Project 2

Benedicte Allum Pedersen, Emil Heland Broll
Fredrik Oftedal Forr

In this project we will have a look at some eigenvalueproblems with numerical calculations.

Project 2a)

We have for three dimentions

$$\hat{U} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix} \qquad \mathbf{v}_i = \begin{bmatrix} v_{i1} \\ v_{i2} \\ v_{i3} \end{bmatrix}$$

This gives us

$$\mathbf{w}_i = \hat{U}\mathbf{v}_i$$
$$\begin{bmatrix} w_{i1} \\ w_{i2} \\ w_{i3} \end{bmatrix} = \begin{bmatrix} u_{11}v_{i1} + u_{12}v_{i2} + u_{13}v_{i3} \\ u_{21}v_{i1} + u_{22}v_{i2} + u_{23}v_{i3} \\ u_{31}v_{i1} + u_{32}v_{i2} + u_{33}v_{i3} \end{bmatrix}$$

If we then do the same for $\mathbf{w}_j$ we get

$$\mathbf{w}_j = \hat{U}\mathbf{v}_i$$
$$\begin{bmatrix} w_{j1} \\ w_{j2} \\ w_{j3} \end{bmatrix} = \begin{bmatrix} u_{11}v_{j1} + u_{12}v_{j2} + u_{13}v_{j3} \\ u_{21}v_{j1} + u_{22}v_{j2} + u_{23}v_{j3} \\ u_{31}v_{j1} + u_{32}v_{j2} + u_{33}v_{j3} \end{bmatrix}$$

wtf................................

If $v_j^T v_i = \delta_{ij}$, where, v_i is an orthogonal basis. If we assume that U is an orthogonal matrix, then $U^T U = 1$. Then an orthogonal unitary transformation becomes

$$\begin{aligned} w_i &= U v_i \\ w_j^T w_i &= (U v_j)^T (U v_i) \\ w_j^T w_i &= v_j^T U^T U v_i \\ w_j^T w_i &= v_j^T v_i = \delta_{ij} \end{aligned}$$

and the dot product and orthogonality are preserved.