

Project 1 - FYS3150

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Project 1a)

$$\begin{aligned} -\frac{u_{i+1} + u_{i-1} - 2u_i}{h^2} &= f_i \\ -(u_{i+1} + u_{i-1} - 2u_i) &= f_i h^2 \\ 2u_i - u_{i+1} - u_{i-1} &= f_i h^2 \end{aligned}$$

This expands to

$$\begin{aligned} 2u_1 - u_0 - u_2 &= f_1 h^2 \\ 2u_2 - u_1 - u_3 &= f_2 h^2 \\ &\vdots \\ 2u_n - u_{n-1} - u_{n+1} &= f_n h^2 \end{aligned}$$

The boundary conditions give us $u_{n+1} = u(1) = 0$ and $u_0 = u(0) = 0$. We also introduce $f_i h^2 = g_i$. Then we can write this expression as

$$\begin{bmatrix} 2 & -1 & 0 & \dots & 0 & 0 \\ -1 & 2 & -1 & \dots & 0 & 0 \\ 0 & -1 & 2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 2 & -1 \\ 0 & 0 & 0 & \dots & -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_{n-1} \\ u_n \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ \vdots \\ g_{n-1} \\ g_n \end{bmatrix}$$

Project 1b)