Project 1 - FYS3150

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Project 1a)

$$
\begin{aligned}
&-\frac{u_{i+1}+u_{i-1}-2u_i}{h^2}=f_i\\
&-(u_{i+1}+u_{i-1}-2u_i)=f_ih^2\\
&2u_i-u_{i+1}-u_{i-1}=f_ih^2
\end{aligned}
$$

This expands to

$$
\begin{aligned}
2u_1-u_0-u_2&=f_1h^2\\
2u_2-u_1-u_2&=f_2h^2\\
&\vdots\\
2u_n-u_{n-1}-u_n+1&=f_3h^2
\end{aligned}
$$

The boundary conditions give us $u_{n+1}=u(1)=0$ and $u_0=u(0)=0$. We also introduce $f_ih^2=g_i$. We can then write this expression as

$$
\begin{bmatrix}
2 & -1 & 0 & \dots & 0 & 0\\
-1 & 2 & -1 & \dots & 0 & 0\\
0 & -1 & 2 & \dots & 0 & 0\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\
0 & 0 & 0 & \dots & 2 & -1\\
0 & 0 & 0 & \dots & -1 & 2
\end{bmatrix}
\begin{bmatrix}
u_1\\ u_2\\ u_3\\ \vdots\\ u_{n-1}\\ u_n
\end{bmatrix}
=
\begin{bmatrix}
g_1\\ g_2\\ g_3\\ \vdots\\ g_{n-1}\\ g_n
\end{bmatrix}
$$

Project 1b)

$$b_1u_1+c_1u_2=\tilde{g}_1 \tag{1}$$
$$a_1u_1+b_2u_2+c_2u_3=\tilde{g}_2 \tag{2}$$
$$a_2u_2+b_3u_3+c_3u_4=\tilde{g}_3 \tag{3}$$
$$\vdots$$
$$a_{n-1}v_{n-1}+a_nv_n=\tilde{g}_n \tag{4}$$

Multiplying equation (1) with $\frac{a_1}{b_1}$, wich gives us.

$$a_1u_1+\tfrac{a_1c_1}{b_1}u_2=\tilde{g}_1\tfrac{a_1}{b_1}$$

We then set equation (2) minus equation (1)

$$\begin{aligned}
a_1u_1 - a_1u_1 + b_2u_2 - \frac{a_1c_1}{b_1}u_2 + c_2u_3 &= g_2 - g_1\frac{a_1}{b_1} \\
\left(b_2 - \frac{a_1c_1}{b_1}\right)u_2 + c_2u_3 &= g_2 - g_1\frac{a_1}{b_1} \\
\tilde{b}_2u_2 + c_2u_3 &= \tilde{g}_2
\end{aligned}$$

The general expressions is

$$\tilde{b}_i = b_i - \frac{c_{i-1}a_{i-1}}{\tilde{b}_{i-1}}, \qquad\qquad \tilde{g}_i = g_i - g_{i-1}\frac{a_{i-1}}{\tilde{b}_{i-1}}$$

Where $\tilde{b}_1 = b_1$ and $\tilde{g}_1 = g_1$