

Oblig 1. matinf3100- Linear optimization

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February 4, 2017

Problem 1

a)

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} -7 \\ 0 \\ 2 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 0 & -3 & 4 \\ 1 & -1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

Now the LP problem given can be written as $\max(\mathbf{c}^T \mathbf{x})$ subject to $\mathbf{Ax} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}$

b)

Introduce slack variables w_1, w_2, w_3 , and get the following dictionary.

$$\begin{aligned} \eta &= \quad - \quad 7x_1 \quad \quad \quad + \quad 2x_3 \\ w_1 &= \quad 1 \quad \quad \quad + \quad 3x_2 \quad - \quad 4x_3 \\ w_2 &= \quad 2 \quad - \quad x_1 \quad + \quad x_2 \\ w_3 &= \quad \quad \quad 3x_1 \quad \quad \quad - \quad x_3 \\ x_1, x_2, x_3, w_1, w_2, w_3 &\geq 0 \end{aligned}$$

Since $2x_3$ is the only variable that we can increase to make the objective value increase. This means x_3 is the entering variable. And by observation on the dictionary we see that w_3 is the leaving variable and x_3 can not increase at all. After change we get the following dictionary.

$$\begin{aligned} \eta &= \quad - \quad x_1 \quad \quad \quad - \quad 2w_3 \\ w_1 &= \quad 1 \quad - \quad 12x_1 \quad + \quad 3x_2 \quad + \quad 4w_3 \\ w_2 &= \quad 2 \quad - \quad x_1 \quad + \quad x_2 \\ x_3 &= \quad \quad \quad 3x_1 \quad \quad \quad - \quad w_3 \\ x_1, x_2, x_3, w_1, w_2, w_3 &\geq 0 \end{aligned}$$

Now we can not increase the objective value any more so the optimal solution is $x_1 = 0, x_2 = 0, x_3 = 0$ with the objective value 0.

c)

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ x_{n+1} \\ \vdots \\ x_{n+m} \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} \quad \mathbf{A}' = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & \ddots & \\ \vdots & & & \ddots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$\mathbf{A} = [\mathbf{A}' \quad \mathbf{I}_m]$$

Now the LP problem given can be written as $\max(\mathbf{c}^T \mathbf{x})$ subject to $\mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq \mathbf{0}$

Problem 2

a)

Start by introducing slack variables and get the following initial dictionary.

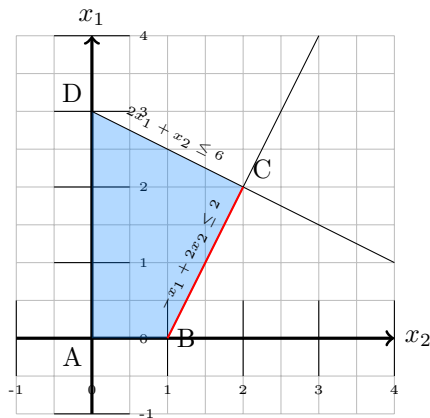
$$\begin{aligned} \eta &= -3x_1 + 6x_2 \\ w_1 &= 6 - 2x_1 - x_2 \\ w_2 &= 2 + x_1 - 2x_2 \\ x_1, x_2, w_1, w_2 &\geq 0 \end{aligned}$$

To increase η let x_2 be the entering variable and let w_2 be the leaving variable. Get the new dictionary.

$$\begin{aligned} \eta &= 6 - 3w_2 \\ x_2 &= 1 + \frac{x_1}{2} - \frac{w_2}{2} \\ w_1 &= 5 - \frac{5x_1}{2} + \frac{w_2}{2} \\ x_1, x_2, w_1, w_2 &\geq 0 \end{aligned}$$

It is impossible to increase η , so we have reached an optimal value, $\eta = 6$. Since x_1 is not in the basis, but missing from the objective function this can have any value as long as it doesn't contradict the restrictions. Thus all the optimal solutions are given by $w_2 = 0$ and $x_1 \leq 2$, this implies that $x_2 \in [1, 2]$.

b)



The red line symbolizes all the optimal solutions. The reason there are multiple solutions is because the objective function is parallel to one of the restrictions.

c)

Start by introducing slack variables and get the following initial dictionary.

$$\eta = \quad \quad 3x_1 \quad + \quad 2x_2$$

$$w_1 = 3 \quad - \quad x_1 \quad + \quad x_2$$

$$w_2 = 2 \quad - \quad x_1$$

$$x_1, x_2, w_1, w_2 \geq 0$$

To increase η let x_1 be the entering variable and let w_2 be the leaving variable. Get the new dictionary.

$$\eta = 6 \quad + \quad 2x_2 \quad - \quad 3w_2$$

$$x_1 = 2 \quad \quad \quad - \quad w_2$$

$$w_1 = 1 \quad + \quad x_2 \quad + \quad w_2$$

$$x_1, x_2, w_1, w_2 \geq 0$$

To increase η further let x_2 be the entering variable, but since we can increase x_2 as much as we want and still not break any restrictions we have that the problem is unbounded.