Oblig2
mat1120

Elsie Mestl

21. oktober 2015

Oppgave 1

i)

Har at:

$$p(t) = a_0 + a_1 t + t^2 \qquad C = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}$$

Regner ut det karektaristiske polynomet til C

$$\begin{aligned} P(\lambda_C) &= \det(C) \\ &= (-\lambda)(-a_1 - \lambda) + a_0 \\ &= a_0 + \lambda a_1 + \lambda^2 \end{aligned}$$

Ser at

$$P(\lambda_C) = p(t), \quad t = \lambda$$

ii)

$$p(t) = a_0 + a_1 t + a_3 t^2 + t^3 \qquad C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix}$$

Regner ut det karektaristiske polynomet til C

$$\begin{aligned} P(\lambda_C) &= \det(C) \\ &= -\lambda(-\lambda(-a_2 - \lambda) + a_1) - a_0(1 - (-\lambda \cdot 0)) \\ &= \lambda^2(-a_2 - \lambda) - \lambda a_1 - a_0 \\ &= -\lambda^3 - \lambda^2 a_2 - \lambda a_1 - a_0 \end{aligned}$$

Ser at

$$P(\lambda_C) = -p(t), \quad t = \lambda$$

Oppgave 2

i)

Har at

$$f'''(t) = 4f(t) + 4f'(t) - f''(t)$$

Vis at $\mathbf{x}(t) = (f(t), f'(t), f''(t))$ er en løsning.

$$\begin{aligned}
\mathbf{x}'(t) &= \mathcal{C}\mathbf{x}(t) \\
&= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 4 & -1 \end{bmatrix} \begin{bmatrix} f(t) \\ f'(t) \\ f''(t) \end{bmatrix} \\
&= \begin{bmatrix} 0 + f'(t) + 0 \\ 0 + 0 + f''(t) \\ 4f(t) + 4f'(t) - f''(t) \end{bmatrix} \\
&= \begin{bmatrix} f'(t) \\ f''(t) \\ f'''(t) \end{bmatrix}
\end{aligned}$$

ii)

Vis at $f(t) = x_1(t)$ er en løsning for (*) når $\mathbf{x}(t) = (x_1(t), x_2(t), x_3(t))$ er en løsning for (**)
Finner egenverdien og egenvektorene til C. Vha matlab, og får at C har egenvektorene:

$$\mathbf{v}_1 = \begin{bmatrix} 0.25 \\ 0.5 \\ 1 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 0.25 \\ -0.5 \\ 1 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

med respektive egenverdier:

$$\lambda_1 = 2 \quad \lambda_2 = -2 \quad \lambda_3 = -1$$

Har videre at:

$$\begin{aligned}
\mathbf{x}(t) &= c_1\mathbf{v}_1 e^{2t} + c_2\mathbf{v}_2 e^{-2t} + c_3\mathbf{v}_3 e^{-t} \\
&= c_1 \begin{bmatrix} 0.25 \\ 0.5 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 0.25 \\ -0.5 \\ 1 \end{bmatrix} e^{-2t} + c_3 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} e^{-t} \\
&= \begin{bmatrix} 0.25c_1e^{2t} + 0.25c_2e^{-2t} + c_3e^{-t} \\ 0.5c_1e^{2t} - 0.5c_2e^{-2t} - c_3e^{-t} \\ c_1e^{2t} + c_2e^{-2t} + c_3e^{-t} \end{bmatrix}
\end{aligned}$$

Siden:

$$\mathbf{x}(t) = (x_1(t), x_2(t), x_3(t))$$

Har vi:

$$\begin{aligned}
x_1(t) &= 0.25c_1e^{2t} + 0.25c_2e^{-2t} + c_3e^{-t} \\
x_2(t) &= 0.5c_1e^{2t} - 0.5c_2e^{-2t} - c_3e^{-t} \\
x_3(t) &= c_1e^{2t} + c_2e^{-2t} + c_3e^{-t}
\end{aligned}$$

Siden

$$f(t) = x_1(t) = 0.25c_1e^{2t} + 0.25c_2e^{-2t} + c_3e^{-t}$$

Setter vi inn for $f(t)$ i (*) ser vi at:

$$\begin{aligned}
&f'''(t) + f''(t) - 4f'(t) - 4f(t) \\
&\qquad = -4(0.25c_1e^{2t} + 0.25c_2e^{-2t} + c_3e^{-t}) - 4(0.5c_1e^{2t} - 0.5c_2e^{-2t} - c_3e^{-t}) \\
&\qquad \quad + c_1e^{2t} + c_2e^{-2t} + c_3e^{-t} + 2c_1e^{2t} - 2c_2e^{-2t} - c_3e^{-t} \\
&\qquad = 0 \quad \text{ved enkel algebra}
\end{aligned}$$

Dermed har vi at $f(t) = x_1(t)$ er en løsning for (*)

iii)

Regnet ut i oppgave 2ii)
Den generelle løsningen er:

$$\mathbf{x}(t) = c_1\mathbf{v}_1e^{2t} + c_2\mathbf{v}_2e^{-2t} + c_3\mathbf{v}_3e^{-t}$$

Har at

$$\mathbf{x}(0) = (1, 0, -2)$$

Setter inn verdien for $t = 0$ og $\mathbf{x}(0)$ inn i den generelle formelen og får:

$$
\begin{align}
1 &= 0.25c_1e^0 + 0.25c_2e^0 + c_3e^0 = 0.25c_1 + 0.25c_2 + c_3 \tag{1}\\
0 &= 0.5c_1e^0 - 0.5c_2e^0 - c_3e^0 = 0.5c_1 - 0.5c_2 - c_3 \tag{2}\\
-2 &= c_1e^0 + c_2e^0 + c_3e^0 = c_1 + c_2 + c_3 \tag{3}
\end{align}
$$

$$
c_3 = 0.5(c_1 - c_2) \tag{2}
$$

$$
\begin{align}
1 &= 0.25(c_1 + c_2) + 0.5(c_1 - c_2) = 0.75c_1 - 0.25c_2 \tag{1+2}\\
-2 &= c_1 + c_2 + 0.5(c_1 - c_2) = 1.5c_1 + 0.5c_2 \tag{2+3}
\end{align}
$$

$$
c_2 = -4 - 3c_1 \tag{2+3}
$$

$$
1 = 0.75c_1 - 0.25(-4 - 3c_1) = 0.75c_1 + 1 + 0.75c_1 = 1.5c_1 \tag{1+2+3}
$$

$$
\begin{aligned}
c_1 &= \frac{2}{3}\\
c_2 &= -6\\
c_3 &= \frac{10}{3}
\end{aligned}
$$

Altså er:

$$
\begin{aligned}
\mathbf{x}(t) &= \frac{2}{3}\mathbf{v}_1e^{2t} - 6\mathbf{v}_2e^{-2t} + \frac{10}{3}\mathbf{v}_3e^{-t}\\
&= \begin{bmatrix} \frac{1}{6}e^{2t} - \frac{3}{2}e^{-2t} + \frac{10}{3}e^{-t} \\ \frac{1}{3}e^{2t} + 3e^{-2t} - \frac{10}{3}e^{-t} \\ \frac{2}{3}e^{2t} - 6e^{-2t} + \frac{10}{3}e^{-t} \end{bmatrix}
\end{aligned}
$$