for the test set using the *fitted* estimator

In [18]:

```
# Create linear regression object
# to fill

# Train the model using the training set
# to fill
```

```
# Make predictions using the test set
# to fill
```

Compute the regression coefficients and the quality measures

- use the attributes `coef_[0]` and `intercept_` of the fitted model
- use functions `mean_squared_error` and `r2_score`

In [ ]:

```
# The coefficient
# to fill

# The root mean squared error
# to fill

# Coefficient of determination = 1 is perfect prediction
# to fill
```

Prepare a `result_summary`

Create a dataframe `result_summmary` with columns `model`, `rmse` and `r2` and store at the end a row with the results for `Univariate Linear`

In [20]:

```
# to fill
```

Second experiment: compute the regression considering all the predicting variables

Now we use the entire data in `X_train` and `X_test` for fitting and predicting

In [21]:

```
# Create linear regression object
# to fill

# Train the model using the training set
# to fill
```

```
# Make predictions using the test set
```

Fit, predict and show the results

Now we see the *regression coefficients* resulting from the fitting.

In particular, *positive coefficients* indicate that the target *increases* with the variable, *negative coefficients* indicate a *decreasing* trend.

The absolute values of the coefficient cannot be considered directly a measure of importance, due to the possibly different orders of magnitude of the data in the different columns (observe above the outputs of `describe`).

In [22]:

```
# Show the coefficients of the predicting variables
# to fill
```

Out[22]:

	Variable	Coefficient
0	adults_n	-0.318682
1	children_n	-0.650924
2	SexHRP	0.383162
3	AgeHRP	0.014913
4	income_hhsize_ratio	0.000989
5	uvmeat	-0.392620

Compute the quality measures

use functions `mean_squared_error` and `r2_score` and store them in the DataFrame `result_summmary` and store the results for `Multivariate Linear` at the end of the `results_summary`

In [23]:

```
# The mean squared error
# to fill

# Coefficient of determination=1 is perfect prediction
# to fill

# update result_summary and show
# to fill
```

Out[23]:

	Model	rmse	r2
0	Univariate Linear	3.886323	0.007595
1	Multivariate Linear	3.665149	0.117338

Decision Tree Multivariate Regresson

In [24]:

```
# Create Decision Tree regression object
from sklearn.tree import DecisionTreeRegressor
```

Fit the tree with default hyperparameters, and find the maximum depth of the unconstrained tree

In [25]:

```
# to fill

print("The maximum depth of the full Decision Tree Regressor is {}".format(max_max_depth))
```

```
The maximum depth of the full Decision Tree Regressor is 34
```

Find the optimal value of the hyperparameter `max_depth` with *cross-validation*

The optimization searches for the *maximum tree depth* guaranteing the smallest mean squared error At the end, this operation returns also the *fitted best tree* `best_estimator_`

In [26]:

```
# to fill
```

```
The optimal maximum depth for the decision tree is 2
```

Compute rmse and append to result summary

For the decision tree regressor the `r2_score` is not significant

In [27]:

```
# Make predictions using the test set
# to fill

# update and show the result_summary
# to fill
```

Out[27]:

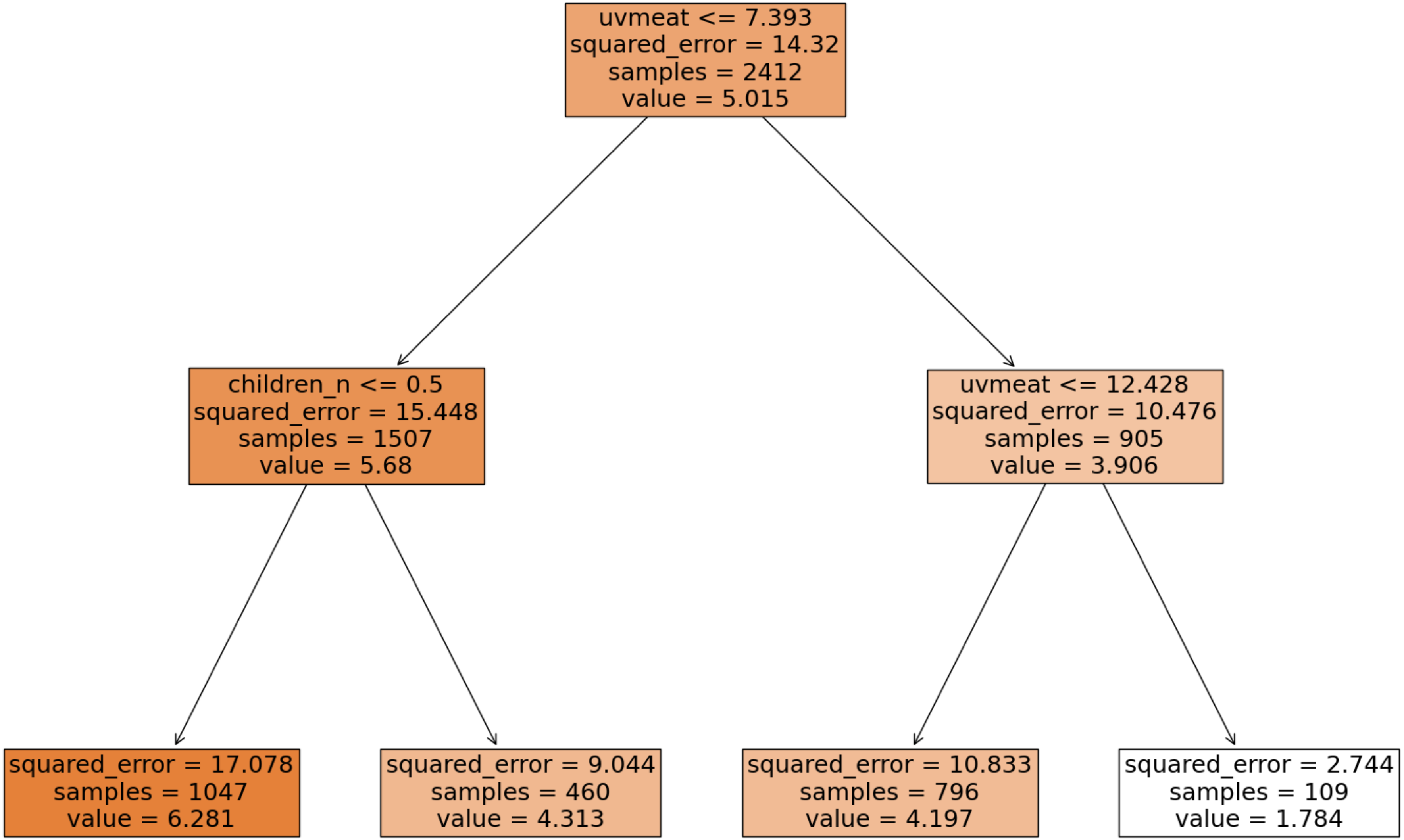
	Model	rmse	r2
0	Univariate Linear	3.886323	0.007595
1	Multivariate Linear	3.665149	0.117338
2	Decision Tree Regression	3.790020	NaN

Show the tree

use `plot_tree`

In [28]:

```
from sklearn.tree import plot_tree
from matplotlib.pyplot import figure
# to fill
```


uvmeat <= 7.393
squared_error = 14.32
samples = 2412
value = 5.015
children_n <= 0.5
squared_error = 15.448
samples = 1507
value = 5.68
uvmeat <= 12.428
squared_error = 10.476
samples = 905
value = 3.906
squared_error = 17.078
samples = 1047
value = 6.281
squared_error = 9.044
samples = 460
value = 4.313
squared_error = 10.833
samples = 796
value = 4.197
squared_error = 2.744
samples = 109
value = 1.784

Random Forest Multivariate Regresson

In [29]:

```
# Create Random Forest regression object
from sklearn.ensemble import RandomForestRegressor
# to fill

# for simplicity, we use as a maximum maximum depth of the tree the value found in
# the unconstrained decision tree fitting
param_grid_rf = {'max_depth': list(range(1,max_max_depth))
}
# create the grid search with cross validation
# look for minimum mean square error
# to fill

# Train the model using the training set
# to fill

# the grid search returns the best estimator, store it in a variable for later use
# to fill
```

In [30]:

```
print("The optimal maximum depth for the trees in the random forest is {}".format(rf.max_depth))
```

```
The optimal maximum depth for the trees in the random forest is 4
```

In [31]:

```
# predict, compute measures, store in result_summary and show
# to fill
```

Out[31]:

	Model	rmse	r2
0	Univariate Linear	3.886323	0.007595
1	Multivariate Linear	3.665149	0.117338
2	Decision Tree Regression	3.790020	NaN
3	Random Forest Regression	3.577735	NaN

Final observations

Linear regression

The multivariate regression with all the predicting variables available with respect to the univariate regression has

- lower RMSE
- higher coefficient of determination
- the p-value suggests the acceptance of both models ### Decision Tree and Random Forest regression
- Decistion Tree has an RMSE slightly higher than multivariate linear regression
- Random Forest has an RMSE slightly lower than multivariate linear regression

Control questions

1. observing the multi-variate experiment, what variable has the higher effect on the target?
2. is there a variable having an almost negligible effect on the target?
3. try to repeat the univariate experiment with one of the other columns at a time and comment the results