Oblig 1.
matinf3100- Linear optimization

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February 5, 2017

Problem 1

a)

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} -7 \\ 0 \\ 2 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad \mathrm{A} = \begin{bmatrix} 0 & -3 & 4 \\ 1 & -1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

Now the LP problem given kan be written as $\max(\mathbf{c}^{\mathrm{T}}\mathbf{x})$ subject to $\mathrm{A}\mathbf{x} \leq \mathbf{b}$, $\mathbf{x} \geq \mathbf{0}$

b)

Introduce slack variables w_1, w_2, w_3, and get the following dictionary.

$$\begin{array}{rcccccc}
\eta = & & - & 7x_1 & & + & 2x_3 \\
\\
w_1 = & 1 & & & + \; 3x_2 & - & 4x_3 \\
w_2 = & 2 & - & x_1 & + \; x_2 & & \\
w_3 = & & & 3x_1 & & - & x_3 \\
\end{array}$$

$$x_1, x_2, x_3, w_1, w_2, w_3 \geq 0$$

Since $2x_3$ is the only variable that we can increase to make the objective value increas. This means x_3 is the entering variable. And by observation on the dictionary we se that w_3 is the leaving variable and x_3 can not increas at all. After change we get the following dictionary.

$$\begin{array}{rcccccc}
\eta = & & - & x_1 & & - & 2w_3 \\
\\
w_1 = & 1 & - & 12x_1 & + \; 3x_2 & + & 4w_3 \\
w_2 = & 2 & - & x_1 & + \; x_2 & & \\
x_3 = & & & 3x_1 & & - & w_3 \\
\end{array}$$

$$x_1, x_2, x_3, w_1, w_2, w_3 \geq 0$$

Now we can not increase the objective value any more so the optimal solution is $x_1 = 0, x_2 = 0, x_3 = 0$ vith the objective value 0.

c)

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ x_{n+1} \\ \vdots \\ x_{n+m} \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} \quad \mathrm{A}' = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & \ddots & & \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$\mathrm{A} = \begin{bmatrix} \mathrm{A}' & \mathrm{I}_m \end{bmatrix}$$

Now the LP problem given kan be written as $\max(\mathbf{c}^{\mathrm{T}}\mathbf{x})$ subject to $\mathrm{A}\mathbf{x} = \mathbf{b}$, $\mathbf{x} \geq \mathbf{0}$

Problem 2

a)

Start by introducing slack variables and get the following intitial dictionary.

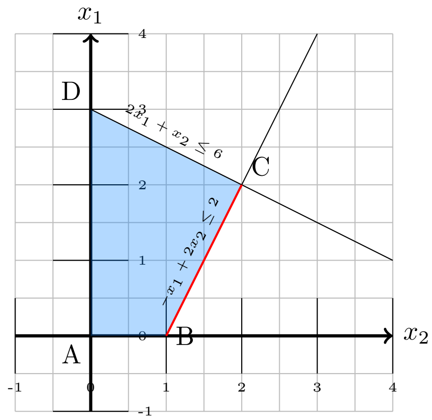
$$\begin{array}{rcccccc} \eta = & & & -3x_1 & + & 6x_2 \\ \\ w_1 = & 6 & - & 2x_1 & - & x_2 \\ w_2 = & 2 & + & x_1 & - & 2x_2 \\ \multicolumn{6}{l}{x_1, x_2, w_1, w_2 \geq 0} \end{array}$$

To increas η let x_2 be the entering variable and let w_2 be the leaving variable. Get the new dictionary.

$$\begin{array}{rcccccc} \eta = & 6 & & & - & 3w_2 \\ \\ x_2 = & 1 & + & \frac{x_1}{2} & - & \frac{w_2}{2} \\ w_1 = & 5 & - & \frac{5x_1}{2} & + & \frac{w_2}{2} \\ \multicolumn{6}{l}{x_1, x_2, w_1, w_2 \geq 0} \end{array}$$

It is imposible to increas η, so we have reached a optimal value, $\eta = 6$. Since x_1 is not in the basis, but missing form the objective function this can have any value as long it doesn't contradict the restrictions. Thus all the optimal solutions are given by $w_2 = 0$ and $x_1 \leq 2$, this implies that $x_2 \in [1, 2]$.

b)



The red line symbolizes alle the optimal solution. The reason there are multiple solitions is beacs the objective function is paralell to one of the restrictions.

c)

Start by introducing slack variables and get the following intitial dictionary.

$$
\begin{array}{rccccc}
\eta = & & & 3x_1 & + & 2x_2 \\
\\
w_1 = & 3 & - & x_1 & + & x_2 \\
w_2 = & 2 & - & x_1 & & \\
\\
\multicolumn{6}{l}{x_1, x_2, w_1, w_2 \geq 0}
\end{array}
$$

To increas η let x_1 be the entering variable and let w_2 be the leaving variable. Get the new dictionary.

$$
\begin{array}{rccccc}
\eta = & 6 & + & 2x_2 & - & 3w_2 \\
\\
x_1 = & 2 & & & - & w_2 \\
w_1 = & 1 & + & x_2 & + & w_2 \\
\\
\multicolumn{6}{l}{x_1, x_2, w_1, w_2 \geq 0}
\end{array}
$$

To increase η futher let x_2 be the entering variable, but since we can increase x_2 as much as we want and still not break any restrictions we have that the problem is unbounded.

Problem 3

a)

$$
\text{minimize:} \quad \sum_i |b_i - \sum_j a_{ij} x_j|
$$

The minimalization problem above can easaly be written as the bellow by introducing $t_i = |b_i - \sum_j a_{ij}x_j|$ for $i = 1, 2, \cdots, n$:

$$\begin{aligned} \text{minimize:} \quad & \sum_i t_i \\ \text{subject to:} \quad & t_i - |b_i - \sum_j a_{ij}x_j| = 0, \quad \text{for } i = 1, 2, \cdots, n \end{aligned}$$

but then $b_i - \sum_j a_{ij}x_j$ has to either be equal to t_i or $-t_i$ hence we can say $b_i - \sum_j a_{ij}x_j$ lies in between the two, for all $i = 1, 2, \cdots, n$. Thus we can rewrite the prevoius to:

$$\begin{aligned} \text{minimize:} \quad & \sum_i t_i \\ \text{subject to:} \quad & -t_i \leq b_i - \sum_j a_{ij}x_j \leq t_i, \quad \text{for } i = 1, 2, \cdots, n \end{aligned}$$

DETTE MÅ FIKSES!!!!! TEKSTEN TIL SLUTT!!!!

b)

Using the following code to compute x using L_2-regression.

```
b = [2, 2, 4, 9, 6, 10, 11, 20, 16, 20];
a = [0: 9];
A = [0, 1; 1, 1; 2, 1; 3, 1; 4, 1; 5, 1; 6, 1; 7, 1; 8, 1; 9, 1];

x = (inv((A.')*A))*(A.')*(b.')

scatter(a, b, 'filled', 'r')
hold on

plot(a, A*x, 'b')
hold on

plot(a, A*[2; 0], 'm')

xlabel('a')
ylabel('b')
legend('datapoints', 'regression using L2', 'regression using L1',...
 'Location', 'northwest')
```

Here $\mathbf{x} = [2.1212, 0.4545]$

Using the followin code to compute x usin L_2-regression.

```
/******************************************
 * OPL 12.5 Model
 * Author: emmestl
 * Creation Date: Feb 5, 2017 at 3:55:31 PM
 ******************************************/
int m = ...;
int n = ...;

range rows = 1..n;
range cols = 1..m;

float b[rows] = ...;
float A[rows][cols] = ...;

dvar float+ x[cols];
dvar float+ t[rows];

minimize sum(i in rows) t[i];

subject to {
        forall (i in rows){
                b[i] - (sum(j in cols) A[i][j] *x[j]) <= t[i];
                -b[i] + (sum(j in cols) A[i][j] *x[j]) <= t[i];
        }
}
```

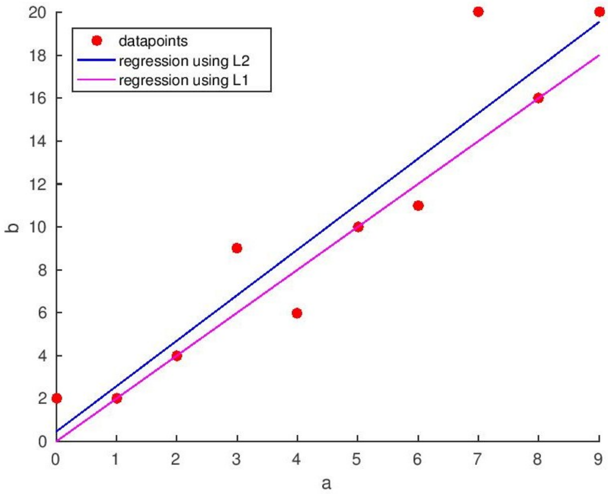
With the given data:

```
/******************************************
 * OPL 12.5 Data
 * Author: emmestl
 * Creation Date: Feb 5, 2017 at 3:55:31 PM
 ******************************************/
m = 2;
n = 10;

b = [2, 2, 4, 9, 6, 10, 11, 20, 16, 20];
A = [[0, 1], [1, 1], [2, 1], [3, 1], [4, 1], [5, 1], [6, 1], [7, 1],...
[8, 1], [9, 1]];
```

Here $\mathbf{x} = [2, 0]$

And by comparing them to the original data we get the following plot:



Yes, the difference is as expected. The L_2 doesn't hit any of the data points but lies more in the middel, where as the L_1 regression hist 4 points but then som of the outliners become quite far of.

I suppose depending on what kind of data-set one has they have both their preferences. As said above the L_1 regression, using the simplex method, does not care about abnormalities in such extent as L_2 so if there is not a lot of data I would say L_1 is to prefer.