

# Oblig 1. matinf3100- Linear optimization

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## Problem 1

a)

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} -7 \\ 0 \\ 2 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 0 & -3 & 4 \\ 1 & -1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

Now the LP problem given can be written as  $\max(\mathbf{c}^T \mathbf{x})$  subject to  $\mathbf{A}\mathbf{x} \leq \mathbf{0}$ ,  $\mathbf{x} \geq \mathbf{0}$

b)

Introduce slack variables  $w_1, w_2, w_3$ , and get the following dictionary.

$$\eta = \quad - \quad 7x_1 \quad \quad + \quad 2x_3$$

$$w_1 = 1 \quad \quad + \quad 3x_2 \quad - \quad 4x_3$$

$$w_2 = 2 \quad - \quad x_1 \quad + \quad x_2$$

$$w_3 = \quad \quad 3x_1 \quad \quad - \quad x_3$$

$$x_1, x_2, x_3, w_1, w_2, w_3 \geq 0$$

Since  $2x_3$  is the only variable that we can increase to make the objective value increase. This means  $x_3$  is the entering variable. And by observation on the dictionary we see that  $w_3$  is the leaving variable and  $x_3$  can not increase at all. After change we get the following dictionary.

$$\begin{aligned}\eta &= & - & x_1 & & - & 2w_3 \\ w_1 &= & 1 & - & 12x_1 & + & 3x_2 & + & 4w_3 \\ w_2 &= & 2 & - & x_1 & + & x_2 \\ x_3 &= & & & 3x_1 & & - & w_3 \\ x_1, x_2, x_3, w_1, w_2, w_3 &\geq 0\end{aligned}$$

Now we can not increase the objective value any more so the optimal solution is  $x_1 = 0, x_2 = 0, x_3 = 0$  with the objective value 0.

c)