EMA 521 (Aerodynamics) Exam 1 Equation Sheet

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Fall 2020

This document aims to include all equations needed for the first exam for EMA 521 at UW-Madison during the Fall 2020 semester. In addition to the equations, I hope to add some notes that assist the reader in using the correct equation.

I welcome feedback on this document. Please feel free to reach out to me by email at egalles@wisc.edu to share your thoughts or concerns. This document was generated on October 19, 2020; for the most recent edition, please visit GitHub (link).

"It is not the critic who counts; not the man who points out how the strong man stumbles, or where the doer of deeds could have done them better. The credit belongs to the man who is actually in the arena, whose face is marred by dust and sweat and blood; who strives valiantly; who errs, who comes short again and again, because there is no effort without error and shortcoming; but who does actually strive to do the deeds; who knows great enthusiasms, the great devotions; who spends himself in a worthy cause; who at the best knows in the end the triumph of high achievement, and who at the worst, if he fails, at least fails while daring greatly, so that his place shall never be with those cold and timid souls who neither know victory nor defeat."

- Theodore Roosevelt

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0 Important terminology and comments

This class uses jargon that is often hard to keep track of. The following is a list of terms we frequently use:

- **Upstream/freestream:** this is the region in front of the wing/object we are studying. It is often denoted by the $_\infty$ subscript.
- **Angle of attack α:** the angle that the leading edge of the airfoil makes with the relative wind. Note that in Figure 0.1, the angle of attack α is positive.
- **Chord c:** the straight-line distance between the leading edge and trailing edge; the length of the chord line from Figure 0.1.
- **Camber:** the distance between the chord line and the camber line.
- **Thickness t:** the actual thickness of the airfoil between the lower and upper surfaces. Note that this value can change depending on where we look at the airfoil!

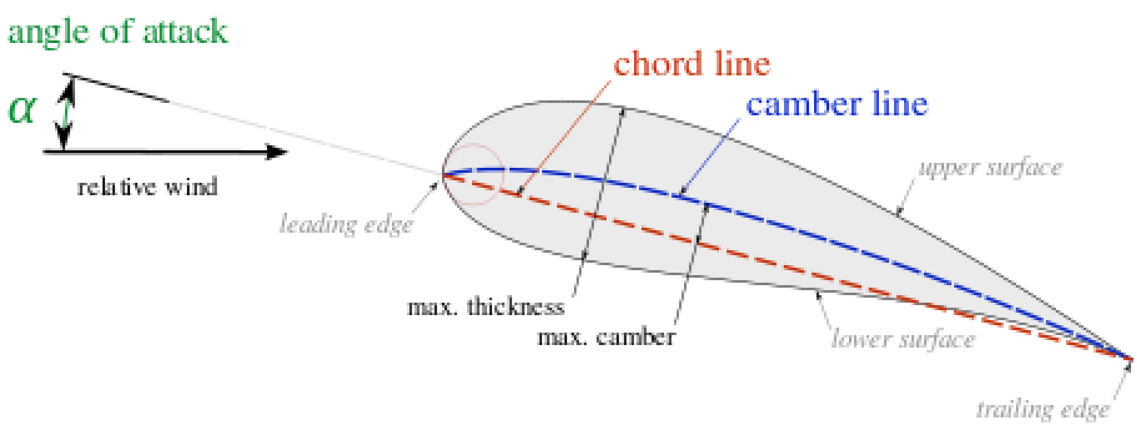

Figure 0.1: 2D airfoil with common terminology

I will aim to explain any more jargon in the sections in which they appear.

1 Aerodynamic forces

The two main forces we study are lift L and drag D. Lift and drag forces are always with respect to the freestream flow! In other words, lift will always be perpendicular to the freestream flow and drag will always be parallel to the freestream flow.

Two other forces we may consider are axial A and normal N. The axial force is oriented from leading edge to trailing edge and the normal force is oriented in a right-hand manner with respect to A such that our thumb points out of the page. These forces will be with respect to the orientation of the object of interest; we don't care about the orientation of the freestream flow.

Lift and drag are related to the normal and axial forces via the following rotation matrix; note how α denotes the angle of attack of the freestream flow.

$$\begin{bmatrix} L \\ D \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} N \\ A \end{bmatrix} \tag{1.1}$$

We also have an aerodynamic moment M, but not much attention has been devoted to it. It's worth noting that the convention for moments is opposite than what we have learned with the typical right-hand rule: negative moments will be pointing out of the page while positive moments will be pointing into the page. Positive moments pitch the object up while while negative moments pitches the object down.

Non-dimensional forces

We look to study non-dimensional forces to give us a better understanding on what causes changes in the forces we care about in this course. We made use of the coefficients of lift and drag up to this point, so those will be given below in both their 3D and 2D forms:

3D:

Note that L and D have units of [force] and S has units of [area].

$$C_L = \frac{L}{\frac{1}{2} \cdot \rho_\infty \cdot V_\infty^2 \cdot S} \tag{1.2}$$

$$C_D = \frac{D}{\frac{1}{2} \cdot \rho_\infty \cdot V_\infty^2 \cdot S} \tag{1.3}$$

We often see the term $\frac{1}{2} \cdot \rho_\infty \cdot V_\infty^2$ appear, so we call this term the dynamic pressure and denote it by the following:

$$q_\infty = \frac{1}{2} \cdot \rho_\infty \cdot V_\infty^2 \tag{1.4}$$

Please note that we can take any force and nondimensionalize it by dividing through by $q_\infty \cdot S$. This allows us to define the axial and normal coefficients as follows:

$$C_A = \frac{A}{q_\infty \cdot S} \tag{1.5}$$

$$C_N = \frac{N}{q_\infty \cdot S} \tag{1.6}$$

2D:

In 2D, we have lift and drag forces L and D in units of $\left[\frac{\text{force}}{\text{length}}\right]$ and since c denotes chord length, it has units of $[\text{length}]$.

$$C_l = \frac{L}{\frac{1}{2} \cdot \rho_\infty \cdot V_\infty^2 \cdot c} \tag{1.7}$$

$$C_d = \frac{D}{\frac{1}{2} \cdot \rho_\infty \cdot V_\infty^2 \cdot c} \tag{1.8}$$

Finding axial and normal forces

We can also define non-dimensional pressure and shear:

$$C_P = \frac{P - P_\infty}{q_\infty} \tag{1.9}$$

$$C_f = \frac{\tau}{q_\infty} \tag{1.10}$$

Using these equations, we can solve for the axial and normal forces A and N. Note that in the following two equations, $\theta \triangleq$ the angle between τ and the the orientation of our object.

$$A = \int_{LE}^{TE} (-P_u \sin\theta + \tau_u \cos\theta) dS_u + \int_{LE}^{TE} (P_l \sin\theta + \tau_l \cos\theta) dS_l \tag{1.11}$$

$$N = -\int_{LE}^{TE} (P_u \sin\theta + \tau_u \cos\theta) dS_u + \int_{LE}^{TE} (P_l \sin\theta - \tau_l \cos\theta) dS_l \tag{1.12}$$

Finding axial and normal coefficients

The following equations come from the textbook. They take our pressure and shear coefficients from (1.9) and (1.10) and find the axial and normal coefficients.

$$C_A = \frac{1}{c}\left[\int_0^c \left(C_{p,u}\frac{dy_u}{dx} - C_{p,l}\frac{dy_l}{dx}\right)dx + \int_0^c (C_{f,u} + C_{f,l})dx\right] \tag{1.13}$$

$$C_N = \frac{1}{c}\left[\int_0^c (C_{P,l} - C_{P,u})dx + \int_0^c \left(C_{f,u}\frac{dy_u}{dx} + C_{f,l}\frac{dy_l}{dx}\right)dx\right] \tag{1.14}$$

2 Governing equations of fluid flow